Assignment 2
Cost & Revenue Functions

Assigned: Monday, February 10, 2003
Due: Tuesday, February 18, 2003

Problems from Text

1. Be sure that you understand all of the elements of 2-3 and 2-37 (basic terminology).

2. Do the following problems and be sure that you get the correct answers:
   - 2-8, 2-16, 2-22, and 2-38

3. Do the following problems and submit them for grading (10% each)
   - 2-9 Determining optimal level of output, using calculus and graphical techniques
   - 2-13 Evaluating alternatives
   - 2-19 Cost-based design
   - 2-33 Risk analysis

4. Fixed Costs, Variable Costs, and Design of a Network (60%)

   Suppose you are designing a new transport network for a region that has a uniform population density. The transportation capabilities are defined by four major factors:
   a. Fixed cost per mile per year to the owner (FC)
   b. Variable cost to the owner for maintenance (VC per vehicle-mile)
   c. Average speed (S)
   d. User Cost on the new network, which is a function of time T and distance D (User cost = U1*T + U2*D)
      (U1 might be the average hourly wage and U2 might be average per mile cost of using an auto)
   e. User Access cost, which is a function of time and distance from the user's home to the network and from the new network to the users destination
      (Access cost = A1*TA + A2*Access distance, where the access distance is expected to be G/2 and the access speed is AS)

   Two additional parameters relate to the potential travel demand:
   f. Trip density (trips per year per square mile = TD)
   g. Average trip length (D)

   Let's assume that the network will be a grid with roads running east to west or north to south at intervals of a uniform distance G. Let's further assume that the objective is to minimize the annual costs of the system to the owner (FC + VC) plus the costs to the users (e.g. the system could be built by the taxpayers for the taxpayers and offered to the public without tolls).

   Questions (20% each):
   a. In qualitative terms, how will G vary with increases in FC, VC, S, U1, U2, TD and D?
   b. Formulate a cost function for the system, defining any additional parameters that you need
   c. Using calculus, find an expression for the optimal value of G as a function of the other variables.
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Extra Credit (up to 20%):

d. Structure a spreadsheet where you can
   i) Plug in values for each of the inputs, i.e.
   
<table>
<thead>
<tr>
<th></th>
<th>Option A</th>
<th>Option B</th>
<th>Option C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed on new net/S</td>
<td>20</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>Value of time for u:U1</td>
<td>$5</td>
<td>$5</td>
<td>$10</td>
</tr>
<tr>
<td>Average Distance (D)</td>
<td>5</td>
<td>50</td>
<td>400</td>
</tr>
<tr>
<td>Average cost per nU2</td>
<td>$0.20</td>
<td>$0.20</td>
<td>$0.20</td>
</tr>
</tbody>
</table>

   ii) In one or a series of steps, calculate the optimal value of G
   
   |                      |          |          |          |
   | Travel time (hours) = Distance | 0.25 | 1.00 | 2.00 |
   | Value of trip time to user     | $1.25   | $5.00  | $20.00 |
   | etc.                            |         |        |        |
   | Optimal Grid                   | G1      | G2      | G3      |

c. Estimate G for a system of arterial roads circa 2000 using estimates of reflecting road and user costs today (e.g. $1 million per mile to build the road, $0.20/car-mile to drive on the road, $0.02/car-mile for road maintenance, average trip length of 10 miles)

d. Estimate G for a system of limited access highways in the Northeast and in the West (make reasonable assumptions about trip density and any other inputs that you need)