

# Economic Activity of Firms and Asset Prices\*

Leonid Kogan<sup>†</sup>    Dimitris Papanikolaou<sup>‡</sup>

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## Abstract

In this paper we survey the recent research on the fundamental determinants of stock returns. These studies explore how firms' systematic risk and their investment and production decisions are jointly determined in equilibrium. Models with production provide insights into several types of empirical patterns, including: i) the correlations between firms' economic characteristics and their risk premia; ii) the comovement of stock returns among firms with similar characteristics; iii) the joint dynamics of asset returns and macroeconomic quantities. Moreover, by explicitly relating firms' stock returns and cash flows to fundamental shocks, models with production connect the analysis of financial markets with the research on the origins of macroeconomic fluctuations.

Keywords: General equilibrium, asset pricing, investment, firm characteristics, stock returns

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<sup>†</sup>NBER and MIT Sloan School of Management, lkogan@mit.edu

<sup>‡</sup>Kellogg School of Management, d-papanikolaou@kellogg.northwestern.edu

# 1 Introduction

In this survey we review the recent developments in the literature that connects the behavior of asset prices to economic activities of firms. The empirical literature has uncovered several patterns in the relations between firm characteristics and stock returns. A few examples of firm characteristics that are correlated with expected stock returns are: market capitalization (Banz (1981)); the market-to-book ratio (Rosenberg, Reid, and Lanstein (1985)); capital expenditures and profitability (see Fama and French (2006) for a literature review). Furthermore, there is evidence of strong comovement in the cross-section of stock returns. As a result, sorting firms on various characteristics generates empirical return factors that help account for the cross-sectional differences in expected stock returns (e.g. Fama and French (1993)). To understand how these and similar patterns arise and their link to the broader properties of the economy, we need to relate firms' stock returns and cash flows to the economic fundamentals, such as the firms' production and investment technologies, their input and output characteristics, macro-economic conditions, agency and asset market frictions, etc. To do so, we need an explicit description of firms' production and investment decisions within asset pricing models.

The fundamental theorem of asset pricing (e.g., Dybvig and Ross (2003)) relates assets' cash flows  $D$  to their prices  $P$  using the stochastic discount factor  $\pi$  as:

$$P_t = E_t \left[ \sum_{s=t+1}^T \frac{\pi_s}{\pi_t} D_s \right] + E_t \left[ \frac{\pi_T}{\pi_t} P_T \right]. \quad (1)$$

This relation links the risk premia in asset returns to their systematic risk, which is captured by the return covariance with the stochastic discount factor:

$$E_t [r_{t+1}] - r_{f,t} = -(1 + r_{f,t}) cov_t \left( r_{t+1}, \frac{\pi_{t+1}}{\pi_t} \right), \quad (2)$$

where  $r_{t+1}$  is the return on a risky asset, and  $r_{f,t}$  is the return on the riskless asset over the same time period. Equation (2) follows directly from the absence of arbitrage without any assumptions on the behavior of households or firms beyond the monotonicity of preferences.

The economic content of (1) is in the explicit relations between the cash flows, the stochastic discount factor  $\pi$ , and the state of the economy. Existing models with production typically take one of two approaches. The partial equilibrium approach takes the specification of  $\pi$  as given, and models firm's endogenous investment decisions. As a result, we can learn which firm characteristics explain the cross-sectional differences in systematic risk of cash flows and stock returns. The general equilibrium approach includes a household sector and thus fully endogenizes the joint distribution of firms' cash flows and the stochastic discount factor. This framework imposes a higher standard of internal consistency than the partial equilibrium approach, since asset prices and macroeconomic quantities are determined endogenously and thus depend on a common set of structural parameters.

General equilibrium models with production nest the endowment economy models reviewed in Campbell (2003). These consumption-based models work off the households' optimizing behavior. For instance, in a frictionless economy, households have complete flexibility in using financial assets to allocate consumption across states of nature, and therefore their consumption choices reveal a valid stochastic discount factor. Models based on endowment economies can tell us whether the pricing relations (1) and (2), applied to the existing financial assets, is consistent empirically with a particular model of household behavior and consumption dynamics, but they cannot explain why some assets have riskier cash flows than others. Thus, while models with production require more explicit assumptions about the economic environment than the traditional consumption-based models, they address a wider range of questions.

We review several key areas of current research in Sections 2 and 3. In Section 4, we outline several directions for future research.

## 2 Aggregate Asset Markets

A substantial portion of the asset-pricing literature attempts to account for the key empirically properties of aggregate asset markets, including the high Sharpe ratio of stock returns, the high volatility of stock returns, and the low and stable risk-free rate, by using models with relatively standard preferences and, preferably, realistic preference parameters. Most of the models have one of the following three features, first introduced in the endowment-economy setting: time-variation in the risk aversion of the representative household (e.g., Constantinides (1990), Campbell and Cochrane (1999)); low-frequency movements in consumption growth (e.g., Parker (2003), Bansal and Yaron (2004)); or rare disasters (Rietz (1988), Barro (2009)). The representative-firm equilibrium models with production deal primarily with the same set of empirical asset pricing facts. However, in addition to their asset pricing implications, these models have nontrivial implications for quantities, such as aggregate consumption and investment. This further limits the set of plausible explanations of observed patterns in asset prices.

We use a version of the stochastic growth model to frame our discussion of the literature (see, e.g., Jermann (1998), Boldrin, Christiano, and Fisher (2001)). We start by describing the production sector, and then introduce households. In our discussion, we emphasize the role of investment adjustment costs and the interaction between technology and preferences in generating a realistic joint dynamics of asset prices and macroeconomic quantities.

All uncertainty in the economy is captured by a stationary Markov process  $\omega_t$ . The financial markets are complete and frictionless, and  $\pi_t$  denotes the stochastic

discount factor (SDF).

## 2.1 Firms

The productive sector consists of a representative competitive firm that produces a single output using physical capital  $K$  and labor  $L$ :

$$Y_t = X_t K_t^\alpha L_t^{1-\alpha}, \quad (3)$$

where  $X_t = X(\omega_t)$  describes the firm profitability process.

The firm accumulates capital through investment:

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (4)$$

where  $\delta$  is the constant depreciation rate. Increasing the capital stock by  $I_t$  units costs

$$\phi(I_t/K_t) K_t, \quad (5)$$

where  $\phi(\cdot)$  is a convex function that allows for decreasing returns to scale in capital installation, i.e., *adjustment costs*. For simplicity, we assume that  $\phi$  is a deterministic function, but it can incorporate additional technological shocks, for instance capital-embodied technical change.

The firm maximizes its market value:

$$V(\omega_0, K_0) = \max_{\{I_s, L_s\}} E_0 \left[ \sum_{s=0}^{\infty} \pi_s D_s \right], \quad (6)$$

where the dividends  $D_t$  are given by

$$D_t = Y_t - \phi\left(\frac{I_t}{K_t}\right) K_t - W_t L_t. \quad (7)$$

$W_t = W(\omega_t)$  is the equilibrium wage process. Without loss of generality, we assume

that the firm is financed by a single share of equity, and refer to the firm value  $V(\omega_0, K_0)$  as its cum-dividend stock price.

The value of the firm satisfies the Bellman equation:

$$V(\omega_t, K_t) = \sup_{I_t, L_t} \left\{ \left[ X_t K_t^\alpha L_t^{1-\alpha} - \phi \left( \frac{I_t}{K_t} \right) K_t - W_t L_t \right] + \text{E}_t \left[ \frac{\pi_{t+1}}{\pi_t} V(\omega_{t+1}, K_{t+1}) \right] \right\}, \quad (8)$$

subject to the capital accumulation constraint (4).

In this setting, due to the constant returns to scale in the production and investment technologies, the marginal value of capital  $\partial V(\omega_t, K_t)/\partial K_t$  is equal to its average value  $V(\omega_t, K_t)/K_t$ . In the language of the  $q$ -theory of investment (e.g., Tobin (1969), Abel (1981), and Hayashi (1982)), the marginal  $q$  equals the average (Tobin's  $q$ ).

The first-order optimality condition of the firm's optimal investment problem<sup>1</sup> relates the investment rate  $I_t^*/K_t$  to the firm value and the state vector as

$$\phi' \left( \frac{I_t^*}{K_t} \right) = \text{E}_t \left[ \frac{\pi_{t+1}}{\pi_t} \frac{\partial V(\omega_{t+1}, K_{t+1})}{\partial K_{t+1}} \right] = \text{E}_t \left[ \frac{\pi_{t+1}}{\pi_t} \frac{V(\omega_{t+1}, K_{t+1})}{K_{t+1}} \right] = \frac{P_t}{K_{t+1}}, \quad (9)$$

where  $P_t$  is the ex-dividend value of the firm at time  $t$ ,  $P_t = \text{E}_t [(\pi_{t+1}/\pi_t)V(\omega_{t+1}, K_{t+1})]$ . The relation (9) between the optimal investment rate of the firm and its marginal  $q$  is a classic example of a theoretical relation between firms' economic activity and financial asset prices.

Equation (9) reveals that investment adjustment costs are essential for the model to produce empirically plausible volatility of aggregate stock returns (e.g., Rouwenhorst (1995)). If  $\phi(I_t/K_t) = I_t/K_t$ , then the unit price of capital is equal to one,  $P_t = K_{t+1}$ . This smooth price of capital is at odds with the data, where the market value of capital is much more volatile than its quantity.

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<sup>1</sup>We assume the interior solution,  $I_t^* > 0$ , and sufficient regularity of the problem ingredients for the value function to be smooth.

To generate realistic stock return volatility, the literature typically assumes convex adjustment costs. One common specification is

$$\phi\left(\frac{I}{K}\right) = \frac{a}{\lambda + 1} \left(\frac{I}{K}\right)^{\lambda+1}, \quad (10)$$

where parameter  $\lambda$  is inversely related to the elasticity of the investment rate with respect to the marginal value of capital (e.g., Jermann (1998)).

Convex adjustment costs reduce the elasticity of the supply of capital. Thus, shifts in the demand for capital are absorbed mostly by changes in the equilibrium price of capital, rather than the quantity of investment (see figure 1). The adjustment cost curvature  $\lambda$  affects the equilibrium dynamics of stock returns and investment rates, which exemplifies the endogenous link between asset prices and macroeconomic quantities in general equilibrium models.

Equations (8) and (9) can be used as partial-equilibrium restrictions to relate stock prices to productivity shocks and the SDF. However, the SDF is endogenous in general equilibrium. We close the model by explicitly modeling the household sector and thus relate the equilibrium stock price and investment explicitly to the exogenous productivity shocks.<sup>2</sup>

## 2.2 Equilibrium

We next introduce a representative household, which demands capital to support its consumption over time. The representative household owns the equity of the

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<sup>2</sup>Some insight into the relation between the equilibrium investment rate and the SDF can be obtained solely from the firm's optimality conditions, without fully specifying the economic environment. Under the production function (3), firms have limited flexibility in allocating output across states, and therefore (9) relates optimal investment only to the moments of the SDF, but not to its realizations state-by-state. Several papers (e.g., Cochrane (1988), Cochrane (1993), Belo (2010), Jermann (2010)) develop alternative specifications of the production function to recover the SDF directly from the firms' investment and production decisions.

representative firm. It behaves competitively and maximizes the utility of lifetime consumption  $U(\{C_0, C_1, \dots\})$  subject to its budget constraint:

$$E_0 \left[ \sum_{s=0}^{\infty} \pi_s (D_s - C_s) \right] = 0. \quad (11)$$

The household also supplies inelastically one unit of labor  $L_t = 1$ .

The equilibrium consumption and investment processes are linked by the market clearing condition:

$$C_t = Y_t - \phi \left( \frac{I_t}{K_t} \right) K_t - W_t L_t. \quad (12)$$

Thus, assumptions on household preferences affect the joint dynamics of both consumption and asset prices. For instance, consider that one way to produce high Sharpe ratios of asset returns in an exchange economy with CRRA preferences is to assume that the representative household has a high degree of risk aversion. However, Benninga and Protopapadakis (1990) show that in economies with production, high risk aversion tends to reduce consumption growth volatility, which makes the equilibrium equity premium less responsive to the household's risk aversion.

In general, the interaction between the equilibrium behavior of asset prices and real quantities is not trivial. Tallarini (2000) describes one significant exception. He considers a standard real business cycle model without investment adjustment costs and with recursive preferences (Epstein and Zin (1989)). He finds that in the case where the elasticity of intertemporal substitution is equal to one, risk aversion has a substantial effect on asset price moments but a much weaker effect on consumption smoothing. However, this separation between quantities and asset prices is only approximate and need not hold in more general settings.

Jermann (1998) and Boldrin et al. (2001) combine adjustment costs with habit formation in preferences. As shown by Constantinides (1990) and Campbell and

Cochrane (1999), habit-formation preferences increase the volatility of households' marginal utility of consumption, allowing for high Sharpe ratios of asset returns despite the low volatility of consumption growth. In a model with production, combining habit formation with adjustment costs helps increase the volatility of the price of capital.

Habit formation enhances the households' propensity to smooth their consumption. Thus, a negative productivity shock translates mostly into a reduction in equilibrium investment, rather than consumption, or into a relatively large shift of the representative household's demand schedule (see figure 1). Convex adjustment costs, in turn, reduce the supply elasticity of capital and ensure that the demand shift is absorbed mostly by a change in the equilibrium price of capital, not the quantity of investment.

General equilibrium models help us analyze the subtle properties of aggregate consumption that are important for asset pricing but are difficult to estimate with purely statistical methods. Consider, for instance, low-frequency fluctuations in consumption growth emphasized by Bansal and Yaron (2004) and related studies on long-run consumption risk. Several recent papers, e.g., Campbell (1994), Kaltenbrunner and Lochstoer (2010), Campanale, Castro, and Clementi (2010), Croce (2010), and Kung and Schmid (2011), identify nontrivial restrictions on the firms' production and investment technologies that one must impose to reproduce the low-frequency consumption dynamics assumed by endowment models with similar household preferences.

We follow Campbell (1994) and combine equation (9) with the household's optimality condition under the CRRA utility function,  $\rho^t C_t^{-\gamma} = \pi_t$ , and then log-linearize around the (de-trended) non-stochastic steady state. Expected log consumption

growth is then proportional to the marginal product of capital

$$E_t [\Delta \ln C_{t+1}] = \text{const} + \psi \left( E_t \left[ \ln \frac{\partial V(\omega_{t+1}, K_{t+1})}{\partial K_{t+1}} \right] - \ln \phi' \left( \frac{I_t^*}{K_t} \right) \right), \quad (13)$$

where the coefficient of proportionality  $\psi = 1/\gamma$  is the elasticity of intertemporal substitution (EIS). Campbell (1994) shows that in the absence of adjustment costs ( $\phi' = 1$ ) and the limit  $\psi \rightarrow 0$ , consumption growth is i.i.d. If households are unwilling to substitute consumption across time, a version of the permanent income hypothesis holds. More generally, the low-frequency component of the consumption process depends on the structural features of the economy, including preferences, the convexity of adjustment costs and the properties of the aggregate productivity process. Thus, an explicit model of production allows us to evaluate the structural assumptions necessary to generate the equilibrium consumption process with the desired low-frequency dynamics.

Disaster risk is a powerful mechanism for generating high and time-varying risk premia. Models featuring disaster risk have been prominent in the recent consumption-based asset pricing literature (e.g., Barro (2009)). Gourio (2011b) explores the effects of time-varying disaster risk on prices and quantities in a general-equilibrium model. In his model, disasters affect both the aggregate productivity and the aggregate capital stock:

$$\begin{aligned} \Delta \ln X_{t+1} &= \mu + \sigma \epsilon_{t+1} + (1 - u_{t+1} b_X), \\ K_{t+1} &= [(1 - \delta)K_t + I_t] (1 - u_{t+1} b_K), \end{aligned}$$

where  $\epsilon_{t+1}$  are IID standard normal shocks, and  $u_{t+1}$  are the independent disaster shocks equal to one with conditional probability  $p_t$  and zero otherwise. The parameters  $b_X$  and  $b_K$  capture the magnitude of the impact of disaster shocks on productivity

and the capital stock respectively.

In this model, disasters raise the equity premium, similar to an endowment economy setting. Moreover, fluctuations in the conditional probability of disasters affect both the risk premia of the financial assets, and the consumption and employment decisions of the representative household. The optimality conditions of the firm are

$$\phi' \left( \frac{I_t^*}{K_t} \right) = \mathbb{E}_t \left[ \frac{\pi_{t+1}}{\pi_t} \frac{\partial V(\omega_{t+1}, K_{t+1})}{\partial K_{t+1}} (1 - u_{t+1} b_K) \right]. \quad (14)$$

With an exogenous stochastic discount factor, an increase in the disaster probability has a negative effect on the stock price and the firm's investment rate. However, in this model the equilibrium feedback effect is important. The SDF is endogenous, hence the effect of disaster risk on investment depends on the representative household's preferences. Gourio shows that disaster risk has a negative effect on investment if the EIS exceeds one. Moreover, the model has a number of testable implications for prices and quantities. As the disaster probability rises, so do the conditional equity premium and the implied volatilities of equity options, while aggregate investment, hours, and output decline.

### 2.3 Remaining Challenges

General equilibrium models with production yield rich testable implications regarding the joint properties of asset returns and aggregate consumption and investment. As performance of these models improves, we see the emphasis in this branch of the literature shifting from matching a standard set of moments towards deriving and testing new implications of these models. Moreover, since the joint dynamics of prices and quantities is driven by a deeper layer of structural shocks, we expect that research in this area will be intimately connected with the broader study of the sources of aggregate fluctuations.

### 3 The Cross-Section of Firms

Much of the asset-pricing literature examines the cross-sectional properties of stock returns. The central focus in this area has been on understanding the sources of differences in risk premia among firms, including the relations between risk premia and firm characteristics. These studies are also making progress on the question of what determines return comovement among firms with similar characteristics, and what this comovement reveals about the broader properties of the economy.

The literature on expected stock returns and firm characteristics considers several sources of firm heterogeneity. Many of these models assume that all firms have identical long-run properties but differ from each other at each time point because of the firm-specific productivity shocks. Other models focus on the structural differences between firms, emphasizing for instance persistent cross-sectional differences in the firms' technologies.

#### 3.1 Firm Characteristics and Stock Returns

##### A Reduced-Form Relation

To begin, we relate expected stock returns to firm characteristics in a partial-equilibrium neoclassical model. We consider the environment described in Section 2 and interpret the firm model as a model of an individual firm.

The key property of this model is the  $q$ -theory equation (9). This equation connects the investment rate of the firm to its market value normalized by its capital stock. Using equations (5) and (9),

$$\ln a + \lambda \ln(I_t^*/K_t) = \ln \frac{P_t}{K_{t+1}} = \ln \frac{P_t}{D_t} - \ln \frac{D_{t+1}}{D_t} + \ln \frac{D_{t+1}}{K_{t+1}}. \quad (15)$$

Next, we apply the Campbell and Shiller (1988) decomposition to the log of the

price-dividend ratio:

$$\ln \frac{P_t}{D_t} \approx \text{const} + E_t \left[ \sum_{j=1}^{\infty} \rho^{j-1} (\Delta \ln D_{t+j} - \ln R_{t+j}) \right], \quad (16)$$

where  $R_t$  denotes the gross stock return, and the constant  $\rho$  depends on the average price-dividend ratio. Thus, we establish a relation between the firm's investment rate and its expected stock returns and profitability:

$$\lambda \ln(I_t^*/K_t) \approx \text{const} + E_t \left[ \ln \frac{D_{t+1}}{K_{t+1}} + \sum_{j=1}^{\infty} (\rho^j \Delta \ln D_{t+j+1} - \rho^{j-1} \ln R_{t+j}) \right]. \quad (17)$$

The first-order condition (17) expresses a relation between three endogenous variables: the optimal investment rate, the expected future firm profitability (measured by a firm's dividends relative to its capital stock), and the expected future stock return. One interpretation of (17) is that, *ceteris paribus*, firm's investment is positively related to its future expected profitability and negatively related to the future expected discount rates. This qualitative relation motivates a number of empirical studies that analyze patterns of cross-sectional correlation between firm's investment, profitability, and expected stock returns. Examples include, among others, Titman, Wei, and Xie (2004), Anderson and Garcia-Feijo (2006), Fama and French (2006), Li, Livdan, and Zhang (2009), and Chen, Novy-Marx, and Zhang (2010).

Cochrane (1991) uses the  $q$ -theory equation (9) and arbitrage arguments to show that the return on the marginal unit of physical investment and the stock market return must coincide state by state. This result has a weaker implication that, under additional restrictions on the model specification, stock returns are positively correlated with changes in investment rate. Cochrane finds empirical support for this prediction in the aggregate time-series data.

Liu, Whited, and Zhang (2009) explore the same theoretical idea at the level of

individual firms. They find supporting evidence for a weaker form of the theoretical prediction, that the conditional expectations of investment returns are positively related to the conditional expectations of stock returns in the cross-section of firms. However, they also find that the relation between realized investment returns and stock returns is weak and sensitive to the relative timing of investment and stock returns.

Cochrane and Liu et al. test the  $q$ -theory of investment in first differences rather than levels. The basic form of the  $q$ -theory of investment in the cross-section of firms has seen limited empirical success (see Chirinko (1993) and Hassett and Hubbard (2002) for extensive surveys of the empirical investment literature). The exact theoretical relation (9) holds only under restrictive assumptions on the firm's technology and needs to be modified to account for realistic frictions, such as fixed costs and time to build (e.g., Caballero and Leahy (1996), Lamont (2000)). Some researchers also emphasize the importance of measurement errors in  $q$  (e.g., Erickson and Whited (2000), and Gomes (2001)). Cochrane (1991) argues that measurement errors may explain why  $q$ -theory may perform better in first differences than in levels. Specifically, he suggests low-frequency changes in the fundamentals as one possible source of measurement errors.

Equation (17) has several limitations as a basis for empirical tests. Most importantly, this equation has no causal content, since it links three endogenous variables. Thus, it can say nothing about the economic causes of the cross-sectional differences in firms' expected returns and their observable characteristics. For instance, empirical tests of the first-order condition (17) cannot differentiate between several different interpretations: that investment responds to market (mis)valuation, (e.g., Morck, Shleifer, and Vishny (1990); Baker, Stein, and Wurgler (2003); Panageas (2005); Gilchrist, Himmelberg, and Huberman (2005); and Polk and Sapienza (2009)); that

market prices affect firm investment due to learning (see Bond, Edmans, and Goldstein (2011) for an extensive review of the literature); or that the accumulation of capital alters the asset composition of the firm and hence affects the properties of stock returns (e.g., Rubinstein (1973); Berk, Green, and Naik (1999); Carlson, Fisher, and Giammarino (2004); and Kogan and Papanikolaou (2010)).

### Endogenous Investment and Risk

To understand how stock returns and firm characteristics are jointly determined by the firm's technology and the macroeconomic environment, we need to solve explicitly for these endogenous variables in terms of the model primitives.

We present the following parameterization of the setting above. The physical time period is  $\Delta t$ . Let the firm's production function be a special case of (3) with  $\alpha = 1$ :

$$Y_t = X_t K_t \Delta t. \quad (18)$$

Assume a standard mean-reverting productivity process  $X_t$  given by

$$X_t = \exp(\bar{x} + x_t) \Delta t, \quad (19)$$

$$x_t = (1 - \theta \Delta t)x_{t-1} + \sigma \sqrt{\Delta t} \varepsilon_t, \quad \varepsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, 1). \quad (20)$$

The productivity shock  $X$  can have an aggregate and an idiosyncratic component.

The firm's capital stock evolves as

$$K_t = (1 - \delta \Delta t)K_{t-1} + I_{t-1} \Delta t. \quad (21)$$

The investment cost function is given by

$$\phi(I/K) = \left( I/K + \frac{a}{\lambda} |I/K|^\lambda \right) \Delta t. \quad (22)$$

Note that in this specification the investment rate can be negative.

Moreover, the interest rate  $r_f$  is constant, and the stochastic discount factor satisfies

$$\pi_t = \pi_{t-1} \exp\left(- (r_f + \eta^2/2)\Delta t - \eta\sqrt{\Delta t} u_t\right), \quad u_t \stackrel{iid}{\sim} \mathcal{N}(0, 1), \quad (23)$$

where  $\text{corr}(\varepsilon_t, u_t) = \rho$ . Hence, the market price of risk attached to  $\varepsilon_t$  is constant.

The firm's optimal investment rate, its  $q$ , and its risk premium depend on the level of log productivity  $x_t$ , which follows an exogenous process. Figure 2 shows that the firm's  $q$  is monotonically increasing in its productivity, and therefore so is its optimal investment rate. Both the conditional beta of stock returns with respect to productivity,  $\beta_x$ , and the discount rate,  $\eta \beta_x \rho \sigma$ , are also increasing functions of the productivity shock.

In this example, the risk premium is positively related to the investment rate and Tobin's  $q$ . This positive relation does not contradict the general relation in (17), since the negative correlation between the expected returns and the investment rate holds only when we control for expected future profitability. Here, the risk premium is positively correlated with productivity, and, as a result, the unconditional correlation between the expected stock returns and the firm's investment rate (or its  $q$ ) is positive.

The qualitative univariate relation between firm characteristics and stock returns is sensitive to the specification of the firm's technology and the stochastic discount factor. To show how the qualitative properties of the model depend on the production function, we add a production cost independent of  $X$ :

$$Y_t = (X_t - c) K_t \Delta t. \quad (24)$$

We assume that the firm has an option to exit the market at zero liquidation value.

The addition of the cost  $cK_t\Delta t$  introduces *operating leverage*. Operating leverage implies that the firm is relatively risky when it operates at low values of productivity because costs do not scale proportionally with sales. In particular, when productivity  $X$  is low, an increase in  $X$  has a substantially larger effect on profitability than when  $X$  is high.

Early examples of formal analysis of the effect of operating leverage on the firm's systematic risk can be found in Rubinstein (1973) and Lev (1974). This concept is also commonly discussed in standard finance textbooks, e.g., Brealey and Myers (1981). In our setting, operating leverage affects the relation between the stock returns and firm's profitability, as we show in Figure 2. In contrast to the model without operating leverage, the expected stock return is decreasing at lower productivity levels.<sup>3</sup>

Several papers combine operating leverage with other modeling assumptions, usually adjustment costs (e.g., Carlson et al. (2004), Zhang (2005), Cooper (2006), Li et al. (2009), Bazdrech, Belo, and Lin (2009), Belo and Lin (2011)). This combination makes it hard to isolate the role of individual assumptions. Some authors argue that asymmetric adjustment costs are the defining feature of these models, because adjustment costs make the firm less flexible in adjusting its capital stock, and thus more risky. However, as we show below, although adjustment costs do play a first-order role in defining the properties of stock returns in some settings, their effect in partial equilibrium is highly sensitive to the details of the model.

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<sup>3</sup>The effect of operating leverage on firms' systematic risk has been studied empirically in many papers. Kothari (2001) provides a survey of the early literature, which includes Lev (1974), Mandelker and Rhee (1984), Subrahmanyam and Thomadakis (1980). The results of the earlier studies are mixed; the conclusions are sensitive to the choice of the empirical measures of operating leverage. Novy-Marx (2011) also provides empirical evidence for the operating leverage mechanism in stock returns by documenting a negative cross-sectional relation between the firms' empirical measure of operating leverage and their subsequent excess stock returns. Gourio (2007) looks for the leverage effect directly in cash flows, and finds that the cash flows of low-productivity firms are indeed more sensitive to the aggregate productivity shocks. In Gourio (2007), operating leverage is generated by the properties of firms' labor costs. Aggregate wages are sticky, implying that firm costs do not move proportionally with firm profits. Thus, the paper provides additional evidence on this particular source of operating leverage.

To illustrate the implications of adjustment costs in our model of the firm, we consider an extreme case of adjustment cost asymmetry: adjustment costs are infinite when disinvesting. Hence, investment is irreversible. We contrast the behavior of the model with and without the irreversibility constraint (omitting the constraint  $I_t \geq 0$ ), and with and without operating leverage ( $c = 0$ ).

Figure 2 summarizes the results. Without operating leverage, the expected stock return is virtually unaffected by the irreversibility constraint. The optimal investment rate is affected by the irreversibility constraint, primarily when the optimal investment in the unconstrained model is negative. When operating leverage is present, asymmetric adjustment costs magnify its effect on risk and expected returns. We can see the effect of investment irreversibility by comparing the solid and dashed lines in the second panel of Figure 2.

We conclude that operating leverage can generate a negative correlation between the expected stock return of a firm and its profitability in our example. There is an interaction effect, through which asymmetric adjustment costs can magnify the effect of operating leverage. However, asymmetric adjustment costs are neither necessary nor sufficient for a negative correlation between investment rates and risk premia.

In addition to operating leverage, the recent literature considers other mechanisms to link firm risk and characteristics. In particular, if the firm owns multiple durable inputs, and the market prices of these inputs have different levels of systematic risk, then the firm's exposure to the aggregate productivity shock depends on the input composition. For instance, recent studies have considered real estate (e.g. Tuzel (2010)), inventories (e.g. Belo and Lin (2011); Jones and Tuzel (2011)), and intangible capital (e.g. Lin (2011); Belo, Vitorino, and Lin (2011)).

The discussion above shows that the specification of the profitability process, e.g.,

comparing (3) with (24), is very important for the asset-pricing implications of the commonly used neoclassical model. In partial equilibrium, the model postulates the profitability process exogenously, which raises the question of whether the modeling overhead associated with describing firms' dynamic investment choices is justified. One way to address this issue is in an equilibrium setting, in which firm profitability is determined endogenously.

### Endogenous Profitability

One way to endogenize firm profitability is to impose market clearing in the product market. The equilibrium price of a good depends on the behavior of firms in the producing sector. Thus, firm profitability is endogenous. A few papers develop standard general equilibrium models with multiple sectors and heterogeneous goods. Some papers gain tractability by using industry equilibrium models.<sup>4</sup>

The asset pricing results in these studies are related to the time-varying supply elasticity, and are analogous to the discussion in Section 2. Specifically, adjustment costs affect stock return risk in equilibrium because they affect the ease with which firms add new capital in response to external shocks, such as shocks to demand for industry output or to firm productivity. When adjustment costs are low, the supply of capital is relatively elastic, largely absorbing exogenous shocks and stabilizing the market value of firms. In contrast, when investment is constrained by adjustment

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<sup>4</sup>A typical industry equilibrium model can be interpreted as a general equilibrium model in which the dynamics outside of the industry of interest are modeled in a reduce-form manner. For example, we consider a two-sector model with two consumption goods, 1 and 2. We define the households' utility over the two goods as

$$E_0 \left[ \sum_{t=0}^{\infty} \pi_t (c_{1,t} + \Theta_t U(c_{2,t})) \right], \quad (25)$$

where  $\pi_t$  and  $\Theta_t$  are preference shocks. We use good 1 as a numeraire. The equilibrium price of good 2 is  $\Theta_t U'(c_{2,t})$ , which corresponds to an inverse demand function with preference shocks  $\Theta_t$ . As a result, the equilibrium stochastic discount factor is equal to  $\pi_t$ , which is exogenously specified in this model.

costs, and thus supply of capital is relatively inelastic, its equilibrium price is relatively sensitive to exogenous shocks. Unlike in partial equilibrium, this mechanism is robust to the exact specification of the production functions of firms.

Kogan (2001; 2004) considers economies with identical firms within a sector; Zhang (2005) adds heterogeneity in productivity. In these models, both investment and disinvestment by firms incurs convex adjustment costs. Hence, the stock return risk of an average firm is non-monotonic in the level of industry profitability.

Novy-Marx (2008), Aguerrevere (2009), Carlson, Dockner, Fisher, and Giammarino (2009), Bena and Garlappi (2011), and Novy-Marx (2011) analyze the effects of imperfect competition. Firms' strategic behavior affects their propensity to invest in response to exogenous shocks, thus changing the elasticity of supply of capital. As a result, the internal organization of the industry matters for the risk of stock returns in equilibrium.

Most of the papers above assume that all firms produce the same output good. Gomes, Kogan, and Yogo (2009) investigate the effect of the durability of output on the cross-section of asset returns. They show that the firms that produce consumer durable goods have different risk characteristics from the firms producing non-durables and services. Services from the durable goods are supplied by both the new goods and the existing stock of durable goods. Because durable goods depreciate relatively slowly compared to non-durables and services, and firms cannot produce a negative amount of durable goods, the supply of the durable goods is downward rigid. Therefore, when the demand for durable goods is low, their supply is relatively inelastic and stock returns of the firms producing durable goods are relatively risky.

## 3.2 Aggregate Shocks and Return Comovement

Many models with heterogeneous firms describe systematic uncertainty as a single aggregate productivity shock. As a result, even if these models account for the first moments in returns, they have difficulty matching second moments. In particular, these models have difficulty replicating the multi-factor structure of return comovement in the data.

Understanding the nature of systematic risk is as important an objective as understanding the differences in risk premia among stocks. In models with a single systematic shock, risk premia of firms are closely aligned with their conditional market betas. As a result, such models have limited ability to account for the empirical failures of the conditional CAPM or differences in conditional Sharpe ratios among various well-diversified portfolios.

To generate a multi-factor structure in stock returns, we need to model multiple sources of aggregate uncertainty that have a heterogeneous impact on the cross-section of asset returns. Models with these features can also help us better identify such shocks using financial data, and provide insights into how these shocks propagate. For instance, such models can tell us how to mimic the fundamental economic shocks using returns on financial assets.

When modeling heterogeneous exposure of firms' stock returns to aggregate shocks, it is convenient to decompose the firm value into the value of assets in place and the present value of future growth opportunities (see Brealey and Myers (1981) for an early textbook reference). Berk et al. (1999) is the first paper to explore quantitatively a structural asset pricing model with differences in systematic risk between growth opportunities and assets in place. They show that firm value composition is related to both its systematic risk exposures and its observable characteristics. For instance, the firm's average  $q$  is positively correlated with the relative value of its

growth opportunities versus assets in place, and thus contains information about the systematic risk of the firm. Even though it is not the main focus of their paper, the model in Berk et al. (1999) features return comovement across firms due to the presence of two aggregate shocks: shocks to average productivity and interest rates.

In Berk et al. (1999), the firm's asset composition changes over time, as the firm acquires new projects, existing projects depreciate, or project productivity changes. This time-series variation in the firm's asset composition gives rise to, among other things, a time-series relation between firms' investment and their risk. This idea is also explored in Carlson et al. (2004). Every time a firm invests, its value of assets in place rises relative to the value of its growth opportunities. Because growth opportunities are relatively risky, higher firm investment predicts lower expected stock returns.

Capital-embodied technological change (e.g., Solow (1960)) is a natural source of comovement among firms with different shares of growth opportunities in firm value. Capital-embodied technological advances get implemented in the new vintages of capital. In contrast to the neutral, disembodied shocks, embodied shocks do not automatically affect the productivity of the older vintages of capital, and therefore they impact the market value of existing assets and future growth opportunities differently. Papanikolaou (2011), Garleanu, Panageas, and Yu (2011), and Garleanu, Kogan, and Panageas (2011) are recent examples of asset pricing models with embodied technological change.

Papanikolaou (2011) explores the implications of investment-specific technology (IST) shocks. IST shocks represent capital-embodied technological change that is typically modeled as shocks to the cost of installing new capital. Several empirical studies have argued that IST shocks account for a substantial part of business-cycle fluctuations and long-run growth (Greenwood, Hercowitz, and Krusell (1997), Greenwood, Hercowitz, and Krusell (2000), Justiniano, Primiceri, and Tambalotti (2011)).

Papanikolaou argues that IST shocks are a systematic risk factor that carries a negative price of risk because households have a higher marginal utility of wealth in states with good investment opportunities. In addition, IST shocks have a positive effect on the value of firms' growth opportunities relative to the value of their assets in place. Therefore, growth firms are attractive to investors despite their low average returns, because they appreciate in value when real investment opportunities improve.

We use a simplified version of the model in Kogan and Papanikolaou (2010) to illustrate how the embodied shocks interact with firm asset heterogeneity.

A firm is a collection of productive units, or projects. Each project  $j$  produces a flow of output  $X_t K_j^\alpha$  per period, where  $\alpha \in (0, 1)$ ;  $K_j$  is the amount of capital irreversibly invested into the project; and  $X_t$  is the common productivity process for all projects. The risk-neutral distribution of productivity growth is

$$X_t = X_{t-1} \exp(\mu_X + \sigma_X \varepsilon_t), \quad \varepsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, 1). \quad (26)$$

Projects expire randomly and independently with probability  $(1 - e^{-\delta})$  per period.

In addition, each firm can invest in additional projects. Investment opportunities arrive randomly and independently. In each period a firm receives an opportunity to invest with probability  $\lambda$ . When a firm creates a new project  $j$  at time  $t$ , it chooses the optimal investment level  $K_j = K_t^*$  and pays the investment cost  $X_t Z_t^{-1} K_j$ . The project starts being productive in the next period.

The cost of capital relative to its productivity depends on the investment-specific productivity process  $Z_t$ , which has the risk-neutral distribution

$$Z_t = Z_{t-1} \exp(\mu_Z + \sigma_Z u_t), \quad u_t \stackrel{iid}{\sim} \mathcal{N}(0, 1) \quad \text{and} \quad \text{corr}(u_t, \varepsilon_t) = 0. \quad (27)$$

Assume the risk-free rate is constant,  $r_f$ . Then, the time- $t$  present value of future

cash flows produced by a single project  $j$  equals

$$V_{j,t} = (K_j)^\alpha E_t \left[ \sum_{s=t+1}^{\infty} e^{-(r_f+\delta)(s-t)} X_s \right] = A(K_j)^\alpha X_t, \quad (28)$$

where  $A$  is a constant. The ex-dividend value of assets in place equals the sum of the values of individual projects owned by the firm:

$$V_{f,t}^A = A \left( \sum_{j \in \{\text{Projects of firm } f\}} (K_j)^\alpha \right) X_t. \quad (29)$$

The value of growth opportunities equals the net present value of future investments in new projects:

$$V_{f,t}^G = E_t \left[ \sum_{s=t+1}^{\infty} \lambda e^{-r_f(s-t)} \{AX_s(K_s^*)^\alpha - Z_s^{-1}X_sK_s^*\} \right] = CX_t, Z_t^{\frac{\alpha}{1-\alpha}}. \quad (30)$$

where  $C$  is a constant.

Kogan and Papanikolaou assume that the two technological shocks,  $\varepsilon_t$  and  $u_t$ , have constant market prices of risk,  $\eta_X$  and  $\eta_Z$ . Based on equations (29) and (30), the present value of growth opportunities has a positive loading on the IST shock  $u_t$ . In contrast, the value of assets in place depends only on the neutral productivity shock  $\varepsilon_t$ . This difference in exposures of assets in place and growth opportunities to the IST shock leads to three main implications.

First, returns on high-growth firms comove with each other because of their common exposure to the IST shock, giving rise to a systematic factor in stock returns that is distinct from the market portfolio. This factor is an innovation in the long-short portfolio of assets in place and pure growth opportunities

$$r_{t+1}^A - r_{t+1}^G - E_t[r_{t+1}^A - r_{t+1}^G] = -\frac{\alpha}{1-\alpha} \sigma_Z u_{t+1}. \quad (31)$$

Second, firms with a higher fraction of growth opportunities in the firm value

(high-growth firms) exhibit different risk premia from those of firms with fewer growth opportunities (low-growth firms). The difference in expected returns between assets in place and growth opportunities equals

$$E_t[r_{t+1}^A - r_{t+1}^G] = -\frac{\alpha}{1-\alpha}\eta_Z\sigma_Z. \quad (32)$$

If IST shocks carry a negative market price of risk ( $\eta_Z < 0$ ), then assets in place earn a higher average return than growth opportunities.

Third, stock return betas with respect to the IST shock reveal cross-sectional heterogeneity in firms' growth opportunities: a firm's beta with the IST shock equals:  $\beta_{f,t}^Z = \text{const} \times (V_{f,t}^G / (V_{f,t}^A + V_{f,t}^G))$ . Thus, firms with higher  $\beta_{f,t}^Z$  exhibit higher average investment rates, and their investment responds stronger to IST shocks.

Kogan and Papanikolaou find empirical support for the predicted relations between the firms' IST-shock betas and their future investment and stock returns. Kogan and Papanikolaou (2011) argue that the same logic applies to the well-documented empirical relations between stock returns and firms' investment rates and profitability.

Several recent papers build equilibrium models with multiple aggregate sources of risk and heterogenous firms. Garleanu, Kogan, and Panageas (2011) model an expanding variety of intermediate goods. In their model, technological advances affect only the production of new types of goods. Garleanu et al. argue that aggregate innovation shocks lead to an inter-generational displacement effect. Innovation benefits new generations of innovators and workers partly at the expense of older generations, whose financial and human capital depreciates as a result of increased competitive pressures created by innovation. Growth firms are those that benefit more from technological innovation, and therefore offer a hedge against displacement shocks. The growth factor in stock returns is thus driven by innovation shocks.

Ai, Croce, and Li (2011) and Ai and Kiku (2011) build equilibrium models with

production in the long-run risk framework of Bansal and Yaron (2004). They argue that growth opportunities are less sensitive than assets in place to long-run risks. In Ai et al., systematic technology shocks do not affect new vintages of capital, hence new firms have lower systematic risk than existing firms. In Ai and Kiku, a new production unit requires both growth opportunities and physical capital. Unexercised growth opportunities do not expire, but can be used later. The relative scarcity of physical capital relative to the stock of growth opportunities implies that the price of physical capital is pro-cyclical. Thus, installed capital is riskier than growth opportunities.

The growth opportunities in Ai et al. and Ai and Kiku are an example of intangible capital, which can differ in its risk properties from physical capital, see e.g., Hansen, Heaton, and Li (2005). Eisfeldt and Papanikolaou (2011) model organization capital, a specific example of intangible capital, as a production factor that is embodied in the firm's management. Shareholders cannot fully appropriate the cash flows from organizational capital. In particular, the division of rents between shareholders and managers depends on the outside option of the managers and changes with the state of the economy. As a result, shareholders who invest in firms with more organization capital are exposed to additional risks.

## **General Equilibrium and Aggregation**

It is challenging to model nontrivial firm heterogeneity in equilibrium. In general, the joint cross-sectional distribution of firm productivity and capital holdings affects the aggregate equilibrium dynamics, creating a curse of dimensionality. One approach is to confront a high-dimensional model head-on, solving for the approximate equilibrium using numerical approximations, e.g., the method of Krusell and Smith (1998). Recent examples of this approach are Zhang (2005), Tuzel (2010), and Favilukis and Lin (2011).

An alternative approach is to model the firms in a way that allows for tractable aggregation. We illustrate the aggregation procedure in the context of the model of Section 3.2. We modify the production function to allow for idiosyncratic project-specific uncertainty:

$$Y_{j,t} = \xi_{j,t} X_t K_j^\alpha, \quad (33)$$

$\xi_{j,t}$  is non-negative stationary process, independent from aggregate productivity, and independently and identically distributed across projects. The conditional mean of  $\xi_{j,t}$  follows a first-order linear process

$$\mathbb{E}_t[\xi_{j,t+s}] - 1 = e^{-\kappa s} (\xi_{j,t} - 1), \quad (34)$$

and therefore  $\mathbb{E}[\xi_{j,t}] = 1$ . The productivity of new projects is drawn from the stationary distribution of  $\xi_{j,t}$ . Thus, the cross-sectional distribution of  $\xi_{j,t}$  is invariant over time.

The present value of future cash flows from an existing project equals

$$V_{j,t} = (K_j)^\alpha \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} e^{-(r_f + \delta)(s-t)} \xi_{j,s} X_s \right] = [A + B(\xi_{j,t} - 1)] K_j^\alpha X_t, \quad (35)$$

where  $A$  and  $B$  are project-independent constants.

When the firm makes an investment decision, it does not observe the initial realization of the idiosyncratic project productivity  $\xi$ , hence the net present value of a new project is equal to  $A K_j^\alpha X_t$ . Thus, the optimal investment is the same for all firms, and equals

$$K_t^* = \operatorname{argmax}_K (A X_t K^\alpha - Z_t^{-1} X_t K) = (A \alpha Z_t)^{\frac{1}{1-\alpha}}. \quad (36)$$

The fact that the firm's investment is independent of its current capital holdings allows for tractable aggregation.

We index the firms by  $m$ , and assume that the set of firms is a unit interval,  $\{m \in [0, 1]\}$ . Let  $\mathcal{J}$  be the set of live projects. Applying the Law of Large Numbers to the cross-section of projects ( $\int_{j \in \mathcal{J}} \xi_{j,t} dj = 0$ ), we find that the aggregate output  $\bar{Y}_t$  is

$$\bar{Y}_t = \int_{j \in \mathcal{J}} Y_{j,t} dj = \int_{j \in \mathcal{J}} \xi_{j,t} X_t K_j dj = X_t \bar{K}_t^\alpha, \quad (37)$$

where  $\bar{K}_t$  denotes the ‘‘aggregate capital stock,’’ defined as

$$\bar{K}_t = \int_{j \in \mathcal{J}} K_j^\alpha dj. \quad (38)$$

The aggregate capital stock changes due to project expiration and aggregate investment  $I_t$ ,

$$\bar{K}_{t+1} = e^{-\delta} \bar{K}_t + I_t, \quad (39)$$

where

$$I_t = \lambda \int_{m \in [0,1]} (K_t^*)^\alpha dm = \lambda (A\alpha Z_t)^{\frac{\alpha}{1-\alpha}}. \quad (40)$$

The triplet of aggregate variables  $(X_t, Z_t, \bar{K}_t)$  follows a Markov process. Aggregate prices of assets in place and growth opportunities are also functions of these variables:

$$\begin{aligned} \bar{V}_t^A &= \int_{j \in \mathcal{J}} V_{j,t} dj = AX_t \bar{K}_t, \\ \bar{V}_t^G &= \int_{m \in [0,1]} \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \lambda e^{-r_f(s-t)} \{AX_s (K_s^*)^\alpha - Z_s^{-1} X_s K_s^*\} \right] dm = CX_t Z_t^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

In general equilibrium,  $A$ ,  $B$  and  $C$  are not constant, and depend on the aggregate state. However, the small number of aggregate state variables implies that we can compute the equilibrium using standard numerical methods. This approach to modeling heterogeneous firms is introduced in Gomes, Kogan, and Zhang (2003), who analyze the cross-sectional relations between stock returns and characteristics in a single-factor general equilibrium production economy. Several other papers rely on a

similar structure for aggregation. Gomes and Schmid (2010) study equilibrium credit spreads. Ai (2010) and Ai et al. (2011) model endogenous creation of investment opportunities.

Furthermore, this type of model produces lumpy investment behavior, consistent with the micro-level evidence of (Cooper and Haltiwanger (2006); Gourio and Kashyap (2007)). In these models, the standard  $q$ -theory of investment does not apply, and  $q$  has limited explanatory power for investment. There is one more distinction between models with lumpy investment and smooth neoclassical models. In the latter, the  $q$ -theory implies that returns on the marginal unit of investment are identical to the stock returns of the firm. In the above model with lumpy investment, this is not the case. The expected return on a new investment equals the expected return on assets in place, which is generally different from the expected stock return of the entire firm. This distinction is important to consider when we interpret the empirical relations between investment and expected stock returns.

Gala (2006) suggests an alternative modeling approach that leads to tractable aggregation. He starts with a neoclassical model and allows firm-level adjustment (5) costs to depend on the aggregate capital stock in the economy. The presence of this type externality in the adjustment cost formulation also leads to a small number of state variables describing the aggregate dynamics.

## 4 Future Directions

Future research on the relations between firms' economic activities and the behavior of asset prices can improve our understanding of the sources of aggregate fluctuations, their propagation mechanisms, and their impact on financial markets. We organize our discussion below around these themes.

## Sources of Aggregate Fluctuations and Information in the Cross-Section of Financial Assets

The business cycle literature considers a number of exogenous sources of economic fluctuations. In addition to disembodied total factor productivity (TFP) shocks, existing models cover embodied technology shocks (Solow (1960), Greenwood et al. (1997)), shocks to monetary policy (Christiano, Eichenbaum, and Evans (2005)), shocks to the agents' information set (Jaimovich and Rebelo (2009), Angeletos and La'O (2011)), shocks to macroeconomic uncertainty (Bloom (2009)), or shocks to the firms' borrowing capacity (Christiano, Motto, and Rostagno (2010), Khan and Thomas (2011)).

The asset-pricing literature has long recognized that shocks to the economy are priced by financial markets according to how these shocks impact the welfare of investors. Thus, the analysis of the joint behavior of asset prices and macroeconomic quantities can shed light on the economic significance of the various sources of aggregate fluctuations.

Most studies that relate financial prices to macroeconomic shocks focus on the ability of the aggregate stock market to predict economic variables (see Stock and Watson (2003) for an extensive survey). However, if the underlying structural shocks affecting the economy have a heterogeneous impact on different financial assets, then the cross-section of asset prices will contain valuable information about the sources of aggregate fluctuations. To extract such information from financial asset prices, we need better models of how different fundamental shocks affect prices of various financial assets.

## Multiple Asset Classes

To better understand the effects of aggregate shocks on asset prices we need to move beyond equity markets. The historical focus on equity markets stems partly from the ready availability of stock price data. It is hard to fully justify this focus from a theoretical perspective, since most models apply to the entire firm and not to the firm's equity. Even under the Modigliani-Miller assumptions of these models, leverage creates a nontrivial distinction between the two. This situation is beginning to change, as market data on corporate debt, credit default swaps, and other types of assets becomes more easily available.

More generally, a systematic analysis of different asset classes would make it possible for researchers to gain insight into the impact of aggregate shocks on different aspects of the economic environment. For instance, the prices of corporate bonds provide useful incremental information about the firms' cost of capital relative to the information revealed by stock prices; the prices of equity options contain useful information about the likelihood and magnitude of disasters; and the term structure of interest rates contains information about the long-run growth of the economy. A few recent examples of using the cross-section of asset prices in various markets to extract information about aggregate shocks include Gilchrist, Yankov, and Zakrajsek (2009), Philippon (2009), Fisher and Peters (2010), Kogan and Papanikolaou (2010), Gilchrist and Zakrajsek (2011), and van Binsbergen, Hueskes, Koijen, and Vrugt (2011).

Going forward, the challenge is to develop coherent models of multiple asset classes, including stocks, bonds, options, currencies, commodities, as well as of heterogeneous assets within asset classes.

## **Real Effects of Financial Market Imperfections**

Thus far, our focus has been on settings in which financial markets are free of frictions and agency problems. Financial market distortions can significantly affect the behavior of the aggregate economy, something financial crises illustrate strikingly well. In particular, agency frictions and financial market imperfections can amplify and propagate real shocks.

One channel through which financial markets can affect the real economy is through the availability of credit (see Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke, Gertler, and Gilchrist (1999); Jermann and Quadrini (2009); and Brunnermeier and Sannikov (2011)). Models with financial frictions can also improve our understanding of the pricing of credit risk (see Gomes and Schmid (2010) and Gourio (2011a) for recent examples).

## **Heterogeneity and Aggregation**

Most tractable general equilibrium models in the asset-pricing literature capture the interplay between the cross-section of asset prices and aggregate dynamics in a “top-down” fashion. In these models, heterogeneity does not factor explicitly into the aggregate dynamics, e.g., Gomes et al. (2003). “Bottom-up” models are generally more cumbersome, but can provide additional valuable insights. In such models, firm heterogeneity plays a key role in shaping the aggregate dynamics, and thus there is a meaningful two-way link between the cross-section of firms and macroeconomic fluctuations. Among recent examples, Khan and Thomas (2008) study the implications of lumpy investment for aggregate dynamics; Bloom (2009) shows how uncertainty shocks coupled with heterogeneous firm-specific productivity gives rise to endogenous fluctuations in the aggregate productivity in the economy; Gabaix (2011) shows how aggregate fluctuations can be generated by firm-specific shocks in an economy with a

heavy-tailed distribution of firm size; and Khan and Thomas (2011) show that firm heterogeneity amplifies the effect of credit shocks.

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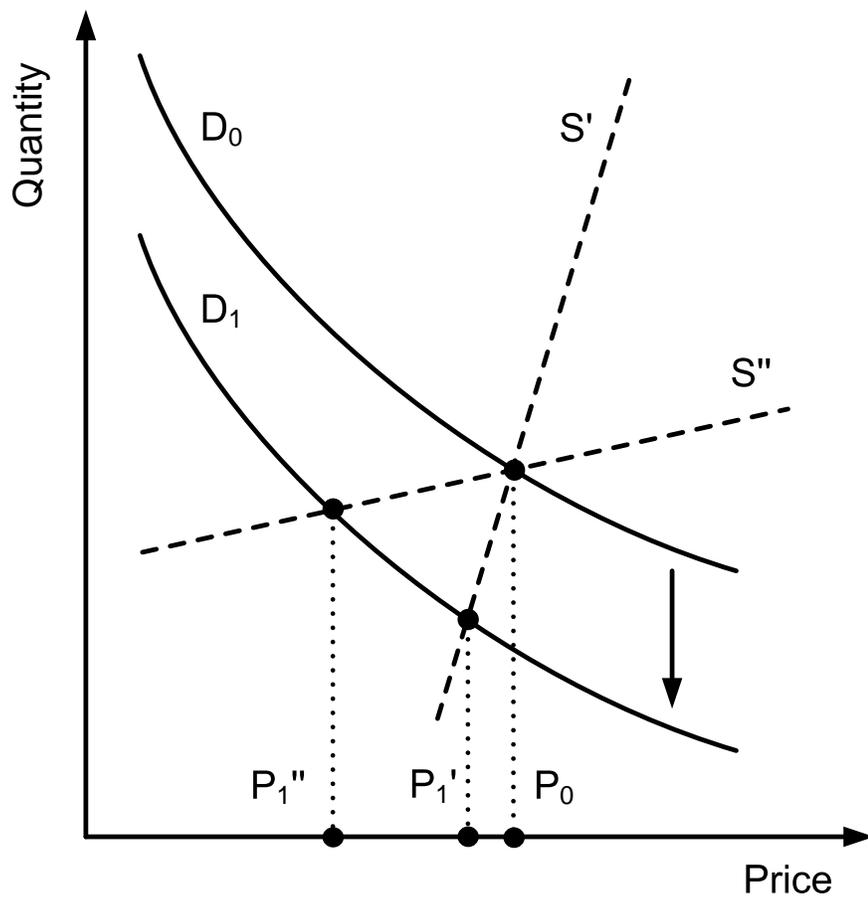
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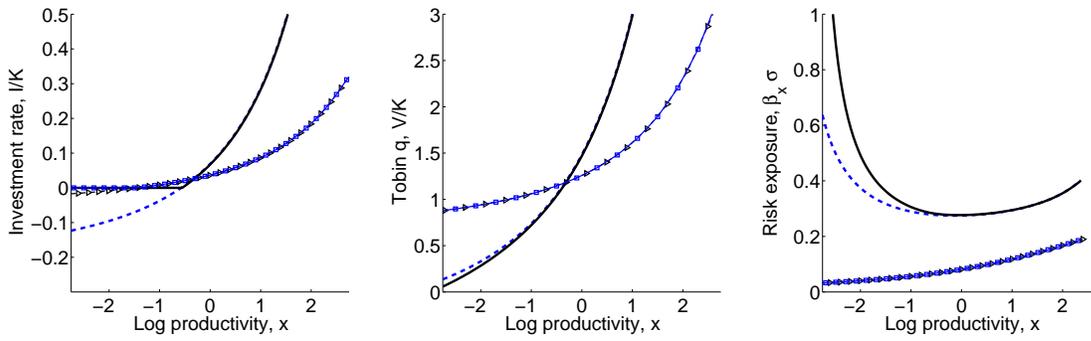
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**Figure 1: Supply and demand for capital.** This figure compares two equilibrium settings with more or less elastic supply curves,  $S'$  and  $S''$ , respectively. In case of a negative demand shock, the demand curve changes from  $D_0$  to  $D_1$ . Under the more elastic supply schedule, the demand shock is absorbed mostly by the change in equilibrium quantity, and the equilibrium price change  $P_0 - P_1'$  is relatively small. Under the less elastic supply, we see the reverse: a smaller impact of the demand shock on the equilibrium quantity with a larger impact on the equilibrium price,  $P_0 - P_1'' > P_0 - P_1'$ . Thus, *ceteris paribus*, lower elasticity of supply makes equilibrium prices more volatile.



**Figure 2:** Solution of the model for  $\lambda = 2$ ,  $a = 7$ ,  $\theta = 0.4$ ,  $\sigma = 0.4$ ,  $\delta = 0.045$ ,  $\eta = 0.30$ ,  $\rho = 0.33$ ,  $r = 0.05$ ,  $c = [0, 0.30]$ ,  $\bar{x} = \ln(0.115 + c)$  in the limit of  $\Delta t \rightarrow 0$ . The graph compares four versions of the model: (a) with operating leverage and investment irreversibility (solid); (b) with operating leverage only (dash); (c) with investment irreversibility only (squares); (d) with no irreversibility or operating leverage (triangles). The first panel shows the optimal investment rate,  $I^*/K$  as a function of log productivity  $x$ . The second panel plots the Tobin's  $q$  of the firm. The third panel plots the conditional risk exposure of the firm, defined as  $\beta_x \sigma$ , where  $\beta_x = \partial \ln(V/K)/\partial x$ .