Reservation Wages and
Unemployment Insurance∗

Robert Shimer
University of Chicago
shimer@uchicago.edu

Iván Werning
Massachusetts Institute of Technology
iwerning@mit.edu

October 3, 2006

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∗We are grateful to seminar participants at Berkeley, Harvard, and MIT and for detailed comments from Raj Chetty, Lawrence Katz, and two anonymous referees. Shimer’s research is supported by a grant from the National Science Foundation. Werning is grateful for the hospitality of the Federal Reserve Bank of Minneapolis and Harvard University.
Abstract

This paper argues that a risk-averse worker’s after-tax reservation wage encodes all the relevant information about her welfare. This insight leads to a novel test for the optimality of unemployment insurance based on the responsiveness of reservation wages to unemployment benefits. Some existing estimates imply significant gains to raising the current level of unemployment benefits in the United States, but highlight the need for more research on the determinants of reservation wages. Our approach complements those based on Baily’s (1978) test.
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1. Introduction

The goal of this paper is to develop a test for the optimal level of unemployment insurance using a minimal amount of economic theory and a minimal amount of data. We approach this by studying a risk-averse worker in a sequential job search setting (McCall, 1970). Our main theoretical insight is that the worker’s after-tax reservation wage—the difference between her reservation wage and the tax needed to fund the unemployment insurance system—encodes all of the relevant information about her welfare. This is true regardless of whether workers are able to borrow and lend to smooth their consumption, or whether they must live hand-to-mouth.

Intuitively, the after-tax reservation wage tells us the take-home pay required to make a worker indifferent between working and remaining unemployed. Since take-home pay translates directly into consumption, it is a valid measure of the worker’s utility. Given the simplicity of the argument, it should not be surprising that this insight turns out to be robust to many variations of our basic model.

To prove this result, we develop a formal dynamic model of job search with risk-aversion. Workers draw wages from a known distribution and accepted jobs last for a fixed amount of time. In order to abstract from wealth effects, we assume workers have constant absolute
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We first consider how workers behave when confronted with an arbitrary level of unemployment benefits and reemployment taxes. We find that a worker's utility while unemployed is a monotone function of her after-tax reservation wage. If she has no access to capital markets, her unemployment utility, measured in consumption equivalent units, is equal to her after-tax reservation wage. If she can borrow and lend, it is equal to her after-tax reservation wage plus the annuity value of her assets. This implies that optimal unemployment insurance—the policy of an agency that chooses actuarially fair unemployment benefits and reemployment taxes to maximize an unemployed worker's utility—simply seeks to maximize the worker's after tax reservation wage.

This insight leads to a novel test for the optimality of unemployment insurance: raising benefits is desirable whenever it raises the after-tax reservation wage. This criteria can be decomposed into two effects. On the one hand, higher benefits reduce the cost of remaining unemployed and therefore raise the pre-tax reservation wage. Thus, if the pre-tax reservation wage is very responsive to unemployment benefits, raising unemployment benefits has a strong positive effect on workers' welfare. However, the increase in benefits must be funded

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1In Shimer and Werning (2005), we show that the behavior and insurance needs of a worker with constant relative risk aversion (CRRA) preferences is similar to that of a worker with the same absolute risk aversion and CARA preferences. Thus one might interpret the results we report here as an approximation for other preferences.
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by an increase in the employment tax. The higher is the unemployment rate or the more responsive it is to unemployment benefits, the greater is the needed increase in the tax. Our optimality condition nets out both effects.

While a large literature studies the responsiveness of unemployment or unemployment duration to unemployment benefits (e.g., Meyer, 1990), there is less research on the responsiveness of reservation wages to benefits. Exceptions include Fishe (1982) and Feldstein and Poterba (1984). Fishe (1982) uses information on actual wages to infer reservation wages, while Feldstein and Poterba (1984) uses direct survey evidence on reservation wages. Both papers find that a $1 increase in benefits may raise pre-tax reservation wages by as much as $0.44. Feldstein and Poterba (1984) interpret this as evidence of the moral-hazard cost of raising unemployment benefits, but our approach turns this logic around, since our theory tells us that the reservation wage measures the welfare of unemployed workers.

If the numbers in Fishe (1982) and Feldstein and Poterba (1984) are correct, we show that the marginal effect of a fully-funded increase in unemployment benefits is large. Of course, these estimates are at best valid for small policy changes; according to the model, sufficiently high unemployment benefits would eventually eliminate all economic activity. Moreover, more recent estimates of the responsiveness of reservation wages to benefits are smaller and imply that current benefit levels are too high. In our view, the uncertainty
around this critical variable calls for more precise estimates of it, but doing so goes beyond the scope of this paper.

Within the public finance literature, the standard approach to measuring optimal unemployment insurance is based on the Baily (1978) test:

“The optimal unemployment insurance benefit level is set when the proportional drop in consumption resulting from unemployment, times the degree of relative risk aversion of workers (evaluated at the level of consumption when unemployed) is equal to the elasticity of the duration of unemployment with respect to balanced budget increases in UI [unemployment insurance] benefits and taxes.” (p. 390)

While this approach is close in spirit to the one we adopt here, we see several advantages to our test. First, our test is entirely behavioral, while the Baily test requires independent estimates of risk-aversion. Indeed, Chetty (2006) argues within a Baily framework that the relevant risk-aversion parameter depends on the context and may be higher for unemployment risk. In light of such concerns, the fact that our test does not requires selecting this, or any other, parameter is particularly convenient.

Second, Chetty (2006) shows that in a dynamic environment, the Baily test requires a long panel data set with information on total consumption. Unfortunately, no such data set
exists, so the best known implementation of the Baily test, Gruber (1997), uses panel data on food expenditure. There are two main limitations to using food expenditure as a proxy for total consumption: recent work by Aguiar and Hurst (2005) shows that the link between food expenditure and food consumption is tenuous because of varying amounts of time spent in household production; and food consumption is likely to react significantly less than total consumption to income or wealth shocks.\(^2\)

Third, our exact test is robust to a number of extensions. In Section 6, we allow for the possibility that jobs differ both in their wage and in their average duration, that the duration of a job is stochastic, that an unemployed worker’s search effort affects the arrival rate of offers, that workers are heterogeneous but there is a single unemployment benefit system, that unemployed workers may be recalled to their previous job, and that unemployed workers are eligible for benefits for only a finite amount of time. None of these extensions affects our basic conclusion that the after-tax reservation wage measures the welfare of the unemployed and therefore none substantially alters our behavioral test for optimal unemployment insurance.

In contrast, although Chetty (2006) shows that extensions of the consumption-based

\(^2\)Indeed, Chetty (2006) extends the consumption test so that it applies to food consumption. Unfortunately, the test then requires setting a parameter for the curvature of the utility function with respect to food, instead of risk aversion.
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Baily test are possible, in our view they may be difficult to implement because they require an empirically challenging comparison of the average marginal utility of consumption during employment with that during unemployment over the worker’s entire lifetime—a moment of consumption data not analyzed by Gruber (1997), for example. Nevertheless, our model can also deliver easily implementable consumption-based tests, but we point out that their derivation uses the full structure of the model, is less robust than the new test we propose here, and requires unexplored consumption measures from panel data.

As mentioned above, one challenge to implementing our behavioral test is that empirical evidence on reservation wages is scarce. Our hope is that this paper, by underscoring its usefulness as a welfare statistic, may lead to greater interest in reservation wage evidence, much as Baily’s (1978) theoretical contribution led to empirical research on how much consumption declines when workers lose their job. Ultimately, the two tests are complementary. Both assess the optimality of unemployment insurance, but exploit different data sources.

Macroeconomists have generally taken a different approach to optimal unemployment insurance, calibrating a stochastic general equilibrium model and then performing policy

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3The Borch-Arrow condition for perfect insurance states that marginal utility should be equalized across all states of nature. Absent full insurance, this condition fails and Chetty’s (2006) comparison provides a metric for the extent of this failure.
experiments within the model (Hansen and Imrohoroglu, 1992; Acemoglu and Shimer, 2000; Alvarez and Veracierto, 2001). An advantage to this approach is that it can address issues we neglect, such as the impact of unemployment insurance policy on capital accumulation. But in order to do that, these papers rely heavily on the entire structure of the model and its calibration, which sometimes obscures the economic mechanisms at work and their empirical validity. This approach also makes evaluating the robustness of the results expensive. In contrast, by focusing on the worker’s partial equilibrium problem—a component in richer general equilibrium models—we are able to highlight, in a tractable way, the main tradeoffs that seem important for understanding optimal unemployment insurance and to point out how the relevant forces can be measured.

A third strand of the literature focuses on the timing of benefits, and in particular, on whether unemployment benefits should fall during an unemployment spell (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997). This paper emphasizes the optimal level of benefits but assumes that benefits and taxes are constant over time.\(^4\) In Shimer and Werning (2005) we argue that, provided workers are given enough liquidity to easily borrow against future

\(^4\)Section 6.9 shows that our test for optimal unemployment insurance is robust even if benefits end after a finite amount of time; however, we do not examine the optimal timing of benefits in that section.
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earnings, constant benefits and taxes are optimal, or nearly so. Besides this difference in emphasis, there are two modeling differences. The first is that here we work in continuous time rather than in discrete time, a superficial change that simplifies the algebra. More importantly, here we allow for separations, so that workers experience multiple unemployment spells. This generalization is important for any quantitative exercise focusing on the level of benefits.

The remainder of the paper proceeds as follows: The next section presents our model of sequential search. Section 3 analyzes how workers behave when confronted with constant unemployment benefits and constant taxes. We consider two financial regimes. In the first, workers have unlimited access to borrowing and lending at a constant interest rate, subject only to a no Ponzi-game condition. In the second, workers must live hand-to-mouth, consuming their income in each period. Section 4 describes the problem of an insurance agency choosing the level of unemployment insurance subject to a budget constraint. Section 5 describes our new test for optimal unemployment insurance and discusses the available empirical evidence that bears on the relevant parameters of that test. Section 6 considers a number of generalizations to our model and shows that our test is unaffected by those changes. Section 7 derives a version of the Baily (1978) test for our model, showing that

\footnote{Such liquidity might be provided by unemployment insurance savings accounts (Feldstein, 2005).}
the exact test depends on all the details of the model and hence is less robust than our behavioral test. We conclude in Section 8.

2. Unemployment and Sequential Search

There is a single risk-averse worker who maximizes the expected present value of utility from consumption,

$$E \int_0^\infty e^{-\rho t} U(c(t)) \, dt,$$

where $\rho > 0$ represents the subjective discount rate in continuous time. We assume throughout the body of the paper that the utility function exhibits CARA, $U(c) = -e^{-\gamma c}$ with coefficient of absolute risk aversion $\gamma > 0$.

At any moment in time a worker can be employed, at some wage $w$ with $t$ periods remaining in the job, or unemployed. An employed worker produces a flow of $w$ units of the single consumption good and pays an employment tax $\tau$. When the job ends, she becomes unemployed. An unemployed worker receives a benefit $b$ and waits for the arrival of job opportunities. The worker receives an independent wage draw from a cumulative
Section 2: Unemployment and Sequential Search

distribution function $F$ with Poisson arrival rate $\lambda$. When a worker gets a wage offer, she observes the wage and decides whether to accept or reject it. If she accepts, employment commences immediately and the job lasts for exactly $T \leq \infty$ periods. If she rejects, she produces nothing and remains unemployed. The worker cannot recall past wage offers. With CARA preferences recall is not optimal, so this last assumption is not binding.

There is an unemployment insurance agency whose objective is to maximize an unemployed worker’s utility by choosing a constant unemployment benefit $b$ and constant employment tax $\tau$, subject to the constraint that the expected cost of the unemployment insurance system is zero when discounted at the interest rate $r = \rho$. Let $B \equiv b + \tau$ denote

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6Section 6.4 shows that our results are robust if a worker’s search effort affects the arrival rate of job offers. That section also allows for the possibility that workers have preferences over consumption and leisure.

7Section 6.2 shows that our main results are robust if the worker draws both a wage and a job duration. Section 6.3 shows they are robust if the duration of a job is uncertain. Section 6.8 shows they are robust if unemployed workers may be recalled to their past wage.

8Section 6.5 shows that our results are robust if the unemployment insurance agency also values the utility of currently-employed workers. Section 6.6 shows they are robust if workers are heterogeneous and the agency has access to lump-sum transfers to address any redistributional issues, while Section 6.7 argues that the after-tax reservation wage is still critical even in the absence of lump-sum transfers.

9In Shimer and Werning (2005) we show that this simple unemployment insurance system is optimal when the worker can borrow and lend at interest rate $r$ and jobs last forever. In any case, Section 6.9 shows that our main results are robust if unemployment benefits decline during an unemployment spell.

10Section 6.1 shows that our main results are robust if the discount rate and interest rate are not equal.
Section 2: Unemployment and Sequential Search

the net subsidy to unemployment, the sum of the benefit a worker receives while unemployed and the employment tax she avoids paying. We show below that a worker’s behavior depends only on the net unemployment subsidy.

We consider two financial environments. In the first, the worker has access to financial markets, namely a riskless borrowing and savings technology, facing only the budget constraint

\[ \dot{a}(t) = ra(t) + y(t) - c(t), \]

and the usual no Ponzi-game condition.\textsuperscript{11} Here \( a(t) \) is assets, \( c(t) \) is consumption, and \( y(t) \) represents current income, equal to the current after-tax wage \( w(t) - \tau \) if the worker is employed, or benefits \( b \), otherwise. The rate of return \( r \) is the same for the worker and the unemployment insurance agency. In the second environment, the worker lives hand-to-mouth. She has no access to a savings technology, \( a(t) = 0 \) for all \( t \), and so must consume her income in each period, \( c(t) = y(t) \).

We study these two extremes because they span the spectrum of financial environments with incomplete markets and because both cases are analytically tractable. The intermediate

\textsuperscript{11}The no-Ponzi condition states that debt must grow slower than the interest rate, \( \lim_{t \to \infty} e^{-rt} a(t) \geq 0 \), with probability one. Together with the budget constraints \( \dot{a}(t) = ra(t) + y(t) - c(t) \), this is equivalent to imposing a single present-value constraint, with probability one.
cases cannot be solved in closed form but could be studied numerically to see whether our two cases provide a good benchmark; doing so goes beyond the scope of this paper.

Finally, define

$$\alpha_t \equiv \frac{1 - e^{-rt}}{r} = \int_0^t e^{-rs} ds.$$  

This is the present value of receiving an additional unit of income for the next $t$ periods. The present value of income from a new job with wage $w$ is $\alpha T w$. Note that in the limit as $r$ converges to zero, $\alpha_t = t$.

3. Worker Behavior

We start by characterizing how a worker behaves when confronted with any constant benefit system $(b, \tau)$. We first consider a worker with no liquidity problems, that is, a worker with access to borrowing and lending at rate $r$. We then turn to the opposite end of the spectrum and consider a hand-to-mouth worker who must consume her current income.
3.1. Workers with Liquidity

We now prove the following results. A worker who can borrow and lend at the interest rate \( r = \rho \) keeps her consumption constant during an employment spell since she faces no uncertainty. She saves, however, gradually accumulating assets while on the job. In contrast, consumption steadily declines during unemployment, because remaining unemployed represents a negative permanent-income shock. This is accompanied by dissavings: assets are run down during unemployment spells. Consumption jumps up when an unemployed worker becomes employed, because finding a job is a discrete positive shock to permanent-income. When unemployed, the worker uses a constant reservation wage policy, accepting jobs above some threshold \( \bar{w} \). Finally, the after-tax reservation wage is a sufficient statistic for the welfare of the unemployed. Formally:

**Proposition 1** Assume a worker has access to financial markets. For a given policy \((b, \tau)\), the lifetime utility of an unemployed worker with assets \( a \) is

\[
V_u(a) = \frac{U(ra + \bar{w} - \tau)}{r}.
\]  

(1)

The consumption of an unemployed worker with assets \( a \) and of an employed worker with...
Section 3: Worker Behavior

assets a, t periods remaining on the job, and a wage w are respectively

\[ c_u(a) = ra + \bar{w} - \tau, \]
\[ c(a, t, w) = r(a + \alpha_t(w - \bar{w})) + \bar{w} - \tau. \]

The reservation wage \( \bar{w} \) is constant and solves

\[ \gamma(\bar{w} - B) = \frac{\lambda}{r} \int_{\bar{w}}^{\infty} (1 + U(r\alpha_T(w - \bar{w})))dF(w). \]

The proof is in the appendix.

For the purpose of this paper, the most important part of this proposition is equation (1). If a worker were to remain unemployed forever her lifetime utility would be \( U(ra)/r \). Thus \( \bar{w} - \tau \) is the value an unemployed worker places on access to the labor market, measured in units of per-period consumption.

To get some intuition for equation (1), suppose a worker could accept a job at wage \( w \) that lasts forever, so her after-tax income would be \( w - \tau \). With the discount rate equal to the interest rate, a worker with a concave utility function \( U \) would keep her consumption constant and so would consume this income plus the annuity value on her assets, \( ra \). That
is, she would consume \( c(a, \infty, w) = ra + w - \tau \), her assets would be constant, \( \dot{a} = 0 \), and her lifetime utility would be \( U(ra + w - \tau)/r \). Now define \( \bar{w} \) so that an unemployed worker is indifferent between continuing to search and working forever at \( \bar{w} \), \( V_u(a) \equiv U(ra + \bar{w} - \tau)/r \). This is equivalent to equation (1). The proof of the proposition in the appendix shows that workers are in fact indifferent about continuing to search or working at \( \bar{w} \) for any amount of time, so \( \bar{w} \) is the reservation wage.\(^{12}\)

### 3.2. Hand-to-Mouth Workers

We now consider worker behavior under an extreme alternative, financial autarky, so a worker must consume her income in each period: \( c_{\text{aut}} = \bar{b} \) while unemployed and \( c_{\text{aut}}(w) = w - \tau \) while employed at wage \( w \). Under financial autarky, a worker’s consumption will typically jump up when she finds a job and down when she leaves her job. Although this is qualitatively different than when the worker has access to financial markets, one critical property is unchanged, the worker’s lifetime utility depends only on her after-tax reservation wage:

\(^{12}\)In general, workers may be willing to take a wage below \( \bar{w} \) for a while, accumulate assets, and eventually quit to search for a higher wage. [Acemoglu and Shimer (1999)] explore this in an environment with decreasing absolute risk aversion. This possibility is absent from our setup because of the CARA utility assumption.
Proposition 2 Assume a worker must consume her income. For a given policy \((b, \tau)\), the lifetime utility of unemployment is

\[
V^\text{aut}_u = \frac{U(\bar{w}^\text{aut} - \tau)}{\rho},
\]

where \(\bar{w}^\text{aut}\) is the reservation wage, the solution to

\[
U(\bar{w}^\text{aut} - \tau) = U(b) + \alpha_T \lambda \int_{\bar{w}^\text{aut}}^{\infty} (U(w - \tau) - U(\bar{w}^\text{aut} - \tau)) dF(w).
\]

The proof is in the appendix. This result is independent of the form of the period utility function \(U\). However, with CARA utility \(U(c_1 - c_2) = -U(c_1)/U(c_2)\), so we can rewrite equation (6) as

\[
U(\bar{w}^\text{aut}) = U(B) + \alpha_T \lambda \int_{\bar{w}^\text{aut}}^{\infty} (U(w) - U(\bar{w}^\text{aut})) dF(w),
\]

which implies that \(\bar{w}\) is determined as a function of \(B = b + \tau\).

It is worth noting that, since the reservation wage summarizes a worker’s utility both under perfect liquidity and financial autarky, the difference in the reservation wage summarizes
the value of access to financial markets. More precisely,

**Proposition 3** A hand-to-mouth worker has a lower reservation wage than a worker with access to capital markets. Moreover, the difference in their reservation wages is the utility gain from access to capital markets, measured in units of per-period consumption.

The proof is in the appendix.

4. Optimal Unemployment Insurance

4.1. The Unemployment Insurance Agency’s Problem

We now turn to the problem of an unemployment insurance agency that chooses actuarially fair unemployment benefits $b$ and employment taxes $\tau$ to maximize the worker’s utility given by equation (1) (if the worker has liquidity) or equation (5) (if the worker is hand-to-mouth). In both cases, this is equivalent to maximizing the worker’s after-tax reservation wage.

To derive the actuarially fair relation between benefits and taxes, consider the net present value cost $C$ of the program, which must satisfy

$$rC = b + \lambda (1 - F(\bar{w})) \left( e^{-rT}C - \alpha_T \tau - C \right).$$
Section 4: Optimal Unemployment Insurance

In words, the flow cost, \( rC \), is comprised of current benefits outlays, \( b \), plus the opportunity of becoming employed, which occurs with arrival rate \( \lambda(1 - F(\bar{w})) \) and reduces costs from \( C \) to \( e^{-rT}C - \alpha_T \tau \), since employed workers pay taxes and become unemployed after some time \( T \). Setting \( C = 0 \) gives the agency’s budget constraint

\[
Db = \alpha_T \tau, \tag{8}
\]

where \( D \equiv 1/\lambda(1 - F(\bar{w})) \) is the expected duration of an unemployment spell. It will prove useful to consider as an approximation the limit as \( r \to 0 \), so that \( \alpha_T = T \) and the budget constraint becomes

\[
ub = (1 - u)\tau, \tag{9}
\]

where \( u \equiv D/(T + D) \) is the fraction of time a worker is unemployed, or equivalently, the unemployment rate.

The unemployment insurance agency recognizes that the worker will set her reservation wage optimally given the chosen policy. Putting this together, the optimal unemployment insurance problem is to choose benefits \( b \), taxes \( \tau \), and a reservation wage \( \bar{w} \) to maximize \( \bar{w} - \tau \) subject to two constraints: the worker sets her reservation wage according to equation (4) (liquidity) or equation (7) (hand-to-mouth); and the insurance must be actuarially fair,
equation (8). Let \( \{b^*, \tau^*\} \) denote the optimal policy.

To see whether unemployment insurance is optimal, all we need to know is how an actuarially fair increase in taxes and benefits affects a worker’s after-tax reservation wage. It is not necessary to make any assumptions about risk-aversion, discount rates, the speed of finding a job, the duration of a job, the distribution of wage offers, or about whether workers have liquidity or must consume hand-to-mouth since workers’ utility is a monotone function of the after-tax reservation wage \( \bar{w} - \tau \).

While this result is theoretically appealing, it may be difficult to implement because it may be hard to discern how much taxes must rise to balance an increase in benefits. In principle this question might be left to a budgetary authority like the Congressional Budget Office, but such an organization would still need to understand how much the increase in benefits raises unemployment duration. Instead, our behavioral test uses information on how unemployment benefits affect the pre-tax reservation wage and on the elasticity of unemployment duration with respect to benefits to characterize how taxes must change and hence to characterize optimal policy.

4.2. A Behavioral Test
Section 4: Optimal Unemployment Insurance

Equation (4) or equation (7) implies that the reservation wage depends on unemployment benefits and taxes, \( \bar{w}(b, \tau) \); let \( D(b, \tau) \equiv 1/\left( \lambda (1 - F(\bar{w}(b, \tau))) \right) \) denote average duration as a function of \( b \) and \( \tau \). It follows that the resource constraint (8) defines taxes as a function of benefits \( \tau(b) \). Differentiate (8) with respect to \( b \) to get

\[
\tau'(b) = \frac{bD_b(b, \tau(b)) + D(b, \tau(b))}{\alpha T - bD_\tau(b, \tau(b))},
\]

where subscripts denote partial derivatives. The denominator is positive if a tax cut reduces the fiscal surplus, i.e., we are on the correct side of the Laffer curve. With CARA utility and either perfect liquidity or hand-to-mouth consumption, the reservation wage and hence unemployment duration depends only on the sum of benefits and taxes (see equations (4) and (7), respectively), so \( D_b = D_\tau \). Then letting \( \varepsilon_{D,b} \equiv bD_b(b, \tau)/D(b, \tau) \) be the the elasticity of unemployment duration with respect to unemployment benefits, we can write the previous equation as

\[
\tau'(b) = \frac{D(b, \tau(b))(1 + \varepsilon_{D,b})}{\alpha T - D(b, \tau(b))\varepsilon_{D,b}}.
\]

Next, since unemployment benefits should maximize \( \bar{w}(b, \tau(b)) - \tau(b) \), a necessary condition
for optimal benefits is
\[
\bar{w}_b(b^*, \tau^*) + \bar{w}_\tau(b^*, \tau^*)\tau'(b^*) = \tau'(b^*),
\]
where as usual subscripts denote partial derivatives. Again, \( \bar{w}_b = \bar{w}_\tau \) under CARA utility, and so combining this equation with equation (10) gives our test for optimal benefits:

**Proposition 4** If unemployment benefits are optimal,
\[
\bar{w}_b(b^*, \tau^*) = \frac{D(b^*, \tau^*)}{\alpha_T + D(b^*, \tau^*)}(1 + \varepsilon_{D,b}(b^*, \tau^*)). \tag{11}
\]

If the left-hand-side of equation (11) is larger than the right-hand-side, a marginal increase in benefits raises the worker’s after-tax reservation wage and so is welfare-improving.

It is convenient to focus on the limit as \( r \) converges to zero. Following the same logic, but starting with equation (9) instead of equation (8) gives
\[
\bar{w}_b(b^*, \tau^*) = u(b^*, \tau^*) \left(1 + \frac{\varepsilon_{u,b}(b^*, \tau^*)}{1 - u(b^*, \tau^*)}\right), \tag{12}
\]
where \( \varepsilon_{u,b} \) is the elasticity of the unemployment rate with respect to benefits. This also follows from equation (11) as \( r \to 0 \) using \( \varepsilon_{u,b} = (1 - u)\varepsilon_{D,b} \). This approximation is good for realistic
values of the discount rate. More importantly, we show in Section 6 that equation (12) is robust to numerous extensions of our basic model.

Our theory provides some guidance on how responsive the reservation wage is to unemployment benefits. Differentiating equation (4) or equation (7), we can prove that $\bar{w}_b \geq D/(\alpha T + D)$, and strictly so if workers are risk averse. However, to see whether equation (11) holds requires looking at the data.

5. Available Evidence

To implement the test proposed in Proposition 4, we need to know five numbers: the interest rate $r$, the duration of a job $T$, the mean duration of an unemployment spell $D$, the elasticity of duration with respect to benefits $\varepsilon_{D,b}$, and the responsiveness of the reservation wage to benefits $\bar{w}_b$. This section starts by examining evidence on the first four numbers, the right hand side of equation (11), and then considers the last number, the left hand side of the equation.
5.1. Threshold for the Response of Reservation Wages to Benefits

The interest rate $r$, the duration of a job $T$, and the mean duration of an unemployment spell $D$ determine $D/(\alpha T + D)$, which is approximately equal to the unemployment rate $u$. Our results are therefore sensitive to the choice of the unemployment rate. We target a 5.6 percent unemployment rate, equal to the average value in the United States between 1948 and 2005. We think of a time period as a week and set the interest rate at $r = 0.001$, equivalent to an annual interest rate of 5.3 percent. The mean duration of an in-progress unemployment spell between 1948 and 2005 was $D = 13.4$ weeks and so we set $T = 225$ to hit the target unemployment rate. Together this implies $D/(\alpha T + D) = 0.062$.

We turn next to the elasticity of duration with respect to benefits, $\varepsilon_{D,b}$, the remaining unknown on the right hand side of equation (11). Perhaps the best-known study of this number is Meyer (1990), who uses administrative data from the Continuous Wage and Benefit History (CWBH). The records cover men who received unemployment benefits in twelve states from 1978 to 1983 and include information on the level and potential duration of benefits, and on pre-unemployment earnings. Katz and Meyer (1990) explain that the source of variation in benefits in Meyer’s data include nonlinearities in benefit schedules, legislative changes, and the erosion of real benefits due to fixed nominal schedules between legislative
changes. By including state fixed effects and past wages in their regressions, both papers effectively control for endogeneity of benefit levels. Moreover, these controls make it unlikely that workers who receive high benefits relative to their past wage and relative to other workers in their state anticipate paying relatively high taxes in the future. Thus these papers convincingly estimate the ‘partial’ elasticity $\varepsilon_{D,b}$ that is needed, holding taxes constant.

In his preferred estimate, Meyer (1990, Table V, specification 5) finds that a one percent increase in unemployment benefits reduces the baseline hazard rate of finding a job by 0.88 percent. Since the hazard rate is the inverse of expected unemployment duration, this implies $\varepsilon_{D,b} = 0.88$. Combined with $D/(\alpha_T + D) = 0.062$, the right hand side of equation (11) evaluates to 0.117. Reasonable parameter changes do not affect this number much. For example, some of Meyer’s (1990) other estimates in Table V show an elasticity as small as 0.53, while Katz and Meyer (1990) find an elasticity of 0.54. Krueger and Meyer (2002, p. 2351) call 0.5 “not an unreasonable rough summary” of the literature on $\varepsilon_{D,b}$. This smaller number would reduce the right hand side to 0.094. Conversely, if unemployment duration were twice as long, $D = 26.4$, but job duration is also twice as long, $T = 450$, leaving the

\[ \varepsilon_{D,b} \]

13 Most of the theoretical literature has interpreted these numbers as the ‘total’ elasticity of duration with respect to an increase in benefits and an actuarially fair increase in taxes. One can show that the partial elasticity $\varepsilon_{D,b} = \hat{\varepsilon}_{D,b}\alpha_T/((1 + \hat{\varepsilon}_{D,b})D + \alpha_T) < \hat{\varepsilon}_{D,b}$, where $\hat{\varepsilon}_{D,b}$ is the total elasticity.
unemployment rate unchanged, the right hand side increases to 0.129.

These numbers represent a threshold for the responsiveness of the reservation wage to benefits that determines whether changes in benefits improve welfare and the best direction of any benefit change. For example, the point estimate 0.117, obtained above from Meyer’s preferred estimate, implies that if a worker’s reservation wage rises by more than $0.117 for every $1 increase in benefits, then such an increase improves welfare. Conversely, if the response is lower then benefits should be decreased. Benefits are locally optimal only when the response of the reservation wage equals this threshold.

5.2. An Ideal Experiment

Before reviewing the available evidence on the responsiveness of the reservation wage to benefits, we discuss two properties that an ideal measure should possess. First, we require an unbiased measure of reservation wages. As we discuss below, our reading of the literature suggests that workers can answer questions about their reservation wage, but estimates of reservation wages using administrative wage data will also always be useful. Second, we require variation in benefits that is orthogonal to any omitted characteristics of the worker.

\[14\] If there are non-wage components of compensation then we require a measure of the reservation wage that holds these job attributes fixed.
Section 5: Available Evidence

or the economic environment. This is achieved most directly via deliberate experimentation; Meyer (1995) documents that U.S. states are sometimes willing to undertake such experiments. In the absence of experiments, however, it should still be possible to exploit nonlinearities in benefit schedules, combined with a rich set of controls for local labor market conditions, to obtain the desired variation. Meyer (1990) and Katz and Meyer (1990) show that this is feasible when measuring the impact of benefits on duration. It should also be feasible when measuring the impact of benefits on reservation wages. Some other properties are also desirable: larger data sets will yield tighter estimates, which is particularly important since reservation wages are measured with error; and if we are concerned with the optimality of the current United States unemployment system, we must use U.S. data since the desired slope parameter $\bar{w}_b$ is likely to vary across countries.

5.3. Direct Evidence on Reservation Wages

15 years ago, Devine and Kiefer (1991, Chapter 4) surveyed the existing evidence on the behavior of reservation wages. The first problem this literature confronted was how to measure reservation wages. Unlike unemployment duration, administrative records do not have any direct information on reservation wages. Instead, Kasper (1967) used data from
the Minnesota Department of Employment Security, which asked workers simply “what wage are you seeking?” Barnes (1975) looked at registered unemployed workers in 12 cities who were asked their “lowest acceptable wage.” Sant (1977) examined the 1966 cohort of the National Longitudinal Survey of Young Males (NLSY), which from 1967 to 1969 asked “what wage are you willing to accept?” Holzer (1987) provides some validation that this type of question contains useful information. Using data from a later cohort of the NLSY, he finds that workers with higher reservation wages are less likely to accept a job offer (Table 4) but earn a higher wage when they do take a job (Table 5). In any case, the variation in sample selection and question design complicates any analysis of reservation wages.

Feldstein and Poterba (1984) were the first to examine how self-reported reservation wages respond to unemployment benefits. They study a supplement to the May 1976 Current Population Survey (CPS) which asked 2,228 unemployment insurance recipients “What is the lowest wage or salary you would accept (before deductions)?” The answers are surprisingly high, on average seven percent above their previous wage. As subsequent authors have noted, this casts some doubt on their results.

In their main analysis, Feldstein and Poterba (1984) regress the ratio of a worker’s reservation wage, \( \bar{w} \), to their last wage, \( w_0 \), on the ratio of their benefit, \( b \), to \( w_0 \) and a number
They run the regression separately for workers reporting different reasons for unemployment. In Table 4, they report that a one percentage point increase in $b/w_0$ raises $\bar{w}/w_0$ by 0.13 and 0.42 percentage points, so $\bar{w}_b \in [0.13, 0.42]$. The lowest slope estimate is for job losers on layoff and the highest is for other job losers; the slope estimate for job leavers is 0.29.

Although this study advanced our understanding of reservation wages, it has some important shortcomings. First, if the last wage is measured with error, the main coefficient estimate is biased towards 1 and hence overstates the true elasticity. Second, the source of variation in their main independent variable, the ratio of benefits to the last wage, is unclear. To the extent that there is a third factor correlated with both the benefit ratio and the reservation wage ratio, their results are biased. Plausible candidates include human capital or any systematic correlation between benefits and local labor market conditions. The authors include control variables to try to soak up such variation, but it seems unlikely that they are able to capture all the relevant dimensions. For example, states that offer higher unemployment benefits may differ systematically along other dimensions. At a minimum,

---

15The controls include age, sex, number of years of education, and dummies for whites, married men, a working spouse, and for the receipt of welfare payments and other supplementary income. The authors note that theory often provides little guidance on how these variables should affect the reservation wage.
Section 5: Available Evidence

taxes must be higher to fund these benefits, which means Feldstein and Poterba (1984) measure the impact of an actuarially fair increase in benefits and taxes on reservation wages, $\bar{w}_b(b, \tau)(1 + \tau'(b))$. But it is also likely that other social insurance programs and the relevant taxes covary with unemployment benefits. In their analysis, the effect of these programs is loaded on to unemployment benefits.

Despite these shortcomings, it is worth understanding the quantitative implications of Feldstein and Poterba’s (1984) estimates. They imply substantial gains from raising unemployment benefits.\footnote{Curiously, Feldstein and Poterba (1984) interpret their estimates of the responsiveness of reservation wages to benefits as an argument for lowering unemployment benefits because of the moral hazard costs. Our model shows that, on the contrary, if the reservation wage is sufficiently responsive to benefits, then benefits must be serving their purpose, improving the welfare of unemployed workers.} A $1$ balanced-budget increase in unemployment benefits raises the after-tax reservation wage by

$$\bar{w}_b(b, \tau)(1 + \tau'(b)) - \tau'(b) = \frac{\bar{w}_b(b, \tau)(\alpha_T + D(b, \tau)) - D(b, \tau)(1 + \varepsilon_{D,b}(b, \tau))}{\alpha_T - D(b, \tau)\varepsilon_{D,b}(b, \tau)},$$

or $0.34$ for job losers not on layoff. In other words, the net welfare gain for an unemployed worker is equivalent to increasing her consumption by 34 cents at all dates in the future. To get a rough sense of the magnitude of this number, there are about 135 million workers in
the United States economy, with about 7.7 million unemployed at any point in time. Giving every unemployed worker, including those not currently collecting unemployment benefits, an extra $1 per week would cost approximately $400 million per year. The net welfare gain is equivalent to (somehow) raising the consumption of all workers by $0.34 per week, for a total of $2.4 billion per year. Of course, even if these estimates are correct, they are only correct locally. Raising benefits by $1000 per week would probably not yield $2.4 trillion per year in additional consumption-equivalent utility.

In any case, more recent studies using different data and sometimes different methodologies have often reached different conclusions. Although it is not the main purpose of their paper, DellaVigna and Paserman (2005, Appendix Table E1) regress the log of the self-reported reservation wage on a dummy for whether the worker received unemployment insurance benefits and numerous controls using data from the NLSY. They find that receiving benefits raises the reservation wage by 4.7 percent, significantly smaller than Feldstein and Poterba (1984) estimate, and they cannot reject the null hypothesis that it has no effect. There are again some serious concerns with using this study for our purpose.

\[17\] There are also numerous studies using non-U.S. data sources; see, for example, Jones (1988), van den Berg (1990), Gorter and Gorter (1993), Jones (2001), and Bloemen and Stancanelli (2001). While these cannot tell us anything about the optimality of the current U.S. unemployment insurance system, they may provide lessons on how to properly measure the impact of unemployment benefits on reservation wages.
First, it is small, with only 1,010 unemployed workers who reported all the necessary information. Second, the measure of benefits is binary, which makes it difficult to compare with the desired slope $\bar{w}_b$. Perhaps more importantly, only 12.9 percent of unemployed workers in the sample receive benefits, much lower than in the population as a whole. This suggests the possibility of a strong selection bias. Finally, although the authors include numerous control variables, they do not attempt to address the possible endogeneity of unemployment benefits or the correlation of benefits and other omitted policy variables.

### 5.4. Indirect Evidence on Reservation Wages

Another approach to measuring reservation wages is to infer them from data on accepted wages. Perhaps the earliest such evidence comes indirectly from Ehrenberg and Oaxaca (1976), who find that workers who receive higher unemployment benefits get higher wage jobs.

To see what this implies about reservation wages requires more structure. Fishe (1982) uses the CWBH files for Florida, a 5 percent sample of state residents from 1971 to 1974. Since this is administrative data, measurement error should be minimal. But since it does not contain any direct information on reservation wages, he has to infer them using a cen-
sored regression model and data on actual wages paid. In his Table 2, he concludes that a $1 increase in potential weekly benefits raises the (unobserved) reservation wage by $0.44, slightly larger than Feldstein and Poterba’s (1984) biggest estimate. There are two main drawbacks to Fishe’s (1982) approach. First, it seems likely that differences in unemployment benefits are driven at least in part by a third factor that is omitted from the regression, biasing his results. Second, the approach requires some parametric assumptions in order to infer how observed wages are related to the unobserved latent wage; Fishe (1982) assumes joint normality of the errors in a wage offer equation and a reservation wage equation.

Once again, more recent data casts doubt on this conclusion. Meyer (1995) studies a number of experiments in which states subsidized workers who found a job quickly and kept it for a specified amount of time; in our framework, this is equivalent to a reduction in the net unemployment subsidy $B$ and hence should lower both unemployment duration and reservation wages. Meyer (1995, p. 96) confirms the first prediction but concludes that “the experiments also tend to show that speeding claimants’ return to work does not decrease total or quarterly earnings following the claim, but the evidence is less strong because the estimates are imprecise.”

There are two ways to interpret this result. If unemployment benefits do not affect the

\[ \text{Card, Chetty and Weber (2006) get similar results using administrative data from Austria.} \]
Section 6: Extensions

We think the most attractive feature of the behavioral test for optimal unemployment insurance is that, while it is theoretically well-grounded, it does not rely on much of the structure of the model. For example, we have already shown that we do not need to know whether workers have easy access to financial markets or no access at all. In this section, we discuss several modifications of, and extensions to, our basic framework in order to establish the robustness of our approach. Each of these modifications alters the formula for how the
reservation wage reacts to benefits, but none of them substantially changes the behavioral test in Proposition 4. To simplify the presentation we discuss each new element separately and keep the mathematical formalities to a minimum.

6.1. Different Interest and Discount Rates

To simplify the exposition we have assumed throughout that the interest rate is equal to the discount rate. While the relationship between $r$ and $\rho$ affects consumption, it is easy to show that with CARA preferences the effect is simply a level-shift in consumption:

$$c_u(a) = ra + \bar{w} - \tau + \frac{\rho - r}{r\gamma},$$

where $\gamma$ is the coefficient of absolute risk aversion. Therefore the objective of the unemployment insurance agency is still to maximize the after-tax reservation wage subject to the budget constraint in equation (8). Thus, the characterization in equation (11) is unchanged.$^{19}$

$^{19}$This argument ignores any possible general equilibrium effects of unemployment benefits on interest rates.
6.2. Sampling Wages and Job Duration

In our baseline model, we assumed that all jobs last for $T$ periods and are heterogeneous only in the wage opportunity. We now prove that our results easily extend to the case when jobs differ both in terms of their wage offer and in terms of their duration.

Suppose that workers sample jobs distinguished by their wage-duration pair $(w, T)$ from some joint distribution function $F(w, T)$. It is straightforward to prove that workers use a reservation-wage rule, accepting all jobs that pay at least $\bar{w}$, independent of $T$. Intuitively, a worker employed at her reservation wage is indifferent about accepting the job and therefore indifferent about how long the job lasts. In particular, an unemployed worker with assets $a$ is indifferent about accepting a job offering her reservation wage forever, and therefore consuming $ra + \bar{w} - \tau$ forever. This pins the value of unemployment, unchanged from equation (1) in the case with liquidity and equation (5) in the case of financial autarky. In both cases, a worker’s utility is still increasing in the after-tax reservation wage $\bar{w} - \tau$.

Optimal unemployment insurance maximizes the after-tax reservation wage subject the resource constraint, $Db = \hat{\alpha} \tau$, where $\hat{\alpha}$ is the expected value of $\alpha_T$ conditional on the wage exceeding $\bar{w}$, a slight extension of equation (8). This leads to the following generalization of
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equation (11):
\[ \bar{w}_b = \frac{D}{\hat{\alpha} + D}(1 + \varepsilon_{D,b} - \varepsilon_{\hat{\alpha},b}), \]  
(13)

where \( \varepsilon_{\hat{\alpha},b} \) is the elasticity of \( \hat{\alpha} \) with respect to benefits. In the limit as \( r \to 0 \), equation (13) further reduces to equation (12), while in the special case where \( w \) and \( T \) are independent, \( \hat{\alpha} \) is a constant and so \( \varepsilon_{\hat{\alpha},b} = 0 \), leaving equation (11) virtually unchanged.

To see why this matters, suppose that higher wage jobs last longer. Then an increase in benefits raises employment duration, \( \varepsilon_{\hat{\alpha},b} > 0 \). This offsets the increase in unemployment duration, reducing the elasticity of the unemployment rate and raising the attractiveness of unemployment insurance. To our knowledge, the existing literature on optimal unemployment insurance has neglected this possibility.

6.3. Job Loss Risk

To focus on the risk of unemployment duration we have abstracted from job loss risk by assuming that the duration of a job is known as soon as the job is accepted. In reality, of course, even after finding a job, a worker faces uncertainty about its length. To be concrete, suppose all jobs end according to a Poisson process with arrival rate \( s \).

The arguments behind Proposition 1 and Proposition 2 are unchanged. A worker em-
ployed at her reservation wage faces no uncertainty and therefore wants to keep her consumption constant. If she has liquidity, this means she consumes the annuity value of her assets plus her after-tax reservation wage. If she is hand-to-mouth, she simply consumes her after-tax reservation wage. This implies that her utility, and therefore the utility of an unemployed worker, is an increasing function of her after-tax reservation wage.

The resource constraint changes slightly when job duration is uncertain, so equation (8) becomes $Db = \tau/(r + s)$. Note that $1/(r + s)$ represents the expected present value of a unit of income until a job ends, analogous to $\alpha_T$ in the baseline model. This introduces a small modification into equation (11):

$$
\bar{w}_b = \frac{D}{r+s} + D(1 + \epsilon_{D,b}).
$$

(14)

Setting $r = 0.001$, $D = 13.4$, $s = 1/T = 1/225$, and $\epsilon_{D,b} = 0.88$, the right hand side evaluates to 0.128, slightly larger than the 0.117 obtained when all jobs last for exactly $T$ periods. Indeed, the only difference between these numbers comes from discounting. If $r = 0$, $D/(T + D)$ and $D/(1/s + D)$ are both equal to the unemployment rate.
6.4. Costly Search

We have so far focused on a worker’s choice of which jobs to accept as the source for the moral-hazard problem. An alternative approach models workers as making a costly search effort choice that affects the arrival rate of homogeneous job opportunities. Reality likely combines both elements; fortunately, so can our model.

To maintain the tractability of our CARA specification, we assume that the search effort is monetary so that the utility function is $U(c - v(e))$ for some disutility of effort function $v(e)$. Effort $e$ improves the arrival of job opportunities $\lambda(e)$.

With this specification, any constant benefit system $(b, \tau)$ induces workers to choose a constant level of effort $e^*$, independent of their wealth level. Effectively this reduces unemployment income by $v(e^*)$. While this naturally alters the reservation wage equation (4), it does not affect the value of an unemployed worker conditional on her reservation wage, which is unchanged from equation (1) and equation (5). Duration is now affected by policy through two channels, the arrival rate of job offers $\lambda(e)$ and the acceptance probability of each $1 - F(\bar{w})$, but this does not modify the agency’s budget constraint equation (8). It follows that Proposition 4 is unchanged by a monetary cost of search: our derivation of equation (11) uses the elasticity of unemployment duration with respect to benefits, but the
reason why benefits affect unemployment duration is immaterial.

We can also allow employed workers to decide how much to work. Suppose that a worker offered a wage $w$ and choosing work effort $e$ earns $ew$ at disutility $v(e)$. Then she optimally sets $e$ to solve $v'(e(w)) = w$ and so the model is isomorphic to one in which she draws income $e(w)w - v(e(w))$ with no disutility of effort.

6.5. Employed Workers and Redistributive Motives

Our analysis has so far focused on optimal insurance for an unemployed worker. We now extend the analysis to consider employed workers. We focus on the case where the worker has perfect access to financial markets.

To start, consider designing insurance for a worker who is initially employed at a wage $w$ with $t$ periods remaining on her job. One simple way to achieve the optimum is to allow the worker to defer entry into the optimal scheme for an unemployed worker. The worker pays no tax until she next becomes unemployed; thereafter, she receives benefit $b$ whenever she is unemployed and pays tax $\tau$ whenever she is employed. If the worker quits her job at time $s \leq t$ to become unemployed, her lifetime utility in consumption units is

$$ra + (1 - e^{-rs})w + e^{-rs}(\bar{w} - \tau). \tag{15}$$
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It follows that the worker keeps her job (sets $s = t$) if $w \geq \bar{w} - \tau$ and quits it immediately otherwise ($s = 0$), which is the socially efficient choice. We conclude that employed workers also wish to maximize $\bar{w} - \tau$ and so demand the same $b^*$ and $\tau^*$ as the unemployed.

Now suppose there is initially a mix of employed and unemployed workers, potentially with different asset levels, wages and remaining job durations. Given the alignment of opinions over benefits, the scheme just described is Pareto efficient. By construction it also involves no redistribution and so everyone agrees on the unemployment benefit level.

Most unemployment benefit schemes offer both insurance and redistribution, which naturally breaks the agreement between employed and unemployed workers. One way to ensure agreement on unemployment benefits is to rely on other taxes to take care of any desired redistribution. To see this, suppose that the unemployment insurance benefits $b$ and taxes $\tau$ take effect immediately, but that workers must also pay an initial lump-sum tax $\kappa(t) - \alpha_t \tau$ conditional on their remaining job duration $t$, which we assume is observable; an unemployed worker has $t = 0$. After some manipulations, the agency’s budget constraint requires

$$
\int_0^\infty \left( \kappa(t) + \frac{e^{-rt}}{r} \left( \frac{\tau \alpha_T - bD}{\alpha_T + D} \right) \right) dG(t) = 0,
$$

where $G(t)$ is the given initial distribution over job durations. For a worker earning a wage
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$w$ with $t$ periods remaining in her job, welfare in consumption units is

$$r(a - \kappa(t)) + (1 - e^{-rt})w + e^{-rt}(\bar{w} - \tau)$$

(17)

Setting $\kappa(t) = 0$ implies that welfare is equivalent to equation (15). Thus, lump-sum transfers can imitate the previous scheme.

Since there is Ricardian equivalence in this environment, we may without loss of generality restrict attention to tax and benefit schemes such that $\int_0^\infty \kappa(t)dG(t) = 0$ so the budget constraint equation (16) is equivalent to equation (8). In this case $\kappa(t)$ is the net lump-sum tax, after each worker has paid her own cost of unemployment insurance. Equation (17) implies everyone agrees on maximizing $\bar{w} - \tau$, independent of the redistribution chosen by $\kappa(t)$. In other words, given these instruments, all Pareto efficient policies maximize $\bar{w} - \tau$, regardless of redistributional concerns. The same is true of more sophisticated lump-sum transfers that may potentially depend additionally on the initial wage $w$ or assets $a$.

This analysis highlights an important point: unemployment insurance is, at best, a crude redistributional tool and will not be used as such if sharper instruments, such as the lump-sum taxes $\kappa(t)$, are available. When this is the case, the optimal level of benefits will not depend on redistributive concerns. For most of the paper, to focus on insurance rather than
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redistribution, we assume either that there is no redistributio nal motive or that the agency has access to lump-sum taxes.

6.6. Worker Heterogeneity with Lump-Sum Transfers

Up to this point we have considered the problem of an insurance agency confronted with a single type of worker. The analysis is also immediately applicable if there are many types of workers, and the agency can tailor the unemployment insurance design to each. We now consider worker heterogeneity but assume that there can be only one unemployment insurance policy that applies to all workers.

Let there be finitely many types of workers denoted by \( n = 1, 2, \ldots, N \) with population fractions \( \pi^n \). We allow the distribution of wages \( F^n(w) \), the duration of jobs \( T^n \), and the risk aversion parameter \( \gamma^n \) to depend on the worker type. We assume the availability of lump-sum taxes that depend on the worker type \( n \) and remaining employment duration \( t \), with unemployment again corresponding to \( t = 0 \). Let \( \kappa^n(t) - \alpha_1 \tau \) denote this lump-sum. As before, Ricardian equivalence allows us to restrict attention to tax and benefit schemes where \( \sum_{n=1}^{N} \pi^n \int_0^\infty \kappa^n(t)dG^n(t) = 0 \), so \( \kappa^n(t) \) represents the redistribution net of that done by unemployment insurance policy.
Let $\tilde{w}^n$ be type $n$’s reservation wage. Following the analysis from Section 6.5 closely, welfare for type $(n, t)$ workers expressed in consumption units is

$$r(a - \kappa^n(t)) + (1 - e^{-rt})w + e^{-rt}(\tilde{w}^n - \tau)$$

and the budget constraint is

$$\sum_{n=1}^{N} \pi^n \omega^n \left( \frac{\tau \alpha T^n - b D^n}{\alpha T^n + D^n} \right) = 0,$$

where $\omega^n \equiv \int_{0}^{\infty} e^{-rt} dG^n(t)$. Because lump-sum transfers $\kappa^n(t)$ can achieve any redistributitional aims, all efficient policies must maximize average consumption and hence must maximize

$$\sum_{n=1}^{N} \pi^n \omega^n (\tilde{w}^n - \tau),$$

while taxes and benefits solve the budget constraint equation (18).

In the limit as $r \to 0$, $\omega^n \to 1$ for any fixed distribution $G^n(t)$ and so equation (19)
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becomes

$$\sum_{n=1}^{N} \pi^n w^n(b, \tau) - \tau,$$

(20)

which we treat as an approximation for \( r > 0 \) but small. Similarly, the budget constraint equation (18) reduces to

$$\sum_{n=1}^{N} \pi^n \left( \frac{\tau T_n - b D^n(b, \tau)}{T_n + D^n(b, \tau)} \right) = 0.$$

Equivalently, a type \( n \) worker spends a fraction \( D^n(b, \tau)/(T^n + D^n(b, \tau)) \) of her life unemployed and the rest employed, so the budget constraint requires that \( u(b, \tau)b = (1 - u(b, \tau))\tau \), as in equation (9), where \( u(b, \tau) \) is the population unemployment rate when the unemployment insurance policy is \((b, \tau)\).

Our behavioral test for optimal unemployment insurance easily generalizes to this environment. Unemployment benefits maximize equation (20), with taxes adjusting to balance the budget. This implies

$$\sum_{n=1}^{N} \pi^n w^n_b = \frac{\tau'(b^*)}{1 + \tau'(b^*)} = u \left( 1 + \frac{\epsilon_{u,b}}{1 - u} \right).$$

(21)
This is a natural generalization of equation (12). We need to know the average response of reservation wages to benefits in the population, the unemployment rate, and the elasticity of the unemployment rate with respect to benefits to perform this test.

6.7. Worker Heterogeneity without Lump-Sum Transfers

We now comment on what occurs if the agency cannot redistribute using a lump-sum tax or any other instrument. For example, suppose there are two types of workers \( n = 1, 2 \), with type 1 having the higher unemployment rate so that unemployment benefits redistribute from type 2 to type 1 workers. Varying benefits \( b \) and adjusting taxes \( \tau(b) \) in a budget balance fashion traces out a utility locus \( (V^1(b), V^2(b)) \). Any point on the northeastern portion of this set of utility points is efficient. The associated set of efficient benefit levels \( b \) may be quite wide and include values that do not lie between the individually optimal levels for both type 1 and 2. This may occur because benefits are no longer necessarily paid by the party that receives them. This discussion illustrates that when redistributive devices are severely limited it is hard to say much about efficient unemployment insurance, as this instrument may have to carry the full weight of redistribution.

Although general conclusions are hard to obtain, if we are willing to quantify the dis-
tributional concerns we can modify our test for optimal unemployment insurance. Suppose for the sake of simplicity that all workers types \( n = 1, 2, \ldots, N \) are initially unemployed, so that their welfare is directly related to \( \bar{w}^n - \tau \), as well as the contribution from assets which unemployment policy does not affect. The welfare criterion is a weighted average of these welfare measures

\[
\sum_{n=1}^{N} \pi^n \lambda^n \bar{w}^n - \tau,
\]

where the weights \( \lambda^n \) are normalized so that \( \sum_{n=1}^{N} \pi^n \lambda^n = 1 \). These weights determine the desire for redistribution: the ratio \( \lambda^j / \lambda^i \) represents the rate at which we are willing to transfer goods from type \( j \) to type \( i \) workers, i.e. the tolerable size of the hole in the “leaking bucket” that takes goods from \( j \) to \( i \). When lump-sum transfers are available this rate must be unity and \( \lambda^n = 1 \).

The optimality condition as \( r \to 0 \) is

\[
\sum_{n=1}^{N} \pi^n \lambda^n \bar{w}^n_b = u \left( 1 + \frac{\varepsilon_{u,b}}{1 - u} \right),
\]

so that some weighted average of the increase in reservation wages must equal the term
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capturing the increase in taxes required to finance a rise in benefits. The test is modified in that the weights are no longer the population weights $\pi^n$ as in equation (21). Equivalently,

$$\sum_{n=1}^{N} \pi^n \bar{w}^n_b - u \left(1 + \frac{\bar{\varepsilon}_{u,b}}{1-u} \right) = \sum_{n=1}^{N} \pi^n (1 - \lambda^n) \bar{w}^n_b.$$

Both sides of this equation are zero when lump-sum taxes are available; when they are not, the right-hand side adds a correction that accounts for the redistributive role unemployment benefits can play. Differences in responsiveness of the reservation wage to benefits $\bar{w}^n_b$ are crucial: workers who gain more from an increase in benefits are those whose reservation wage rises most. Reservation wages emerge again as an element in the diagnostic of unemployment insurance policy, even when an allowance is made for its distributive role.

6.8. Temporary Layoffs

We have assumed throughout this paper that unemployed workers never return to their old employer. In reality, many unemployment spells end when a worker is recalled to her previous job. Following Burdett and Mortensen (1978), Pissarides (1982), and Katz (1986), we allow for this possibility by distinguishing unemployed workers according to their last
wage $w_0$ and allowing that an unemployed worker may be recalled to that wage according to a Poisson process with arrival rate $\mu$. We continue to assume that unemployed workers get new wage offers at rate $\lambda$. Once a new offer is accepted, the worker loses access to her previous wage.

The possibility of recall implies that a worker’s reservation wage is an increasing function of her past wage, $\bar{w}(w_0)$; the precise formula is a slight complication of equation (4) (liquidity) or equation (7) (hand-to-mouth). More importantly for us, the utility of an unemployed worker is still equal to the annuity value of her assets plus her after-tax reservation wage. It follows that if lump-sum transfers are available to address any distributional concerns, all workers agree on maximizing the average after-tax reservation wage. In other words, temporary layoffs introduce ex post heterogeneity, but the test for the optimality of unemployment insurance is identical to the one in an economy with ex ante heterogeneous workers.

6.9. Finite Benefit Eligibility

We have assumed until now that benefits last forever. In reality, most unemployment benefit systems pay only for a specified amount of time; in the United States, this is typically 6 months. After this, a worker must be employed for some months before she is eligible to
collect benefits again. Our analysis readily extends to this case.

When benefits fall with the duration of unemployment, there are two reasons why a worker is willing to take a job: she earns a wage and she resets her eligibility for benefits. To see why this matters, recall our intuition for why the after-tax reservation wage summarizes a worker’s utility: an unemployed worker is indifferent between remaining unemployed and working forever at the after-tax wage $\bar{w} - \tau$; and the utility of a working forever at a constant wage is just the annuity value of assets plus the wage. But an unemployed worker may be willing to take a bad job just long enough to reset her benefit eligibility but unwilling to keep the job forever. That is, she has two different reservation wages, a low one for a job that she can quit when she resets her benefit eligibility and high one for a job that she is willing to keep past that point. Only the higher reservation wage informs us about her utility.

For a newly unemployed worker with maximum eligibility, the two reservation wages are the same. A worker with maximum eligibility takes a job only if she is willing to keep it forever since taking a job and later quitting does not increase her benefit eligibility. This implies that when benefits fall with the duration of unemployment, we should examine the responsiveness of a newly unemployed worker’s after-tax reservation wage to benefits.

\footnote{A similar logic and similar conclusions hold if the search environment is nonstationary for other reasons, for example because the arrival rate of job offers declines during an unemployment spell.}
While we do not know of any direct evidence on how the responsiveness of reservation wages to benefits changes with unemployment duration, search theory suggests that when benefit duration is finite, newly unemployed workers should be more responsive to a change in benefits than workers who have used up part or all of their eligibility (van den Berg, 1990). For example, consider a worker who has been unemployed for a long time and is no longer eligible for benefits. Raising benefits will actually lower such a worker’s reservation wage since it encourages her to accept a mediocre job in order to renew eligibility.

7. A Consumption-Response Test

The goal of this section is to link our model with existing tests for optimal unemployment insurance which are based on the response of consumption to becoming unemployed (Baily, 1978; Gruber, 1997; Chetty, 2006). To do this, we return to the benchmark model of Section 2 and show how we can use the full structure to derive a test linking the decline in consumption during an unemployment spell to risk aversion and the elasticity of unemployment duration with respect to benefits. Our exact test depends on whether workers have liquidity. If

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21 Most of the studies summarized in Section 5 measure the average response of reservation wages to benefits; van den Berg (1990) is an exception.
they do, our test looks at the average drop in consumption during an unemployment spell. In the hand-to-mouth model our test examines the difference in consumption between an unemployed worker and a worker employed at her reservation wage. Each of the extensions analyzed in Section 6 would further modify our consumption-response tests since, in contrast to our behavioral test, these tests build on the full structure of the model including the determinants of consumption and reservation wages.

7.1. Workers with Liquidity

We start with the case when workers have access to financial markets. In this case, our consumption-response test relates the speed of decline in consumption to the elasticity of unemployment duration with respect to unemployment benefits:

**Proposition 5** Assume workers have access to financial markets. If unemployment benefits are chosen optimally, the expected absolute decline in consumption during an unemployment spell is

\[
\frac{1}{\gamma} \frac{\alpha_T + D}{\alpha_T} \frac{\varepsilon_{D,b}}{1 + \varepsilon_{D,b}}.
\]

Equivalently, the expected percentage decline in consumption during an unemployment spell
should be

\[
\frac{1}{\sigma} \frac{\alpha_T + D}{\alpha_T} \frac{\varepsilon_{D,b}}{1 + \varepsilon_{D,b}},
\]

where $\sigma$ is the coefficient of relative risk aversion evaluated at the consumption level at the start of the unemployment spell, $\sigma = \gamma c_u(a_0)$.

The proof is in the appendix.

As in most consumption-response tests, this optimality condition relates the average decline in consumption to the elasticity of duration with respect to benefits, but there are some important differences: (i) we use the partial elasticity $\varepsilon_{D,b}$ whereas previous studies have used the total elasticity defined in footnote 13; (ii) the expression describes the average decline in consumption during an unemployment spell; and (iii) the elasticity expression is somewhat different than in previous work.

These points need clarification. First we use the partial elasticity holding taxes fixed because we believe this corresponds to the empirical evidence on the responsiveness of unemployment duration to unemployment benefits summarized in Section 5. Turning now to point (ii), Baily’s (1978) original analysis and Gruber’s (1997) subsequent work focus on the discrete drop in consumption between employment and unemployment. For example, in his empirical implementation of Baily’s (1978) test, Gruber (1997) uses PSID data to look
at the drop in food consumption for a worker who is employed in year $t$ and unemployed in year $t+1$. Point (iii) is now easily explained. In these papers the optimality condition equates some measure from consumption data to the elasticity of duration. Instead, we find an expression involving the elasticity, but not equal to it. Since the consumption measures differ, it should not be surprising that the optimality conditions call for equating these to different expressions involving the elasticity.

To implement this test, we plug the usual values $r = 0.001$, $T = 225$, $D = 13.4$, and $\varepsilon_{D,b} = 0.88$ into equation (22). In addition, assume that the coefficient of relative risk aversion at the start of the unemployment spell is $\sigma = 2$. Then the model predicts that consumption should decline by 25 percent during an unemployment spell if the unemployment benefit level is optimal. If instead the observed decline in consumption is smaller, a decrease in unemployment benefits would raise welfare.

We know of no direct evidence on the magnitude of the decline in consumption during an unemployment spell, but there is some indirect evidence based on food consumption and expenditure. Gruber (1997) reports that food expenditures fall by about 6.8 percent when a worker is employed one year and unemployed the next. Aguiar and Hurst (2005) find that the unemployed spend 19 percent less on food than do the employed using cross-sectional data; however, because of an increase in time spent on shopping and food preparation, this
translates into only a 5 percent drop in food consumption. Of course, since the income elasticity of food consumption is less than 1, it seems likely that the expenditure on and consumption of other goods declines more than this during an unemployment spell. In addition, even if food consumption could proxy for total consumption, these measures do not generally represent the average decline during a spell.

### 7.2. Hand-to-Mouth Workers

We now turn to hand-to-mouth workers. In this case, our test relates the difference in consumption between a worker at the reservation wage, \( \bar{w}^{\text{aut}} - \tau \), and an unemployed worker, \( b \), to the elasticity of unemployment duration:

**Proposition 6** Assume workers must consume their income in each period. If unemployment benefits are chosen optimally, the difference between the consumption of an employed worker at the reservation wage and the consumption of an unemployed worker is

\[
\frac{1}{\gamma} \log(1 + \varepsilon_{D,b})
\]

Equivalently, the percentage drop in consumption when a worker loses a job paying her reser-
A Consumption-Response Test

The reservation wage should be

\[ \frac{1}{\sigma} \log(1 + \varepsilon_{D,b}), \]

where \( \sigma \) is the coefficient of relative risk aversion evaluated at the consumption level of a worker earning the reservation wage, \( \bar{w}_{aut} - \tau \).

The proof is in the appendix.

Once again, there are three important differences between our condition and most existing formulas based on the response of consumption to unemployment: (i) we use the partial elasticity \( \varepsilon_{D,b} \); (ii) we use the difference between the lowest acceptable level of consumption while employed and consumption while unemployed, rather than the average difference; and (iii) the final expression is slightly different than in previous work, with \( \log(1 + \varepsilon) \) rather than \( \varepsilon \).

Given the usual values of \( \varepsilon_{D,b} = 0.88 \) and \( \sigma = 2 \), the critical question in the hand-to-mouth model is whether the consumption of a worker employed at her reservation wage is 32 percent more than the consumption of unemployed workers. To measure this, we need to know both the drop in consumption following unemployment and the worker’s reservation wage. Data on food expenditures and consumption from Gruber (1997) and Aguiar and Hurst (2005) suggest that many workers are willing to take jobs that raise their
consumption by less than 32 percent, so workers are currently over-insured.

In our view, there are three drawbacks to the consumption-response tests we have presented here. The first is that the moments of the consumption data that we should look at depend on the structure of financial markets. The second is the unavailability of reliable, high frequency consumption data for goods other than food. Finally, the behavioral test is robust to assumptions like the predictability of job loss and the extent of heterogeneity. Introducing these modifications is likely to further change the consumption-response tests.

Chetty (2006) derives another consumption-based test that does not rely heavily on the structure of the model and is identical in the hand-to-mouth and liquidity cases. This test tells us to compare the average lifetime marginal utility of a worker when employed and when unemployed. To implement such a test, we either need a very rich data set on the lifetime path of consumption for a large panel of individuals or we have to make some assumption about the economic environment so that we can extrapolate the desired moments from a limited data set.
8. Conclusions

This paper argues that the after-tax reservation wage measures the well-being of unemployed workers. Any policy that raises the average after-tax reservation wage is therefore beneficial, and the benefit can be measured by the average increase in the after-tax reservation wage. While we have applied this mainly to thinking about optimal unemployment insurance, the insight is more general. For example, Proposition 3 shows that the after-tax reservation wage encodes the value of liquidity. Going beyond this paper, when evaluating any policy towards the unemployed—examples include severance payments, reemployment bonuses, training subsidies, and job search centers—the key question is whether the policy raises the after-tax reservation wage.

We have assumed CARA preferences throughout the body of this paper. This assumption is convenient but probably not essential. Proposition 2 shows that the after-tax reservation wage measures a hand-to-mouth worker’s welfare regardless of her preferences. Moreover, in our companion paper Shimer and Werning (2005), we argue that the behavior of a worker with constant relative risk aversion (CRRA) preferences is quantitatively similar to that of a worker with CARA preferences and the same coefficient of risk aversion if both workers have access to liquidity. Indeed, our intuition for the proof of Proposition 1 explains why this is true: the only reason the after-tax reservation wage would not measure the welfare
of an unemployed worker is if workers are willing to take jobs temporarily but not permanently. While this is a theoretical possibility, we doubt that the phenomenon is quantitatively important.

Finally, our paper implies that a key empirical issue is the responsiveness of the reservation wage to unemployment benefits or other labor market policies. Some existing estimates suggest that reservation wages are very responsive, implying huge gains from increasing unemployment benefit levels. Other estimates are much smaller and imply current benefit levels are too high. An important goal for future research should be to obtain more precise estimates of how labor market policies affect reservation wages.
Appendix A: Proof of Proposition 1

Appendix

A. Proof of Proposition 1

A Convenient CARA Property. We start by proving that

\begin{equation}
V_u(a) = \frac{1}{r} U(c_u(a)) \quad \text{and} \quad V(a, t, w) = \frac{1}{r} U(c(a, t, w)),
\end{equation}

where $V(a, t, w)$ is the value of an employed worker with assets $a$, $t$ periods remaining on the job, and a wage $w$. We prove this for the general case where $r$ and $\rho$ are not necessarily equal. With additively separable utility, we have an Euler equation

\begin{equation}
U'(c_s) = e^{(r-\rho)(s'-s)} \mathbb{E}_u U'(c_{s'}) \quad \forall s' > s,
\end{equation}

where the expectation is taken using all the information available when $c_s$ is chosen. With CARA, $U'(c) = -\gamma U(c)$, so the Euler equation implies per-period utility is a random walk with drift:

\begin{equation}
U(c_s) = e^{(r-\rho)(s'-s)} \mathbb{E}_u U(c_{s'}). 
\end{equation}
Appendix A: Proof of Proposition 1

Now consider the lifetime utility $V_s$ at time $s$ of a worker facing some stochastic future consumption path at all future dates $s'$:

$$V_s = \int_s^\infty e^{-\rho(s' - s)} \mathbb{E}_s U(c_{s'}) ds' = \int_s^\infty e^{-\rho(s' - s)} e^{-(r-\rho)(s' - s)} U(c_s) ds' = \frac{1}{r} U(c_s).$$

The second equation uses equation (25) while the third equation solves the integral.

**Shape of the Consumption and Value Functions.** The shapes of the consumption and value functions follow immediately from equation (24). It is feasible for a worker with assets $a$ to consume $ra$ more than a worker with assets 0 and vice-versa, assuming the two have the same employment duration and wage. This implies

$$c(a, t, w) = ra + c(0, t, w).$$

(26)

Next, consider two employed workers, one at a wage $w$ and another at a wage $w'$. If each has $t$ periods remaining in his job, the present value (as of the end of the previous period) of the difference in earnings is

$$(w - w') \int_0^t e^{-rs} ds \equiv \alpha_t (w - w').$$
Appendix A: Proof of Proposition 1

If the present value difference happens to equal the difference in the two workers’ asset levels, they have the same resources and will behave the same:

\[ c(a, t, w) = c(a + \alpha_t(w - w'), t, w') \, . \]

Combining with equation (26) gives

\[ c(a, t, w) = r(a + \alpha_t(w - w')) + c(0, t, w') \] (27)

for any \( w' \).

Note that if the job is finished, \( t = 0 \) and \( \alpha_0 = 0 \), the worker is unemployed so \( c(a, 0, w) = c(a, 0, w') \) for all \( w \) and \( w' \). It is convenient to define \( c_u(a) \equiv c(a, 0, w) \) as the consumption of a worker who starts a period unemployed and \( V_u(a) \equiv V(a, 0, w) \) as her value function.

**Reservation Wage.** Consider a worker who accepts a job at wage \( w \). Her value function is \( V(a, T, w) \) and so she takes the job if \( V(a, T, w) \geq V_u(a) \). Using equation (24), this is equivalent to \( c(a, T, w) \geq c_u(a) \), which by equation (26) implies a reservation wage rule, independent of assets, satisfying

\[ c(0, T, w) = c_u(0) \, . \] (28)
Appendix A: Proof of Proposition 1

Combine equation (28) with equation (27), evaluated at $w' = \bar{w}$, to get a convenient expression for the consumption of a newly employed worker:

$$c(a, T, w) = r(a + \alpha_T(w - \bar{w})) + c_u(0).$$

(29)

**Behavior of the Employed.** A worker who starts a period with $t \geq 0$ periods remaining in her job faces no uncertainty until the job ends and therefore keeps consumption constant. That is, for any $t > 0$,

$$\frac{dc(a(t), t, w)}{dt} = 0,$$

where $\dot{a}(t) = ra + w - \tau - c(a(t), t, w)$ is the rate of increase in assets. Differentiating gives

$$c_a(a, t, w)(ra + w - \tau - c(a, t, w)) = c_t(a, t, w),$$

where subscripts denote partial derivatives. Note from equation (27) that $c_a(a, t, w) = r$, so this is a differential equation for $c$ as a function of $t$ with terminal condition equation (29). The solution is

$$c(a, t, w) = ra - (w - \tau)(e^{r(T-t)} - 1) + e^{r(T-t)}(\alpha_T(w - \bar{w}) + c_u(0))$$

(30)
Appendix A: Proof of Proposition 1

This provides an alternate expression for $c(a, 0, w)$, which we know is equal $ra + c_u(0)$. Simplifying this equality pins down the constant in the consumption function,

$$c_u(0) = \bar{w} - \tau.$$  \hfill (31)

Substituting equation (31) into equation (29) yields the consumption functions for unemployed and employed workers found in equation (2) and equation (3), while substituting these into equation (24) gives the value of an unemployed worker in equation (1). All that remains is to determine the worker’s reservation wage.

**Behavior of the Unemployed.** Expected marginal utility for an unemployed worker is a Martingale. This implies

$$U''(c_u(a))c_u'(a)\dot{a} + \lambda \int_{\bar{w}}^{\infty} \left(U'(c(a, T, w)) - U'(c_u(a))\right) dF(w) = 0,$$

where $\dot{a} = ra + b - c_u(a) = B - \bar{w}$ using equation (31). Since $U''(c) = -\gamma U'(c) = \gamma^2 U(c)$ and $c_u'(a) = r$, we can rewrite this as

$$\gamma rU(c_u(a))(B - \bar{w}) = \lambda \int_{\bar{w}}^{\infty} \left(U(c(a, T, w)) - U(c_u(a))\right) dF(w).$$
Appendix B: Proof of Proposition 2

Next, use \( U(c_1)/U(c_2) = -U(c_1 - c_2) \) and \( U(0) = -1 \) to get

\[
\gamma r (\bar{w} - B) = \lambda \int_{\bar{w}}^{\infty} (U(c(a, T, w) - c_u(a)) + 1) dF(w).
\]

Simplifying using equation (29) yields equation (4). This completes the characterization of worker behavior in Proposition 1.

B. Proof of Proposition 2

We use a pair of recursive equations. Let \( V_{\text{aut}}^{\text{u}} \) denote the expected utility of an unemployed worker living under autarky and let \( V_{\text{aut}}^{\text{v}}(w, T) \) denote the corresponding value for a newly-employed worker at a wage \( w \). These solve

\[
\rho V_{\text{aut}}^{\text{u}} = U(b) + \lambda \int_{0}^{\infty} \max \{ V_{\text{aut}}^{\text{v}}(w, T) - V_{\text{aut}}^{\text{u}}, 0 \} dF(w)
\]

\[
V_{\text{aut}}^{\text{v}}(w, T) = \int_{0}^{T} e^{-\rho t} U(w - \tau) dt + e^{-\rho T} V_{\text{aut}}^{\text{u}}
\]
Appendix C: Proof of Proposition 3

The flow value of an unemployed worker comes from her current utility $U(b)$. In addition, at rate $\lambda$ she gets a wage draw $w$ which she may accept, giving capital gain $V^\text{aut}(w, T) - V^\text{aut}_u$, or reject. An employed worker in a new job earns $U(w - \tau)$ for the next $T$ periods and then has continuation value $V^\text{aut}_u$.

The Bellman equation for a newly-employed worker implies

$$V^\text{aut}(w, T) - V^\text{aut}_u = \alpha_T (U(w - \tau) - \rho V^\text{aut}_u),$$

since $\rho = r$, so the reservation wage solves $U(\bar{w}^\text{aut} - \tau) = \rho V^\text{aut}_u$. Equivalently, the lifetime utility of an unemployed worker is given by equation (5). Substituting this into the Bellman equation for an unemployed worker gives equation (6) for the reservation wage. □

C. Proof of Proposition 3

We start with two inequalities. At any $w \geq \bar{w}$,

$$-U(B - \bar{w}) - 1 = \exp(\gamma(\bar{w} - B)) - 1 > \gamma(\bar{w} - B),$$

$$1 + U(r \alpha_T(w - \bar{w})) \geq r \alpha_T(1 + U(w - \bar{w})).$$
Appendix C: Proof of Proposition 3

The first equality uses the definition of $U$ and the first inequality uses convexity of the exponential function. To prove the second inequality, note that $1 - y \geq e^{-xy} - ye^{-x}$ when $x > 0$ and $y \in [0, 1]$. When $x = 0$, this is trivially true. Moreover, the derivative of the right-hand-side with respect to $x$ is $y(e^{-x} - e^{-xy})$. Since $y \in [0, 1]$ and $x \geq 0$, $x \geq xy$ and hence $e^{-x} \leq e^{-xy}$, so the right-hand-side is decreasing in $x$. Hence the inequality holds for any positive $x$. If $x = \gamma(w - \bar{w}) > 0$ and $y = r\alpha_T \in [0, 1]$, this is equivalent to the desired inequality.

Now suppose $\bar{w}$ solves equation (4). The previous inequalities imply

$$-U(B - \bar{w}) - 1 > \gamma(\bar{w} - B) = \frac{\lambda}{r} \int_{\bar{w}}^\infty (1 + U(r\alpha_T(w - \bar{w})))dF(w) \geq \alpha_T \lambda \int_{\bar{w}}^\infty (1 + U(w - \bar{w}))dF(w).$$

It is easy to confirm that the first expression is decreasing in $\bar{w}$ and the last expression is increasing, so the solution to

$$-U(B - \bar{w}^{\text{aut}}) - 1 = \alpha_T \lambda \int_{\bar{w}^{\text{aut}}}^\infty (1 + U(w - \bar{w}^{\text{aut}}))dF(w)$$

requires $\bar{w}^{\text{aut}} < \bar{w}$. Under CARA utility, $U(c_1 + c_2) = -U(c_1)U(c_2)$, so this is equivalent to
Appendix D: Proof of Proposition 5

the reservation wage equation (7).

Finally, under financial autarky, equation (5) shows that an unemployed worker’s utility is $U(\bar{w}^{\text{aut}} - \tau)/\rho$. With access to financial markets, equation (1) shows that it is $U(ra + \bar{w} - \tau)/\rho$. The worker is indifferent to the scenarios if $a = (\bar{w}^{\text{aut}} - \bar{w})/r$, a reduction in assets that lowers the worker’s consumption by $ra = \bar{w}^{\text{aut}} - \bar{w}$ in every future period. □

D. Proof of Proposition 5

First take the partial derivative with respect to $b$ of both sides of equation (4), holding fixed the tax rate $\tau$:

$$\gamma(\bar{w}_b - 1) = -\bar{w}_b\alpha_T \lambda \int_{\bar{w}}^{\infty} U'(r\alpha_T(w - \bar{w}))dF(w).$$

Since $U'(c) = -\gamma U(c)$, we can eliminate the integral using equation (4). Solving this expression for $B = b + \tau$ gives

$$B = \bar{w} + \frac{1}{r\alpha_T\gamma} \left( \frac{1}{\bar{w}_b} - \frac{\alpha_T + D}{D} \right).$$

Second, note that while a worker is unemployed, assets fall at rate $\dot{a} = ra + B - c_u(a) = B - \bar{w}$, where the second equality uses equation (2). Since a unit decrease in assets reduces $c_u(a)$ by $r$, consumption falls linearly during an unemployment spell, $\dot{c}_u = r(B - \bar{w})$. Substitute
Appendix D: Proof of Proposition 5

$B$ from the previous equation.

\[
\dot{c}_u = \frac{1}{\alpha T \gamma} \left( \frac{1}{\bar{w}_b} - \frac{\alpha T + D}{D} \right).
\]

This holds for any tax and benefit policy. At the optimal policy, we can eliminate $\bar{w}_b$ using equation (11) to get

\[
D \dot{c}_u = -\frac{1}{\gamma} \frac{\alpha T + D}{\alpha T} \frac{\varepsilon_{D,b}}{1 + \varepsilon_{D,b}}.
\]

Finally, if an unemployment spell lasts for $t$ periods, the drop in consumption is $\dot{c}_u t$. The density of the duration of an unemployment spell is $e^{-t/D}/D$, so the expected drop in consumption during an unemployment spell is

\[
\int_0^\infty \frac{e^{-t/D} \dot{c}_u t}{D} \, dt = D \dot{c}_u.
\]

Combining these equations completes the proof. \(\Box\)
E. Proof of Proposition 6

Totally differentiate equation (6):

\[
\left( U'(\bar{w}^{\text{aut}} - \tau) + \alpha_T \lambda \int_{\bar{w}^{\text{aut}}}^{\infty} U'(\bar{w}^{\text{aut}} - \tau) dF(w) \right) \left( \bar{w}_b^{\text{aut}} + \bar{w}_\tau^{\text{aut}} \tau'(b) - \tau'(b) \right) \\
= U'(b) - \alpha_T \lambda \int_{\bar{w}^{\text{aut}}}^{\infty} U'(w - \tau) dF(w) \tau'(b).
\]

The left-hand-side is zero if benefits are chosen optimally, to maximize \( \bar{w}^{\text{aut}}(b, \tau(b)) - \tau(b) \). Then use equation (10) to eliminate \( \tau'(b) \) from the right-hand-side:

\[
\frac{U'(b)}{\mathbb{E}(U'(w - \tau)|w \geq \bar{w}^{\text{aut}})} = \frac{\alpha_T(1 + \varepsilon_{D,b})}{\alpha_T - D\varepsilon_{D,b}},
\]

where the denominator on the left-hand-side is the expectation of the marginal utility of consumption conditional on the wage drawn from \( F \) exceeding \( \bar{w}^{\text{aut}} \).

Under CARA utility, this simplifies further since the ratio of marginal utility is the same as the ratio of utility,

\[
\frac{U(b)}{\mathbb{E}(U(w - \tau)|w \geq \bar{w}^{\text{aut}})} = \frac{\alpha_T(1 + \varepsilon_{D,b})}{\alpha_T - D\varepsilon_{D,b}}.
\]
Appendix E: Proof of Proposition 6

Since equation (6) implies

\[ \frac{U(b)}{\mathbb{E}(U(w - \tau) \mid w \geq \bar{w}^{\text{aut}})} = \frac{\alpha_T}{(D + \alpha_T) U(\bar{w}^{\text{aut}} - \tau)} D, \]

the previous two equations give

\[ \frac{U(b)}{U(\bar{w}^{\text{aut}} - \tau)} = 1 + \varepsilon_{D,b}. \]

Since \( U(c) = -e^{-\gamma c} \), the result follows immediately. \( \square \)
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