How Many Designs Are There and Where Are They?

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- Design spaces
- Design space (D)   representation

  (   ) X transformation (Γ)
- D = Γ (   )
- What representations can there be?
- What transformations can there be?

Where are all the designs?

- Are they in ?
  [embedded vs disparate]
- Are they in Γ ?
  [homogeneous vs inhomogeneous]
- Are they in $D = \Gamma(\ )$?
  [possible emergence in D]

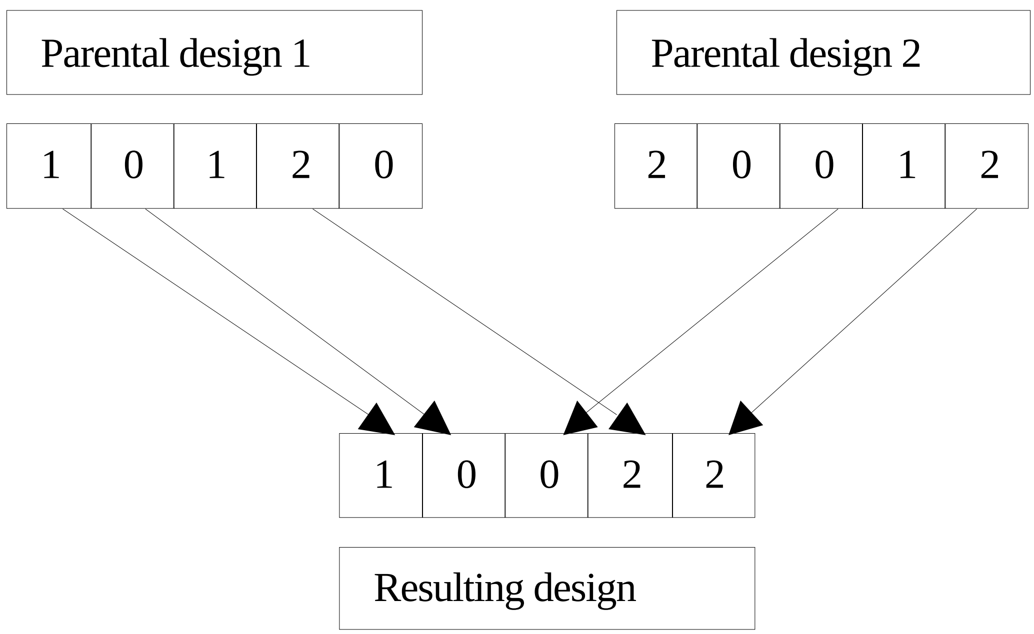
The goal is to construct such computational operators that are able to use input points in a current design space and to generate output points in a superspace - an expanded design space that encompasses the current design space

Design transformation processes

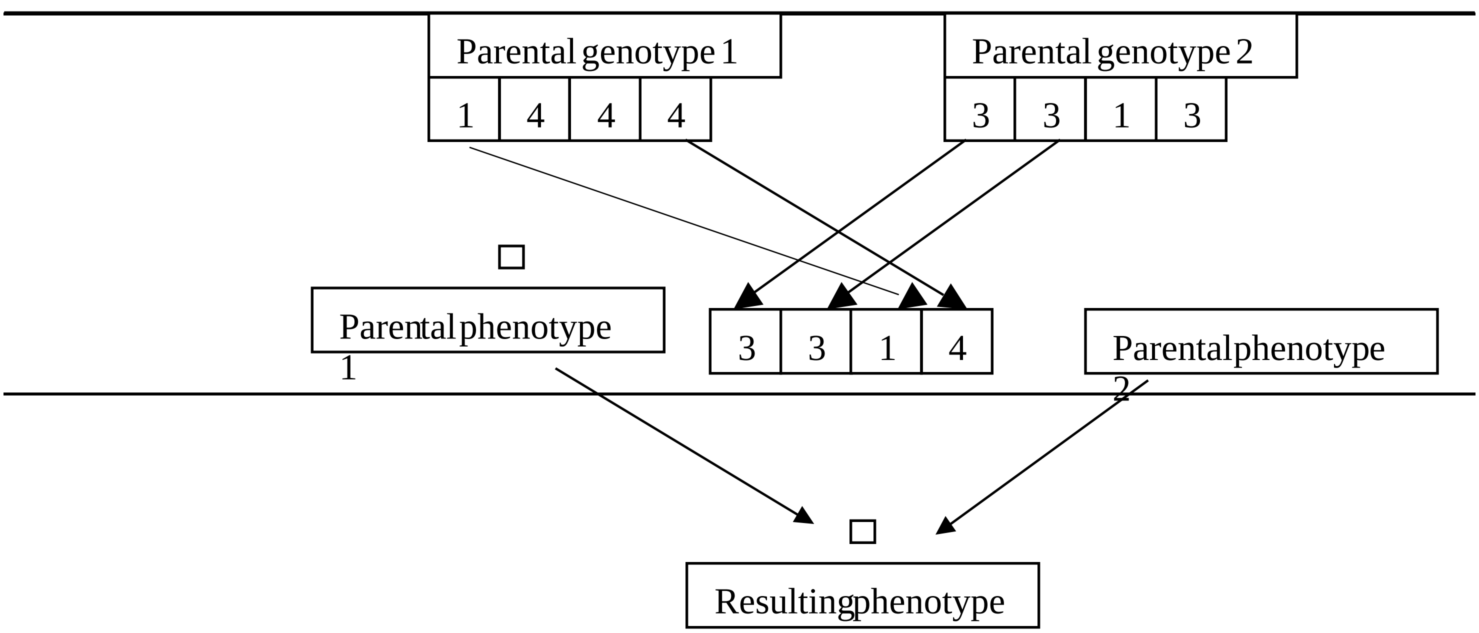
All replacement transformations

- **combination**
- **mutation**
- **analogy**
- **first principles**
- **emergence**

Genetic crossover as combination operator



Transformation = Genetic crossover for homomorphic genotype and phenotype spaces



Genotypic interpolation $\longleftrightarrow$ Phenotypic interpolation

- $C(g_1, g_2) \rightarrow g_c$
- $g_c(t) = f(t) g_1 + (I - f(t)) g_2$
- $g_c(t) = c_1(t) g_1 + c_2(n-t) g_2$,
- $g_c(t) = FG(t, g_1, g_2)$
  $FG(0, g_1, g_2) = g_1$
  $FG(1, g_1, g_2) = g_2$

- $C(p_1, p_2) \rightarrow p_c$
- $p_c(t) = \underline{f}^c(t) p_1 + \underline{q}^c(n-t) p_2$
- $p_c(t) = \underline{f}(t) p_1 + \underline{q}(n-t) p_2$
- $p_c(t) = F(t, p_1, p_2)$
  $F(0, p_1, p_2) = p_1$
  $F(1, p_1, p_2) = p_2$

GA's crossover is equivalent to the random sampling of a particular case of phenotype-phenotype interpolation

$$p_c = \mathbf{F}(\mathbf{t}, p_1, p_2),$$

$$\mathbf{t} = [0,1] \qquad \mathbf{F}(0, p_1, p_2) = p_1$$

$$\mathbf{F}(1, p_1, p_2) = p_2$$

Possible generalizations

GENETIC CROSSOVER

Phenotypic crossover-based interpolation

$\mathbf{p_c}=\mathbf{F}(t,\mathbf{p}_1,\mathbf{p}_2)$, $t=[0,1]$

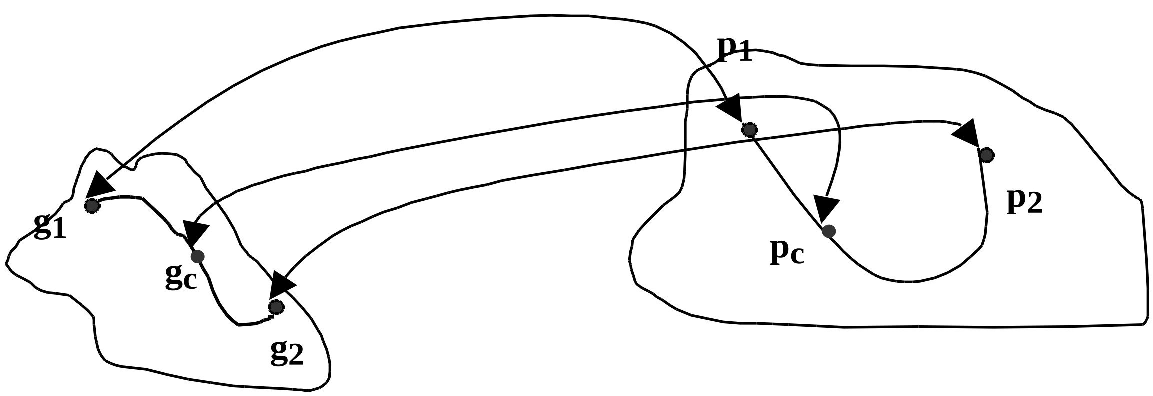
Phenotypic crossover-based extrapolation

$\mathbf{p_c}=\mathbf{F}(t,\mathbf{p}_1,\mathbf{p}_2)$, $t=[-a,0)$ $t=(1,a]$

Phenotypic non crossover-based interpolation and extrapolation

$\mathbf{p_c}=\mathbf{FM}(t,\mathbf{p}_1,\mathbf{p}_2)$, $t=[-a,a]$ $a >=1$

Crossover as an interpolation

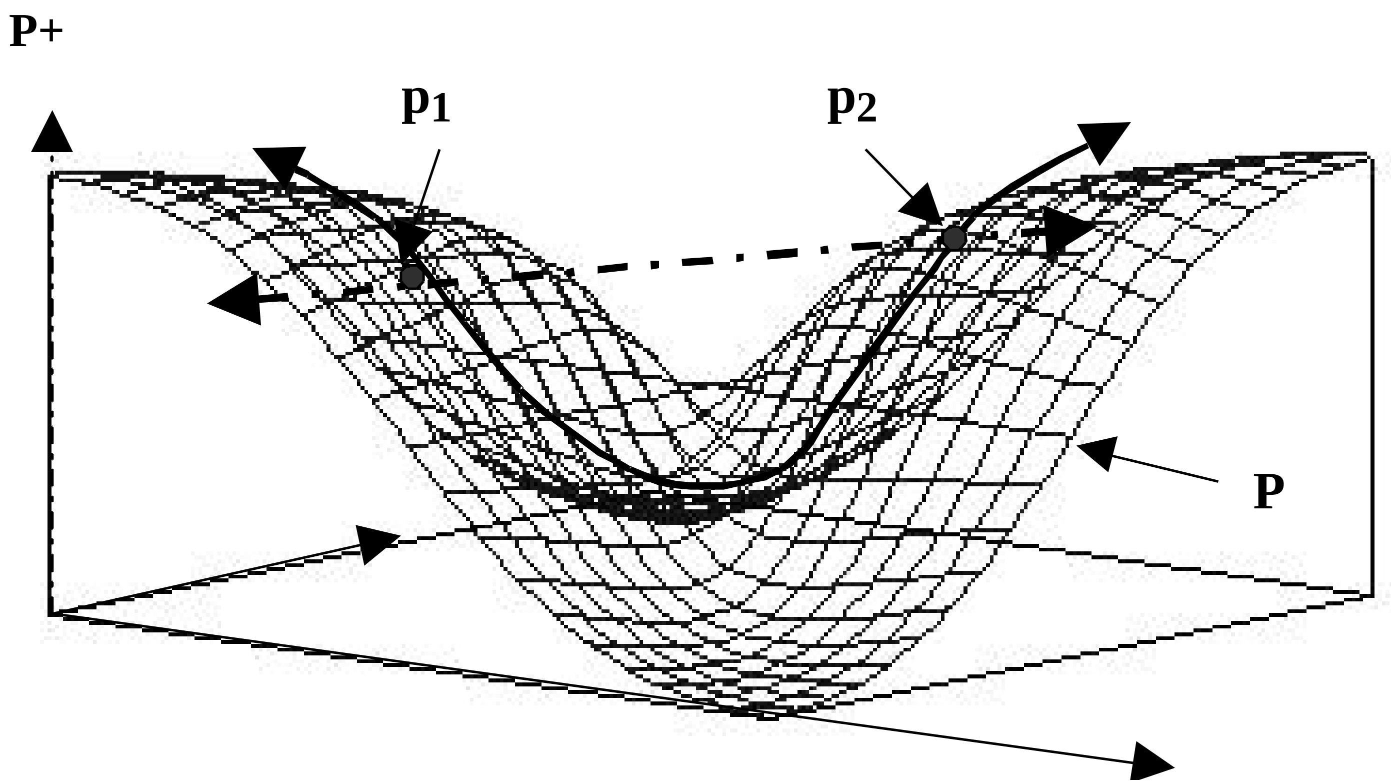


Phenotypic space P

Genotypic space G

$$\mathbf{C}(\mathbf{g}_1,\mathbf{g}_2) \rightarrow \mathbf{g}_\mathbf{c}$$

$$\mathbf{C}(\mathbf{p}_1,\mathbf{p}_2) \rightarrow \mathbf{p}_\mathbf{c}$$


P+
p1
p2
P

Example

- Design space of 2-D shapes
- Functional representation

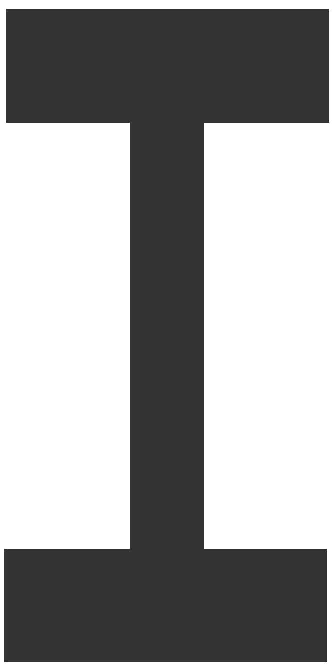
  $F(x)>=0$, x belong to shape S

  $F(x)<0$, x outside of S
- Phenotype-phenotype interpolation

  $F_c(t,x')=tv(x')\ F_1(x') + (1-t)w(x')\ F_2(x')$,

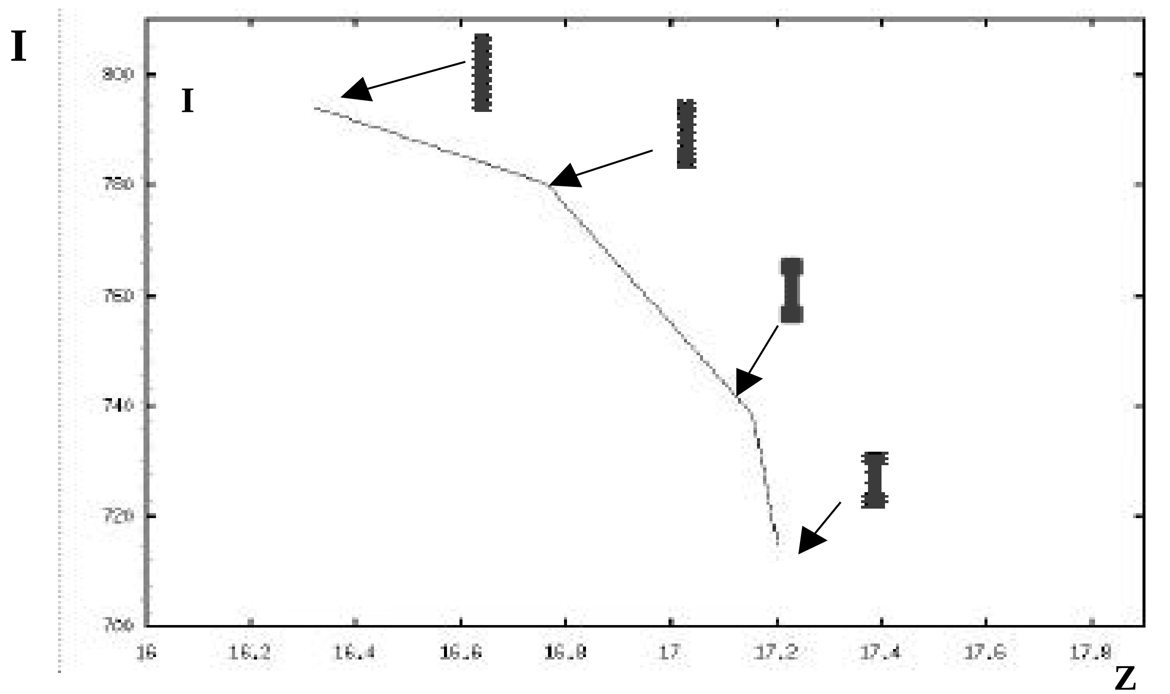
  $x'(t,x): D \rightarrow D$, $t =[0,1]$

Designing a beam

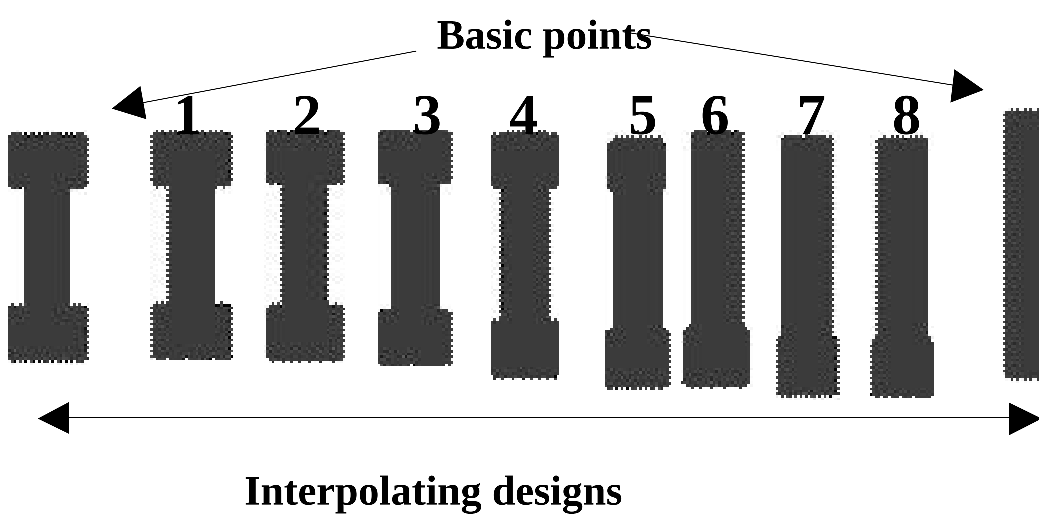


- FITNESS=(I,Z)
- I is the moment of inertia
- Z is the section modulus

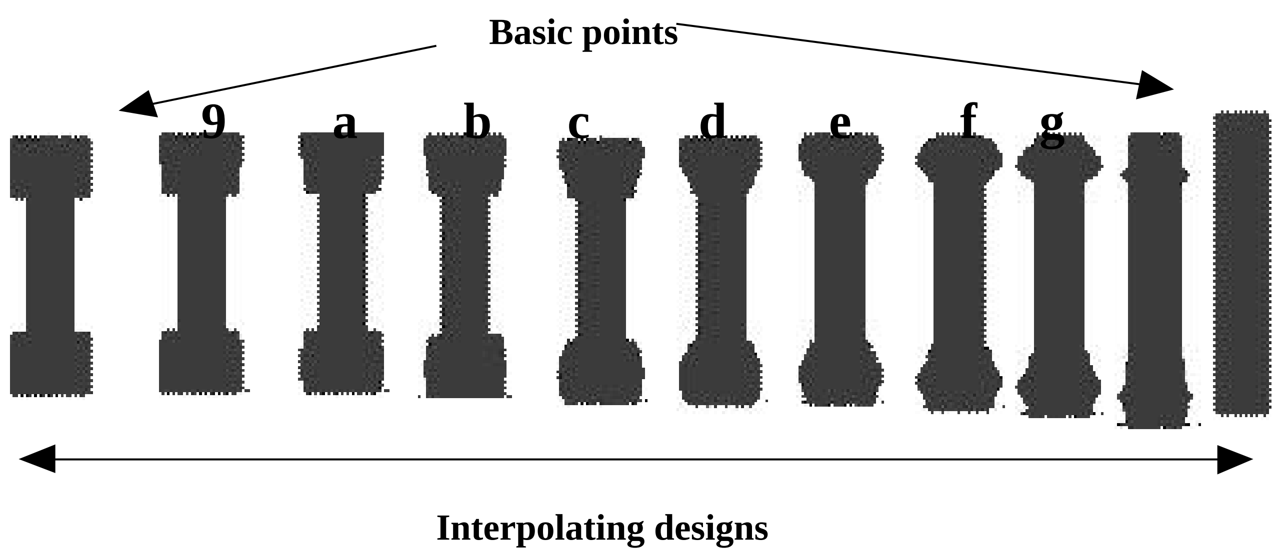
Pareto optimal set from standard GA



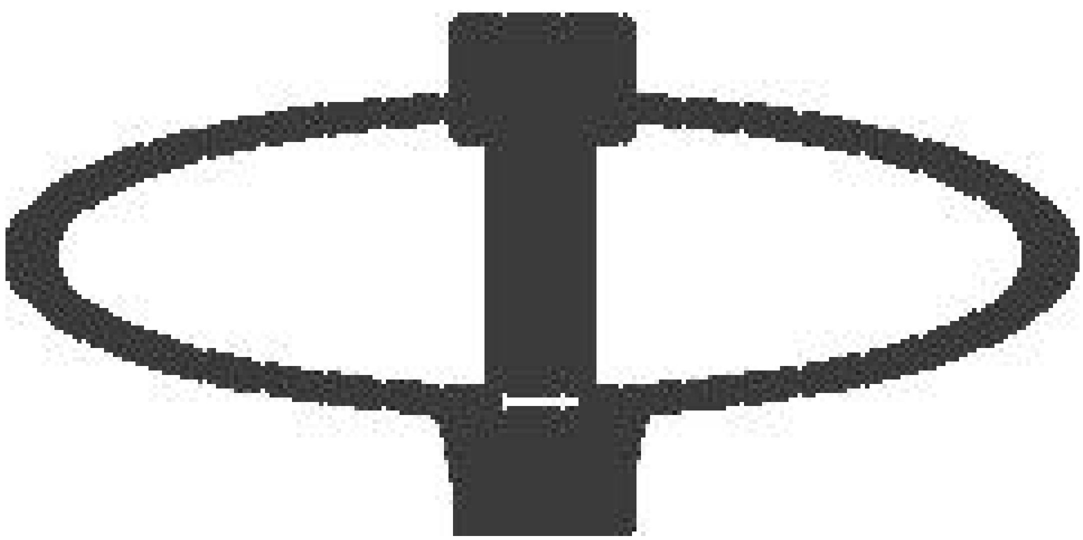
Linear interpolation



Non-linear interpolation



Unexpected design from a nonlinear interpolation



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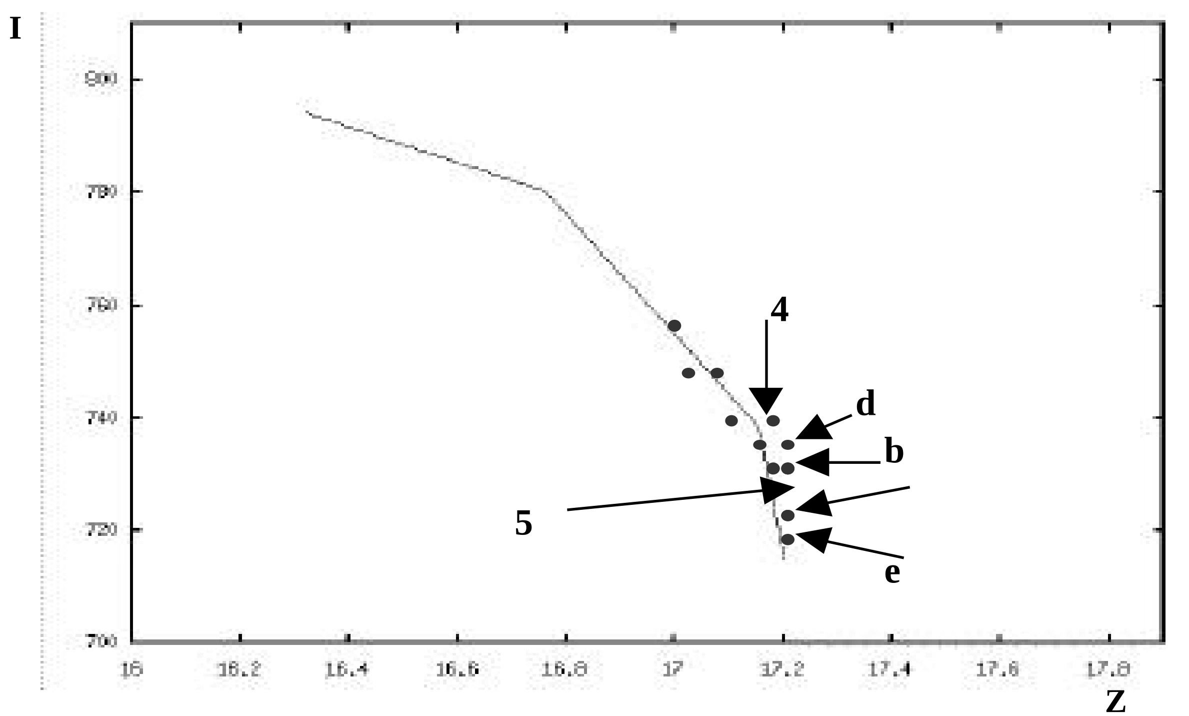
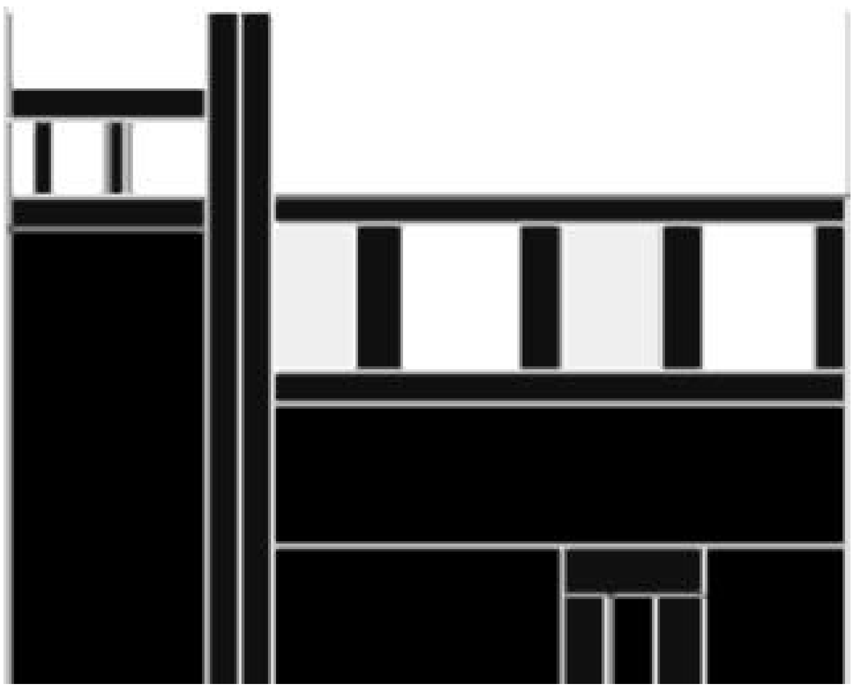
Pareto optimal set and interpolations



Interpolation



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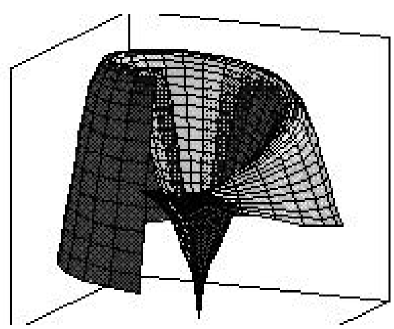

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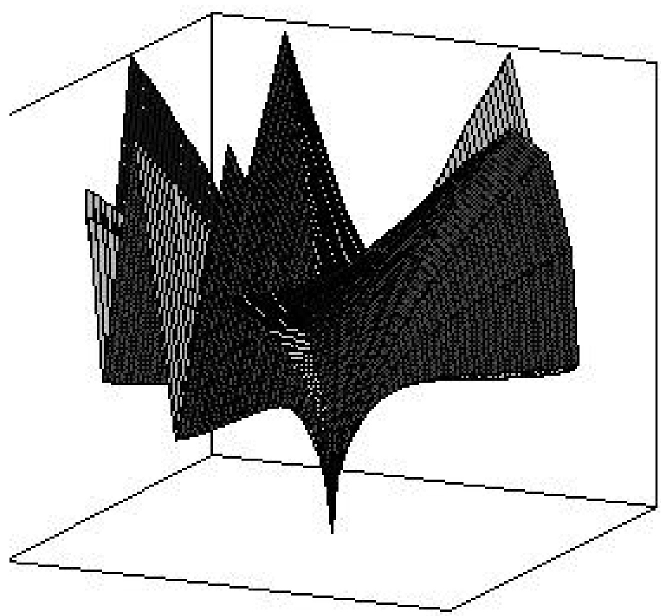
Extrapolation



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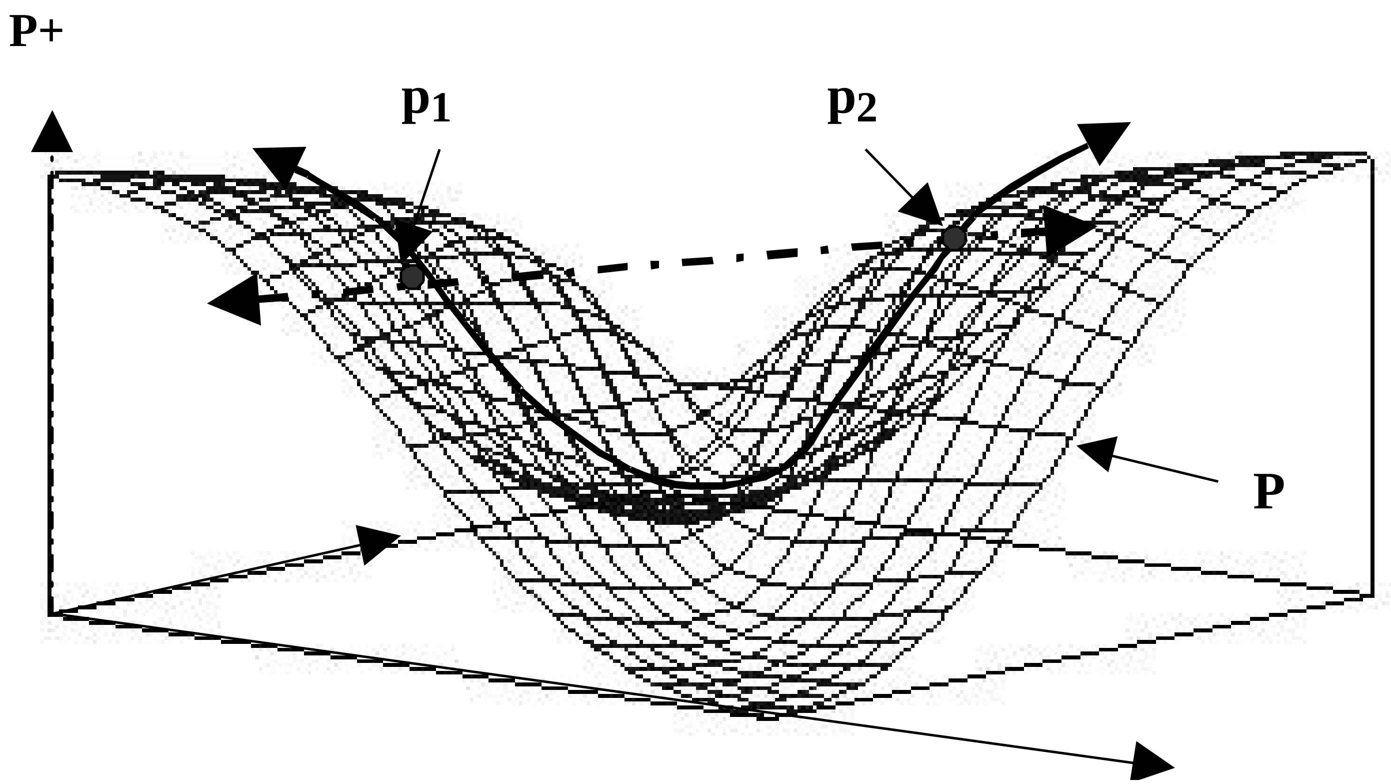
P+
p1
p2
P

Combination is an exploration process, ie generates new design spaces

Generalisation of genetic crossover produces unexpected results due to unexpected design spaces being generated

Rich area of study