Renormalization Group Improved Prediction for Higgs Production at Hadron Colliders

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arXiv:0808.3008 and arXiv:0809.4283

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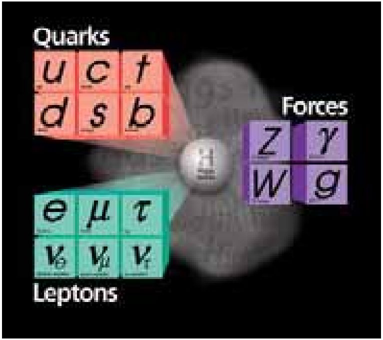
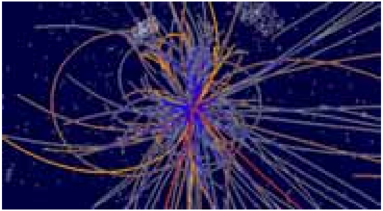
Outline

The Higgs boson in the Standard Model



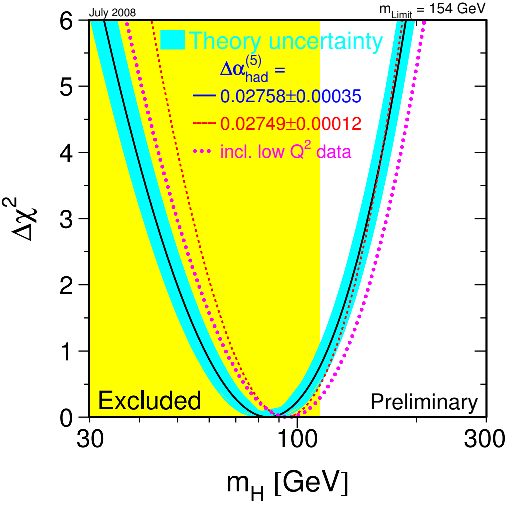
The "God" particle

- The minimal way to implement electroweak symmetry breaking
- Generates masses
- Still missing — one of the main reasons that we have the Large Hadron Collider now



CMS simulation of Higgs events

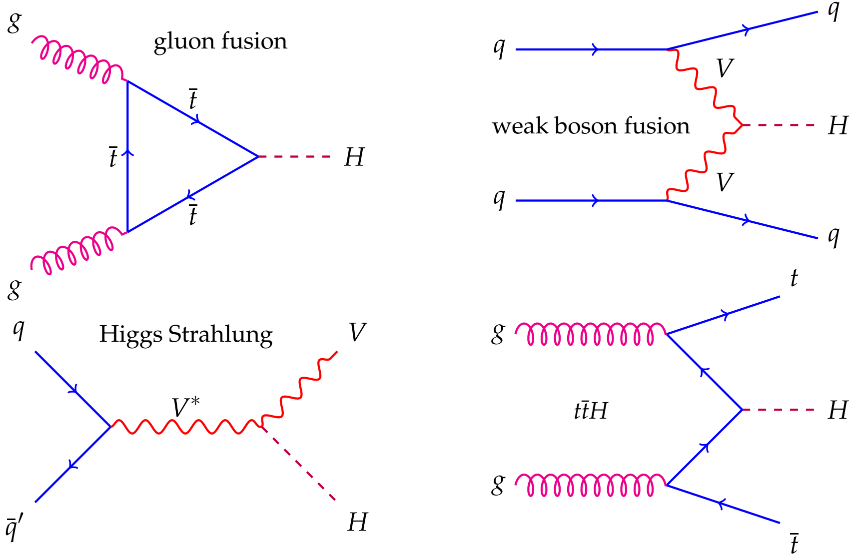
The mass of the Higgs boson



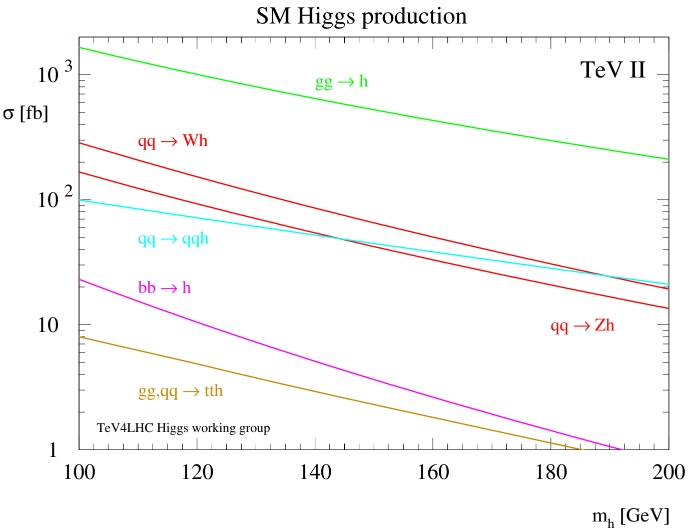
[Figure from the LEP EWWG]

- Direct search limit:
  $m_H > 114$ GeV at 95%CL
- Electroweak global fit:
  $m_H < 154$ GeV at 95% CL
  or
  $m_H < 185$ GeV when combined

Higgs production at hadron colliders

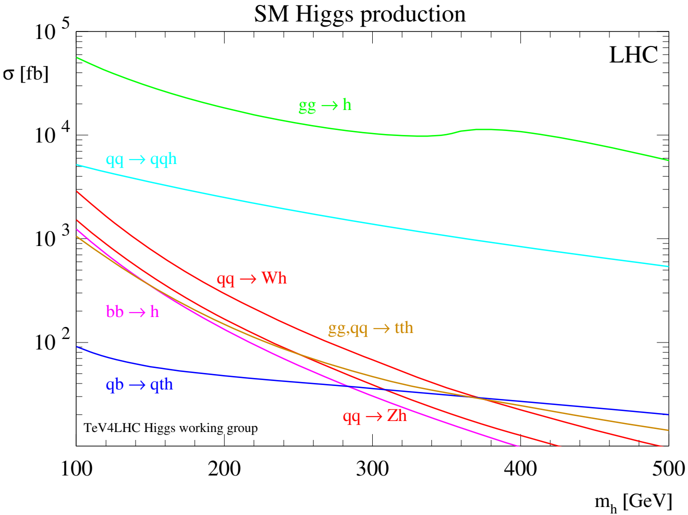


Cross sections at the Tevatron Run II



[Figure from arXiv:hep-ph/0612172]

Cross sections at the LHC



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The gluon fusion channel

- Dominant production channel — important at the LHC
- One of the best theoretically studied process
  - NLO QCD: [Dawson '91], [Djouadi, Spira and Zerwas '91]
    large K-factor observed: $\sim 70\%$ increase!
  - NNLO QCD: [Harlander and Kilgore '02], [Anastasiou and Melnikov '02], [Ravindran, Smith and van Neerven '03]
    correction smaller, but scale uncertainty still large ($\sim \pm 15\%$)!
  - NNLL threshold resummation: [Catani, de Florian, Grazzini and Nason '03]
    reduced scale uncertainty while no improvement on convergence!
  - N^3LO soft approximation: [Moch and Vogt '03]
    N^3LO soft correction turns out to be small (a few percent).
- What we do:
  - Threshold resummation from an effective theory point of view
  - Momentum space approach to avoid the Landau pole problem
  - A new choice of the hard matching scale which significantly improves convergence

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Fixed-order cross section

The total cross section is given by the convolution ($\tau = m_H^2/s$)

$$\sigma = \sigma_0 \sum_{i,j} C_{ij}(z, m_t, m_H, \mu_f) \otimes f_{i/N_1}(x_1, \mu_f) \otimes f_{j/N_2}(x_2, \mu_f)$$
$$= \sigma_0 \sum_{i,j} \int_\tau^1 \frac{dz}{z} \int_{\tau/z}^1 \frac{dx}{x} C_{ij}(z, m_t, m_H, \mu_f) f_{i/N_1}(x, \mu_f) f_{j/N_2}(\tau/z/x, \mu_f)$$

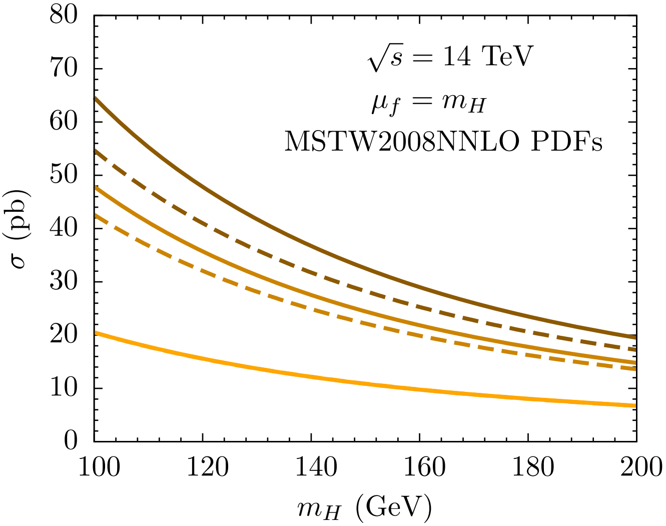
The hard scattering kernel C_{ij} is known to NNLO in the large m_t limit

- C_{gg} contains leading singular terms at partonic threshold ($z = m_H^2/\hat{s} \to 1$)

$$C_{gg}(z, m_t, m_H, \mu_f) \equiv C(z, m_t, m_H, \mu_f) + \text{regular terms}$$
$$= \delta(1-z)$$
$$+ \frac{\alpha_s}{\pi} \left[6 \left[\frac{1}{1-z} \ln \frac{m_H^2 (1-z)^2}{\mu_f^2 z} \right]_+ + \delta(1-z) \left(\frac{11}{2} + 2\pi^2 \right) + \text{regular terms} \right]$$
$$+ \left(\frac{\alpha_s}{\pi} \right)^2 \left[9 \left[\frac{1}{1-z} \ln^3 \frac{m_H^2 (1-z)^2}{\mu_f^2 z} \right]_+ + \cdots + \text{regular terms} \right] + \mathcal{O}(\alpha_s^3)$$

- C_{gq} and $C_{q\bar{q}}$ only contain regular terms

Fixed-order cross section



- Corrections are large and dominated by the leading singular terms in C_{gg}.

The threshold logarithms

In general, the singular part of C_{gg} can be written symbolically as

$$\begin{aligned} C = 1 & \qquad \text{[LO]} \\ + \alpha_s(L^2 + L + 1) & \qquad \text{[NLO]} \\ + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) & \qquad \text{[NNLO]} \\ + \alpha_s^3(L^6 + L^5 + \cdots + 1) & \qquad \text{[N}^3\text{LO]} \\ + \cdots & \end{aligned}$$

[$L \sim \ln(1-z)$. I treat $1/(1-z)$ as a log and denote $\delta(1-z)$ by 1.]

Convergence bad if L^2 large!

However, this is not the only way to do the expansion...

Resummation as a reorganization

Convergence would be better if one could rewrite the perturbative series as (illustration only)

$$\begin{aligned} C = {} & (1 + \alpha_s L^2 + \alpha_s^2 L^4 + \alpha_s^3 L^6 + \cdots) && [\mathrm{LO}'] \\ & + (\alpha_s L + \alpha_s^2 L^3 + \alpha_s^3 L^5 + \cdots) && [\mathrm{NLO}'] \\ & + (\alpha_s^2 L^2 + \alpha_s^3 L^4 + \cdots) && [\mathrm{NNLO}'] \\ & + \cdots \end{aligned}$$

Resummation is a reorganization of the perturbative series in order to improve the convergence.

The problem is: how to make this kind of reorganization concrete and how to sum the infinite number of terms at each new order?

Resummation from factorization

Let's forget about the "reorganization" and "sum to all orders" stuff at this moment and take a different viewpoint

- The presence of double logs in the threshold region is due to the presence of two distinct scales:

$$Q_{\text{hard}} \sim m_H \gg Q_{\text{soft}} \sim \sqrt{\hat{s}}(1-z)$$

  The logarithms are actually $\ln(Q^2_{\text{hard}}/Q^2_{\text{soft}})$.
- Can get rid of the logs by separating these scales: factorization
  - Traditional method: diagrammatic approach
    [Collins, Soper, Sterman, Korchemsky, Catani and many others]
  - We use effective theory method: field and operator approach
    SCET: [Bauer, Fleming, Pirjol, Stewart, Rothstein, Beneke, Chapovsky, Diehl, Feldmann and many others]
- Resummation is automatically achieved by the evolution factor between the scales — will be justified later

Factorization using effective theories

- In the threshold region, the relevant energy scales are

$$2m_t \gg \sqrt{\hat{s}} \sim m_H \gg \sqrt{\hat{s}}(1-z) \gg \Lambda_{\rm QCD}$$

- We construct a sequence of effective theories

$$\boxed{\begin{array}{c}\text{SM}\\ n_f=6\end{array}} \xrightarrow{\mu_t} \boxed{\begin{array}{c}\text{SM}\\ n_f=5\end{array}} \xrightarrow{\mu_h} \boxed{\begin{array}{c}\text{SCET}\\ hc,\overline{hc},s\end{array}} \xrightarrow{\mu_s} \boxed{\begin{array}{c}\text{SCET}\\ c,\bar{c}\end{array}}$$

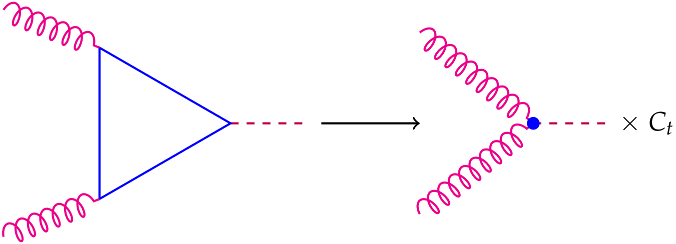
$$C_t(m_t^2,\mu_t^2) \qquad H(m_H^2,\mu_h^2) \qquad S(\hat{s}(1-z)^2,\mu_s^2)$$

- The resulting factorization formula reads

$$\sigma = \sigma_0 \, [C_t(m_t^2,\mu^2)]^2 \, H(m_H^2,\mu^2) \\ \times S(\hat{s}(1-z)^2,\mu^2) \otimes f_{g/N_1}(x_1,\mu) \otimes f_{g/N_2}(x_2,\mu)$$

  - Any single choice of μ^2 leads to large logs, especially double logs in the hard function H and the soft function S
  - Solution: choose the appropriate scale for each function and use RG evolution to connect them

First step: integrating out the top quark



$$C_t(m_t^2, \mu^2) = 1 + \frac{11}{4}\frac{\alpha_s(\mu^2)}{\pi} + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 \left[\frac{886}{9} - \frac{137}{3}\ln\frac{m_t^2}{\mu^2}\right] + \cdots$$

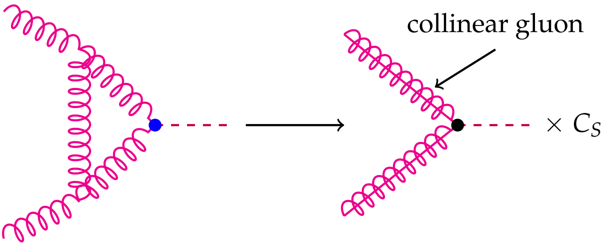
$$\approx 1 + 0.094 + 0.007 + \cdots \quad \text{for } \mu = m_t$$

$$\approx 1 + 0.085 + 0.010 + \cdots \quad \text{for } \mu = 2m_t$$

$$\approx 1 + 0.103 + 0.003 + \cdots \quad \text{for } \mu = m_t/2$$

Only single log at NNLO, good convergence for natural choice $\mu = \mu_t \approx m_t$, small scale dependence (and running effect).

Second step: integrating out the hard modes



$$
\begin{aligned}
H(m_H^2, \mu^2) &= |C_S(-m_H^2 - i\epsilon, \mu^2)|^2 \\
&= 1 & \text{[LO]} \\
&\quad + \frac{\alpha_s(\mu^2)}{4\pi}\left[-6\ln^2\frac{m_H^2}{\mu^2} + 7\pi^2\right] & \text{[NLO]} \\
&\quad + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2\left[18\ln^4\frac{m_H^2}{\mu^2} + \frac{46}{3}\ln^3\frac{m_H^2}{\mu^2} + \cdots\right] & \text{[NNLO]}
\end{aligned}
$$

(valid for $\mu^2 > 0$)

Suppose that we want to choose $\mu \sim Q_{\text{soft}} \ll m_H$ to get rid of the logs in the soft function, then we have large logs in the hard function.

RG evolution of the hard function

The hard function obeys an evolution equation (valid for $\mu^2 > 0$)

$$\frac{d}{d\ln\mu} H(m_H^2, \mu^2) = 2\left[\Gamma_{\rm cusp}^A(\alpha_s)\ln\frac{m_H^2}{\mu^2} + \gamma^S(\alpha_s)\right] H(m_H^2, \mu^2)$$

The solution is

$$H(m_H^2, \mu^2) = \exp\left[4S(\mu_h^2, \mu^2) - 2a_\Gamma(\mu_h^2, \mu^2)\ln\frac{m_H^2}{\mu_h^2} - 2a_{\gamma^S}(\mu_h^2, \mu^2)\right] H(m_H^2, \mu_h^2)$$

$$S(\nu^2, \mu^2) = -\int\limits_{\alpha_s(\nu^2)}^{\alpha_s(\mu^2)} d\alpha \frac{\Gamma_{\rm cusp}^A(\alpha)}{\beta(\alpha)} \int\limits_{\alpha_s(\nu^2)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')}, \quad a_\Gamma(\nu^2, \mu^2) = -\int\limits_{\alpha_s(\nu^2)}^{\alpha_s(\mu^2)} d\alpha \frac{\Gamma_{\rm cusp}^A(\alpha)}{\beta(\alpha)}$$

Now H can be evaluated at some hard scale μ_h for good convergence and then evolved down to some soft scale $\mu \sim Q_{\rm soft}$ to match the soft function. This gives what we call
"Renormalization Group Improved (RGI) perturbation theory"

RG-Improved perturbation theory and resummation

RGI Pert. Theory	Log Approx.	Γ^A_{cusp}	γ^S, ...	H, ...
LO	NLL	2-loop	1-loop	tree-level
NLO	NNLL	3-loop	2-loop	1-loop
NNLO	NNNLL	4-loop	3-loop	2-loop

Now let's see what happens when we want to evaluate the hard function at some soft scale $\mu \sim Q_{\text{soft}}$ in RGI perturbation theory.

We use a "natural" choice $\mu_h^2 = m_H^2$, and re-expand the leading order expression in RGI perturbation theory in powers of $\alpha_s(\mu^2)$:

$$H(m_H^2, \mu^2) = \exp\left[4S(m_H^2, \mu^2) - 2a_{\gamma^S}(m_H^2, \mu^2)\right]_{\text{LO}}$$

$$= 1 - \frac{\alpha_s(\mu^2)}{4\pi} 6 \ln^2 \frac{m_H^2}{\mu^2} + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 \left[18 \ln^4 \frac{m_H^2}{\mu^2}\right.$$

$$\left. + \frac{46}{3} \ln^3 \frac{m_H^2}{\mu^2} + \left(-\frac{302}{3} + 6\pi^2\right) \ln^2 \frac{m_H^2}{\mu^2}\right] + \cdots \quad \text{[RGI-LO]}$$

So indeed the RGI-LO expression sums infinite number of terms in fixed-order perturbation theory via the evolution factor.

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So indeed the RGI-LO expression sums infinite number of terms in fixed-order perturbation theory via the evolution factor.

Is the choice $\mu_h^2 = m_H^2$ good enough?

Now the remaining question is whether or not $H(m_H^2, \mu_h^2)$ has good convergence for $\mu_h^2 \sim m_H^2$. For $m_H = 120$ GeV,

$$\begin{aligned} H(m_H^2, m_H^2) &\approx 1 + 5.50\alpha_s(m_H^2) + 17.24\alpha_s^2(m_H^2) + \cdots \\ &\approx 1 + 0.618 + 0.218 + \cdots \end{aligned}$$

Not so good... But why?

We know $H(m_H^2, \mu^2) = |C_S(-m_H^2 - i\epsilon, \mu^2)|^2$, with

$$\begin{aligned} C_S(-m_H^2, \mu^2) &= 1 && \text{[LO]} \\ &+ \frac{\alpha_s(\mu^2)}{4\pi}\left[-3\ln^2\frac{-m_H^2}{\mu^2} + \frac{\pi^2}{2}\right] && \text{[NLO]} \\ &+ \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2\left[\frac{9}{2}\ln^4\frac{-m_H^2}{\mu^2} + \frac{23}{3}\ln^3\frac{-m_H^2}{\mu^2} + \cdots\right] && \text{[NNLO]} \end{aligned}$$

The double logs leave behind π^2 terms for the choice $\mu^2 = m_H^2$

— maybe a better choice is $\mu^2 = -m_H^2 - i\epsilon$?

This requires evaluating the running $\alpha_s(\mu^2)$ at negative argument.

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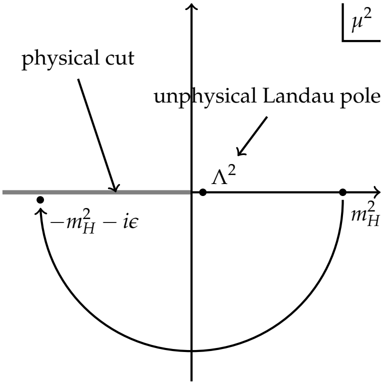
Running of α_s in the complex μ^2 plane



$$\int_{\alpha_s(\mu^2)}^{\alpha_s(-\mu^2)} \frac{d\alpha}{\beta(\alpha)} = -\frac{i\pi}{2}$$

For $m_H = 120$ GeV

$$\alpha_s(m_H^2) \approx 0.112$$
$$\alpha_s(-m_H^2 - i\epsilon) \approx 0.107 + 0.024i$$

The "π^2 resummation"

Now we can evaluate

$$H(m_H^2, -m_H^2) = |C_S(-m_H^2, -m_H^2)|^2 \approx 1 + 0.0840 - 0.0015 + \cdots$$

Convergence much better!

And we use RG evolution to obtain

$$H(m_H^2, m_H^2) = \exp\left[2\mathrm{Re}\left(2S(-m_H^2, m_H^2) - a_{\gamma^S}(-m_H^2, m_H^2)\right)\right] H(m_H^2, -m_H^2)$$

Again we re-expand the leading order expression in RGI perturbation theory in powers of $\alpha_s(m_H^2)$:

$$\begin{aligned} H(m_H^2, m_H^2) &= \exp\left[2\mathrm{Re}\left(2S(-m_H^2, m_H^2) - a_{\gamma^S}(-m_H^2, m_H^2)\right)\right]_{\mathrm{LO}} \\ &= 1 + \frac{\alpha_s(m_H^2)}{4\pi} 6\pi^2 + \left(\frac{\alpha_s(m_H^2)}{4\pi}\right)^2 \left[12\pi^4 + \frac{302}{3}\pi^2\right] + \cdots \end{aligned}$$

[RGI-LO]

Note that the π^2 terms are not chosen arbitrarily, but are generated automatically from RG evolution.

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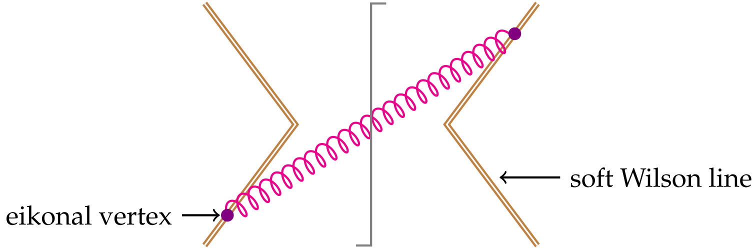
Final step: the soft function

The soft function is related to the vacuum expectation value of a Wilson loop constructed from soft gluon fields



$$S(\hat{s}(1-z)^2, \mu^2) = \sqrt{\hat{s}}\, W_{\text{Higgs}}(\sqrt{\hat{s}}(1-z), \mu)$$

$$= \delta(1-z) + \frac{\alpha_s}{\pi}\left[6\left[\frac{1}{1-z}\ln\frac{m_H^2(1-z)^2}{\mu^2 z}\right]_+ + \delta(1-z)\left(\frac{3}{2}\ln^2\frac{m_H^2}{\mu^2} + \frac{\pi^2}{4}\right)\right] + \cdots$$

RG evolution of the soft function

- The soft Wilson loop satisfies an evolution equation

$$\frac{dW_{\rm Higgs}(\omega,\mu)}{d\ln\mu} = -\left[4\Gamma^A_{\rm cusp}(\alpha_s)\ln\frac{\omega}{\mu} + 2\gamma^W(\alpha_s)\right] W_{\rm Higgs}(\omega,\mu) - 4\Gamma^A_{\rm cusp}(\alpha_s)\int_0^\omega d\omega' \,\frac{W_{\rm Higgs}(\omega',\mu) - W_{\rm Higgs}(\omega,\mu)}{\omega-\omega'}$$

- Conventional method is to solve this equation in Mellin moment space and then numerically transform back to momentum space
- An analytical solution directly in momentum space is given with the help of an associated soft function $\widetilde{s}$ (which is the Laplace transformation of the soft function) [Becher and Neubert '06]:

$$W_{\rm Higgs}(\omega,\mu) = \exp\left[-4S(\mu_s^2,\mu^2) + 2a_{\gamma^W}(\mu_s^2,\mu^2)\right] \times \widetilde{s}_{\rm Higgs}(\partial_\eta,\mu_s^2)\,\frac{1}{\omega}\left(\frac{\omega}{\mu_s}\right)^{2\eta}\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

Choice of the soft scale

- The naive choice $\mu_s \sim \sqrt{\hat{s}}(1-z)$ does not work because z is integrated over:

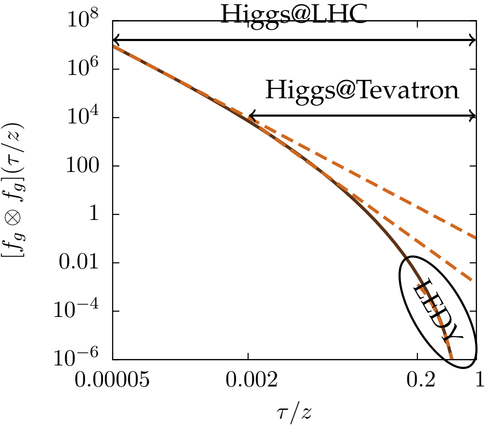
$$\begin{aligned}\sigma &\propto S(\hat{s}(1-z)^2,\mu^2) \otimes f_{g/N_1}(x_1,\mu) \otimes f_{g/N_2}(x_2,\mu) \\ &\propto \int_\tau^1 \frac{dz}{z} S(\hat{s}(1-z)^2,\mu^2) \left[f_{g/N_1} \otimes f_{g/N_2}\right](\tau/z,\mu)\end{aligned}$$

- Another guess $\mu_s \sim m_H(1-\tau)$ (based on the hadronic threshold) does not take into account the fall-off of the parton luminosity: the partonic threshold region is important if
  1. $\tau = m_H^2/s \to 1$ (the hadronic threshold region is reached);

     or

  2. The parton luminosity $f \otimes f$ is a rapidly decreasing function in the integration range (dynamical threshold enhancement).
     It has been shown to be the case in low energy Drell-Yan process [Becher, Neubert and Xu '07]

Choice of the soft scale



- Low energy Drell-Yan (LEDY): moderate to large τ

$$\sigma \sim \int_\tau^1 \frac{dz}{z} \left(\frac{1-\tau/z}{1-\tau}\right)^{11} S(1-z)$$

- Higgs production at Tevatron: small τ

$$\sigma \sim \int_\tau^1 \frac{dz}{z} z^{2.5} S(1-z)$$

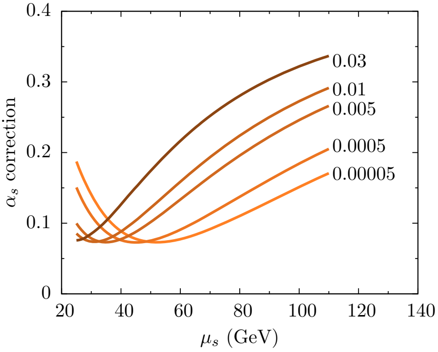
- Higgs production at LHC: very small τ

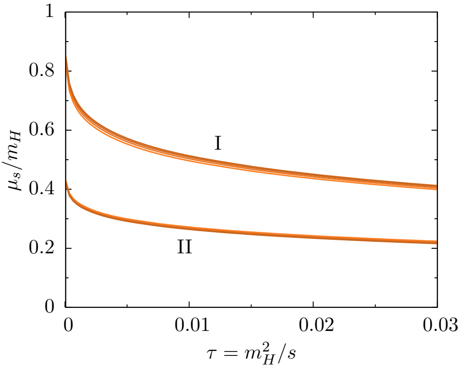
$$\sigma \sim \int_\tau^1 \frac{dz}{z} z^{1.8} S(1-z)$$

In the Higgs case, the fall-off of the parton luminosity is not strong enough — no large threshold logarithms expected.

Choice of the soft scale

- Nevertheless, we will follow the Drell-Yan paper and choose an effective soft scale μ_s so that the convolution $S \otimes f \otimes f$ has good convergence.





- Two criteria characterizing good convergence: $\mu_s^{II} < \mu_s < \mu_s^{I}$.
- $\mu_s \sim m_H/2$ not so small — no large threshold logarithms.

Putting things together

Now everything is in place, and different scales are connected by the evolution factor U:

$$\sigma^{\text{resummed}} = \sigma_0 \, [C_t(m_t^2, \mu_t^2)]^2 \, H(m_H^2, \mu_h^2) \, U(m_H, \mu_t, \mu_h, \mu_s, \mu_f) \\ \times \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \, \widetilde{s}_{\text{Higgs}} \left(\ln \frac{m_H^2 (1-z)^2}{\mu_s^2 z} + \partial_\eta, \mu_s^2 \right) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \\ \otimes f_{g/N_1}(x_1, \mu_f) \otimes f_{g/N_2}(x_2, \mu_f)$$

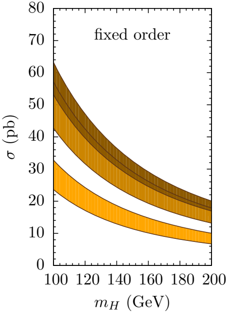
We define our final RG improved cross section with matching to fixed-order result:

$$\sigma^{\text{RGI}} = \sigma^{\text{resummed}} \Big|_{\mu_t, \mu_h, \mu_s, \mu_f} + \left(\sigma^{\text{fixed order}} \Big|_{\mu_f} - \sigma^{\text{resummed}} \Big|_{\mu_t = \mu_h = \mu_s = \mu_f} \right)$$

Predictions for the cross section

Comparison of uncertainties and convergence

LHC, MRST2001LO/2004NLO/2004NNLO PDF sets



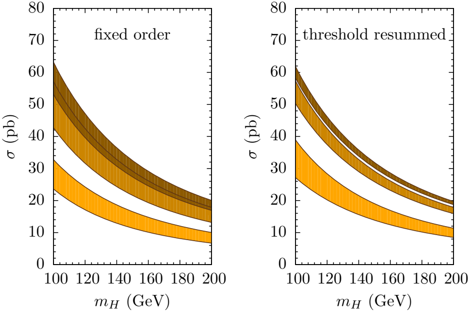
fixed order

$$m_H/2 < \mu_r = \mu_f < 2m_H$$

Predictions for the cross section

Comparison of uncertainties and convergence

LHC, MRST2001LO/2004NLO/2004NNLO PDF sets



threshold resummed

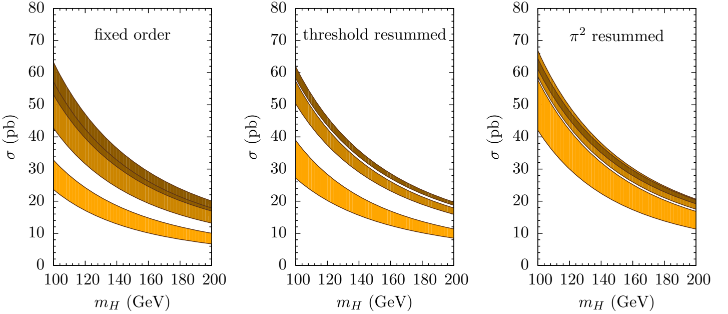
$$m_t/2 < \mu_t < 2m_t, \quad \mu_s^{II} < \mu_s < \mu_s^I$$

$$(m_H/2)^2 < \mu_h^2 < (2m_H)^2, \quad m_H/2 < \mu_f < 2m_H$$

Predictions for the cross section

Comparison of uncertainties and convergence

LHC, MRST2001LO/2004NLO/2004NNLO PDF sets



π^2 resummed

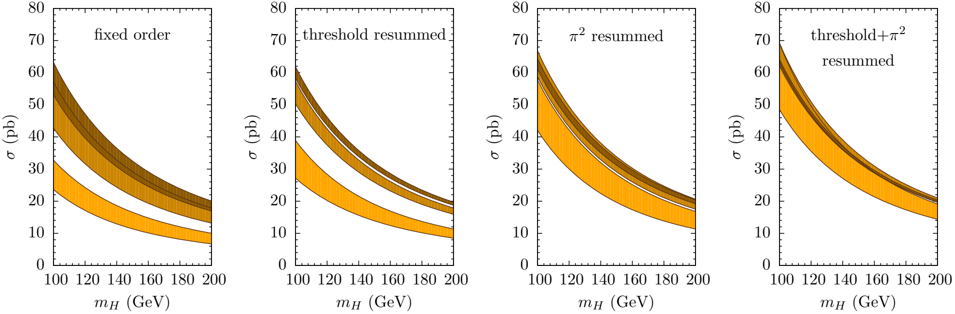
$$m_H/2 < \mu_t = \mu_s = \mu_f < 2m_H$$

$$\mu_h^2 = -\mu_f^2$$

Predictions for the cross section

Comparison of uncertainties and convergence

LHC, MRST2001LO/2004NLO/2004NNLO PDF sets



threshold + π^2 resummed

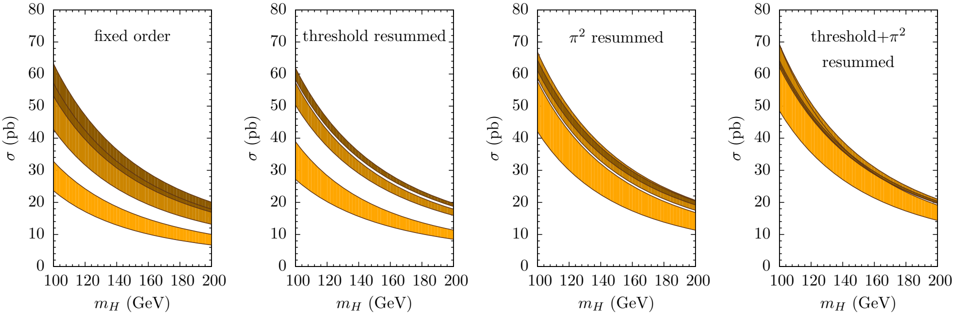
$$m_t/2 < \mu_t < 2m_t, \quad \mu_s^{II} < \mu_s < \mu_s^{I}$$

$$(m_H/2)^2 < -\mu_h^2 < (2m_H)^2, \quad m_H/2 < \mu_f < 2m_H$$

Predictions for the cross section

Comparison of uncertainties and convergence

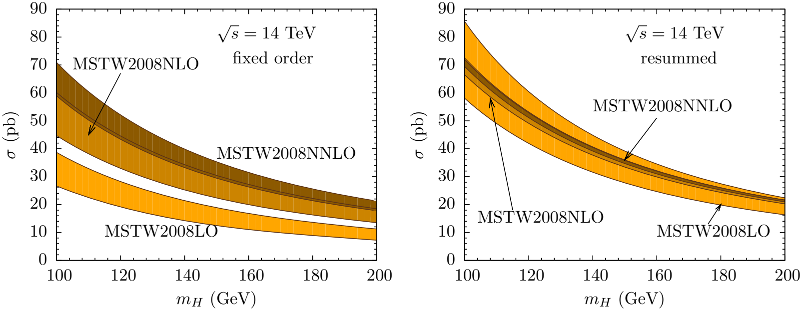
LHC, MRST2001LO/2004NLO/2004NNLO PDF sets



- Resumming threshold logs reduces scale dependence, but does not improve convergence;
- Resumming π^2 leads to faster convergence and smaller scale dependence;
- Both effects increase the cross section.

Predictions for the cross section

MSTW2008 PDF sets



Predictions for the cross section

Scale and PDF uncertainties

LHC, $m_H = 120$ GeV, MSTW2008NNLO PDF set

	fixed order	threshold	π^2	threshold + π^2
LO	$15.5^{+2.4+0.4}_{-2.1-0.5}$	$17.8^{+3.3+0.4}_{-2.7-0.6}$	$27.1^{+4.0+0.6}_{-3.8-0.8}$	$31.2^{+5.7+0.8}_{-4.8-1.0}$
NLO	$35.5^{+5.9+0.8}_{-4.6-1.1}$	$37.7^{+3.6+0.9}_{-1.2-1.2}$	$45.0^{+3.0+1.1}_{-3.3-1.4}$	$46.6^{+2.5+1.1}_{-1.1-1.5}$
NNLO	$47.6^{+4.5+1.1}_{-4.2-1.5}$	$48.5^{+2.5+1.2}_{-0.5-1.5}$	$51.4^{+1.7+1.2}_{-1.6-1.6}$	$51.4^{+1.4+1.2}_{-0.3-1.6}$

Tevatron, $m_H = 120$ GeV, MSTW2008NNLO PDF set

	fixed order	threshold	π^2	threshold + π^2
LO	$0.281^{+0.105+0.018}_{-0.071-0.019}$	$0.389^{+0.062+0.023}_{-0.046-0.024}$	$0.491^{+0.180+0.031}_{-0.127-0.033}$	$0.681^{+0.105+0.040}_{-0.080-0.042}$
NLO	$0.650^{+0.172+0.041}_{-0.131-0.044}$	$0.764^{+0.077+0.045}_{-0.026-0.048}$	$0.855^{+0.125+0.053}_{-0.130-0.056}$	$0.954^{+0.046+0.055}_{-0.022-0.059}$
NNLO	$0.901^{+0.126+0.056}_{-0.124-0.060}$	$0.961^{+0.048+0.058}_{-0.012-0.062}$	$1.003^{+0.051+0.061}_{-0.074-0.065}$	$1.022^{+0.025+0.061}_{-0.005-0.065}$

- Scale uncertainty is smaller than PDF uncertainty after including both effects.
- Uncertainty from α_s not shown here ($\sigma \propto \alpha_s^2$).

Application to other time-like processes

- With Sudakov double logs
  - Drell-Yan process near partonic threshold
- Without Sudakov double logs
  - Total cross section for $e^+e^- \to$ hadrons
  - Hadronic τ decay
  - Hadronic Higgs decay

Drell-Yan process

- The π^2 terms arising from analytic continuation of Sudakov form factor was noticed before [Parisi '80], [Magnea and Sterman '90]
- The summation of π^2 terms in the Drell-Yan case was achieved via the relation between the form factor for time-like and space-like momentum transfer evaluated at the same renormalization scale:

$$\left|\frac{C_V(-Q^2,\mu^2)}{C_V(Q^2,\mu^2)}\right|^2 \approx \exp\left(\frac{C_F\alpha_s\pi}{2}\right)$$

- In our approach everything is a proper choice of scale followed by RG evolution, and the relation is between the form factor for the same momentum transfer evaluated at time-like and space-like renormalization scale:

$$\left|\frac{C_V(-Q^2,\mu^2)}{C_V(-Q^2,-\mu^2)}\right|^2 \approx \exp\left(\frac{C_F\alpha_s\pi}{2}\right)$$

Appearance similar, but conceptually different.

Drell-Yan process

- Similar to Higgs production, but with different color factors (fundamental instead of adjoint representation)
- The hard function for $Q = 8$ GeV

$$|C_V(-Q^2, Q^2)|^2 = 1 + 0.0845 + 0.0292 + \cdots$$

$$|C_V(-Q^2, -Q^2)|^2 = 1 - 0.1451 - 0.0012 + \cdots$$

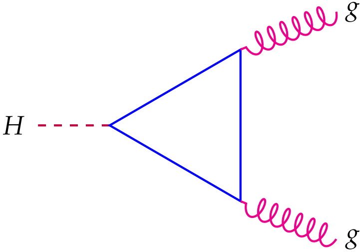
  - Surprisingly small NLO correction for $\mu^2 > 0$
  - The corrections change sign when go from $\mu^2 > 0$ to $\mu^2 < 0$
- Compare to the Higgs case

$$|C_V(-Q^2, Q^2)|^2 = 1 + \frac{\alpha_s(Q^2)}{\pi}\left[\frac{C_F \pi^2}{2} - \left(\frac{16}{3} - \frac{\pi^2}{9}\right)\right] + \cdots$$

$$|C_S(-m_H^2, m_H^2)|^2 = 1 + \frac{\alpha_s(m_H^2)}{\pi}\left[\frac{C_A \pi^2}{2} + \frac{\pi^2}{4}\right] + \cdots$$

  - The Sudakov π^2 terms smaller by C_F/C_A
  - The smallness of NLO correction for $\mu^2 > 0$ (and the change of sign) results from a coincidental cancellation

Hadronic Higgs decay



- Looks similar to Higgs production at first sight, but has intrinsic difference: no Sudakov double logs, therefore no associated π^2 terms!

Hadronic Higgs decay

$$\Gamma(H \to gg) = \frac{G_F m_H^2}{36\pi^3\sqrt{2}}\, K_t(m_t^2)\, K_H(m_H^2)$$

$K_H(m_H^2)$ in fixed-order perturbation theory:

$$K_H(m_H^2) \approx \alpha_s^2(m_H^2)\left[1 + \frac{\alpha_s(m_H^2)}{4\pi}\, d_1^H + \cdots\right]$$

$K_H(m_H^2)$ in RG-improved perturbation theory:

$$K_H(m_H^2) \approx 0.962\,\alpha_s^2(m_H^2)\left[1 + \frac{\alpha_s(m_H^2)}{4\pi}\left[0.962\, d_1^H - 0.088\,\frac{\beta_1}{\beta_0}\right] + \cdots\right]$$

- The only effect of choosing $\mu^2 < 0$ comes from the running of α_s, which is small at such high energy.
- RG improvement reproduces so-called "contour-improved perturbation theory".

Summary

- Higgs production via gluon fusion is an important process at hadron colliders. However its prediction in perturbative QCD has poor convergence. Also the scale dependence is still large even at NNLO.
- We performed an analysis of Higgs production near partonic threshold using effective field theory:
  - All matching scales are chosen properly to ensure the perturbative convergence of the corresponding quantities;
  - Especially a time-like hard scale $\mu_h^2 < 0$ is chosen to eliminate the π^2 terms associated with Sudakov double logs in the hard function;
  - Different scales are connected by renormalization group evolution, which sums certain terms in fixed-order perturbation theory to all orders.
- Numerically no large threshold logarithm is found. Threshold resummation reduces scale dependence but does not improve convergence of the cross section.
- On the other hand, we find large corrections from Sudakov π^2 terms, and the new choice $\mu_h^2 < 0$ significantly improves convergence and leads to more reliable predictions.

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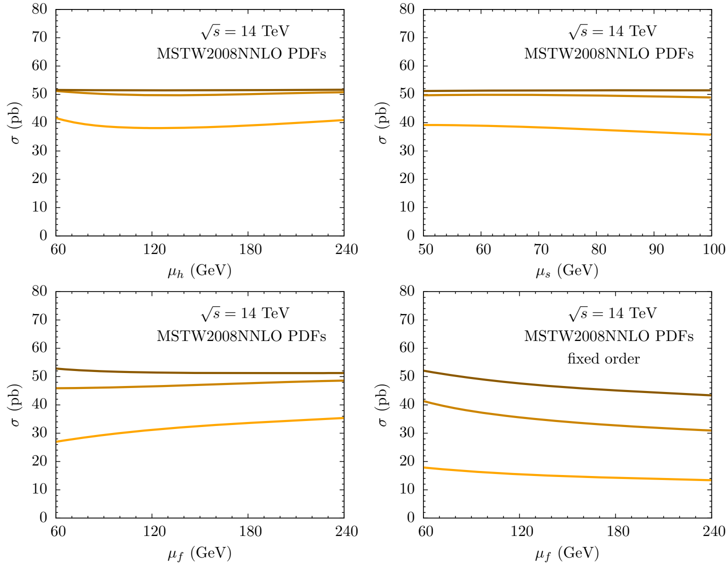
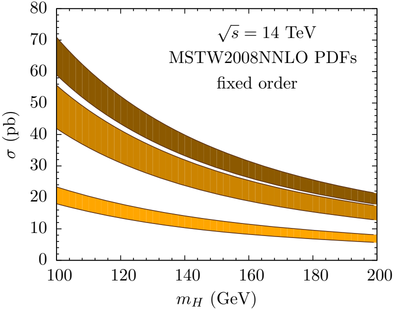
Predictions for the cross section

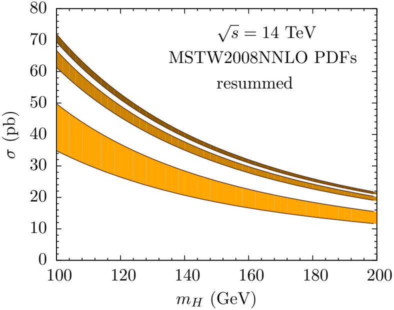
Scale dependence



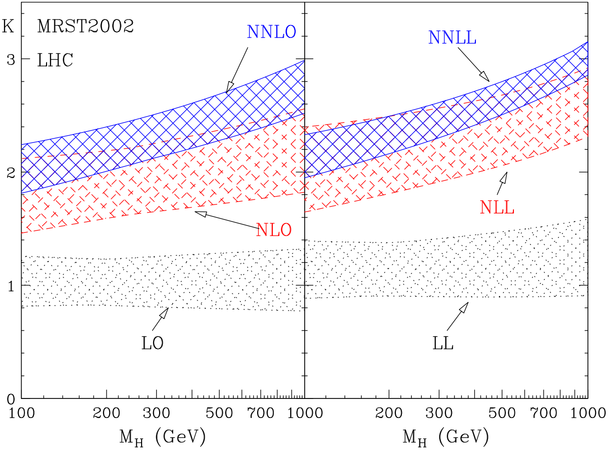
Predictions for the cross section

MSTW2008NNLO PDF set





Traditional threshold resummation



[Figure from arXiv:hep-ph/0306211]