Cost Terminology

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1.011 Project Evaluation

Fixed vs. Variable Costs

- **Fixed Costs**
  - Unaffected by changes in activity level over a feasible range of operations for a given capacity or capability over a reasonable time period
  - For greater changes in activity levels, or for shutdowns, the fixed cost can of course vary
  - Examples: insurance, rent, CEO salary

- **Variable Costs**
  - Vary with the level of activity
  - Examples: construction labor, fuel costs, supplies

- **Incremental Costs**
  - Added costs for increment of activity

Total Cost (V) = Fixed Cost + f(volume)

Avg. Cost (V) = Fixed Cost/V + f(volume)/V

Incremental Cost(V0;V1) = f(V1) - f(v0)

Marginal Cost (V) = d(Total Cost)/dV = f'(V)

(Assuming we in fact have a differentiable function for variable costs!)

A Simple, Linear Cost Function:

TC = a + bV = 50 + V, 10 < V < 100

Anc. Cost = a/V + b = 50/V + 1

Marginal Cost (V) = d(TC)/dV = b = 1

Classic Tradeoff: Can we afford higher fixed costs in order to get lower variable costs?

Breakeven point B is where TC1 = TC2

TC2 = 95 + V/2

TC = 50 + V

Marginal Cost

Average Cost

Volume

Cost

Breakeven point B is where TC1 = TC2

Marginal Cost

Average Cost

Volume

Cost

Breakeven point B is where TC1 = TC2
**Breakeven Volume**

If \( b_1 < b_0 \) and \( a_1 > a_0 \), then there is a volume \( V^* \) where the total costs are equal:

\[
a_0 + b_0(V^*) = a_1 + b_1 (*)
\]

\( V^* = \frac{(a_1-a_0)}{(b_0-b_1)} = \frac{\text{FC}}{\text{Reduction in VC}} \)

If VC savings are minor, or if the increase in fixed costs is high, then you need higher volume to justify the high fixed cost option.

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**Which Site is Best for Asphalt Mixing Plant?**

Cheaper (A) or Closer (B)?

*(Example 2-2, EE, pp. 27-28)*

<table>
<thead>
<tr>
<th>Site</th>
<th>Site A</th>
<th>Site B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rental</td>
<td>4 mo x $1000/mo</td>
<td>4 mo x $5,000/mo</td>
</tr>
<tr>
<td>Setup</td>
<td>$15,000</td>
<td>$25,000</td>
</tr>
<tr>
<td>Flagman</td>
<td>0</td>
<td>$96/day</td>
</tr>
<tr>
<td>Total FC</td>
<td>$19,000</td>
<td>$53,000</td>
</tr>
<tr>
<td>Transport Cost/Cu. Yd.</td>
<td>6 mi x $1.15/mi</td>
<td>4.3 mi x $1.15/mi</td>
</tr>
<tr>
<td>Transport, total</td>
<td>$345,000</td>
<td>$247,250</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$364,000</td>
<td>$300,410</td>
</tr>
</tbody>
</table>

Breakeven Vol. = \((53,160 - 19,000) / (1.7 x 1.15)\) = 17,473 cu yd

**Comments on Breakeven Analysis**

- YOU (the contractor) know YOUR cost and YOUR technology
- You (the analyst) can build an algebraic expression to represent YOUR costs
- You can substitute variables to reflect options and technologies available
- You can find breakeven points quite readily
- Hints of other problems
  - Need for flagman suggests there may be congestion
  - Is your haulage cost model correct?
  - Does higher rent suggest a tonier neighborhood where you may be regarded as a nuisance?

**More Comments - CEE Projects**

- Typical major projects reduce both marginal and average costs per unit of capacity
- Will there be sufficient demand to allow prices that cover average costs?
- In general, smaller projects will be better at low volumes until poor service and congestion hurt performance

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**Non-Linear Cost Functions**

\[
\text{Avg Cost} = \frac{C(V)}{V} \\
\text{Marginal Cost} = \frac{dC}{dV} \\
\text{Incremental Cost} = \frac{(C(V_1) - C(V_0))}{(V_1 - V_0)}
\]

If marginal cost exceeds average cost, then average costs are rising
Some Other Cost Terminology

- **Opportunity Cost**
  - A key economic concept! What else could be done with these resources?

- **Sunk Cost**
  - Expenditures that cannot be recovered and that are common to all options and therefore can be ignored ("focus on the differences")

- **Direct, Indirect, and Standard Costs**
  - **Direct** - easily related to a measurable activity or output
  - **Indirect** (or overhead or burden) - their costs related to the overall operation
  - **Standard costs** - used in budgeting, estimating & control

- **Recurring vs. Non-recurring costs**
  - **Recurring** - repetitive; could be fixed or variable
  - **Non-recurring** - typically the one-time expense of getting started

- **Cost vs. Expense**
  - "Expense" is a specific cash or other expenditure that can be followed in the accounting system
  - Depreciation is a non-cash expense - according to tax rules
  - Repayment of principal on a loan is definitely cash, but not a current expense item
  - "Cost" can refer to non-financial matters, such as lost time, aggravation, or pollution

Even More Cost Terminology

- **Lifecycle Cost** - A Key Concept for CEE Project
  - Design
  - Construct
  - Expand
  - Operate
  - Decommission
  - Salvage

- **Lifecycle Cost - Greatest Potential For Lifecycle Savings is in Design!**

  - Easy to modify design and materials
  - Limited ability to modify infrastructure or operation
  - Few options - cost already incurred

Basic Economic Concepts - Differing Perspectives of Economists and Engineers

- **Production functions**
  - Economists either assume this is known or try to estimate a very aggregate model based upon actual performance
  - Engineers are constantly trying to "improve productivity", i.e. find better ways to use resources to produce more or better goods and services

- **Cost functions**
  - Both use total, average, variable, and marginal costs; engineers go into much greater detail than economists
  - Short-run and long-run cost functions
  - Economists typically focus on effects of volume and prices
  - Engineers typically focus on costs and capacity

- **Duality of production and cost functions**

Production Function

The production function describes the technology of a system, i.e. the maximum output that will be given by a given set of inputs. This can be expressed by a simple (but not very descriptive!) equation:

\[ F(q, x; \theta) = 0 \]

where

- \( F \) = some as yet unspecified functional relationship
- \( q \) = vector of outputs
- \( x \) = vector of inputs
- \( \theta \) = vector of service quality factors (if not included in \( q \))
Using a Production Function

- If we have the prices of the inputs, then we can find the most efficient (i.e., minimum cost) way to produce a given level of output.
- If we have an excellent understanding of the production function, then we can predict what resources will be needed to provide different levels of output.
- Even if we only have a rather aggregate understanding of the production function, we may still be able to understand how resource requirements will change at different levels of production (e.g., are there economies of scale, scope, or density?).

Duality of Production Functions and Cost Functions

- We can study productivity and other issues by considering either the production function or the cost function.
  - With a production function and output prices, we can find the best use of a given level of inputs.
  - Maximize production subject to resource constraints.
  - With a cost function, we can find the least cost means of producing a given level of output.
  - Minimize cost subject to providing the desired level of output.
- Since costs are easier to observe than technological possibilities, much economic research and most managerial decisions deal with cost functions rather than production functions.
- Engineers are often immersed in technology, which in effect is the production function, as they seek better ways of providing a service.

Cost Functions

- Minimum cost for producing output q given:
  - Production function
  - Supply relationships for inputs (i.e., prices for the required inputs as a function of the volume and location of the inputs required).
- A special case: linear costs, where w is the cost of each input x.
  - \( C(q, w; \theta) = \sum w_i x_i = \min(\sum w x) \)
  - s.t. \( P(q, w; \theta) = 0 \)

Long-Run & Short-Run Costs

- Long-run costs:
  - All inputs can vary to get the optimal cost.
  - Because of time delays in reaching equilibrium and the high costs of changing transportation infrastructure, this may be a rather idealized concept in many systems!
- Short-run costs:
  - Some (possibly many) inputs are fixed.
  - The short-run cost function assumes that the optimal combination of the optional inputs are used together with the fixed inputs.

Methods of Estimating Costs

- Accounting:
  - Allocate expense categories to services provided using:
    - Detailed cost data from accounting systems.
  - Activity data from operating MIS.
- Engineering:
  - Knowledge of technology (possibly new technology) and operating capabilities.
  - Prices of inputs.
- Econometric:
  - Knowledge of total costs for a varied set of firms or conditions.
  - Aggregate data representing inputs and system characteristics.

Engineering Costs

- Engineers need to examine the costs of different technologies and operating strategies, so historical costs may not be relevant.
- When pushing the limits of technology (e.g., heavy axle loads or congested highways), it is necessary to include some science in the cost models.
- Engineering models can go to any required level of detail, so long as there is some scientific or historical evidence available.
- Most researchers work with some sort of engineering models as they examine the performance of complex systems.
Accounting Costs

Every company and organization will have some sort of accounting system to keep track of expenses by (very detailed) categories. These costs can readily (and possibly correctly or at least reasonably) be allocated to various activities, such as:
- number of shipments
- number of terminal movements
- vehicle-miles

This allows a quick way to estimate the average costs associated with each activity, which can be used to build a cost model (which can be quite useful even though they tend to be disparaged by both engineers and economists!)

Refinements can reflect which elements of expense are fixed and which are variable.

Econometric Cost Models

Deal with the complexity problem by assuming a simplified, more aggregate cost model

- Calibrate using available data
- Structure so that it can be calibrated using standard regression analysis
- Structure so that its parameters are in themselves interesting, e.g. the marginal product of labor
- Focus on specific parameters of interest in current policy debates.

Engineering Cost: Cost Elements to Consider

Cost = Owner's Cost + User's Cost + Externalities

<table>
<thead>
<tr>
<th>Elements</th>
</tr>
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<tbody>
<tr>
<td>Construction</td>
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<tr>
<td>Maintenance</td>
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<tr>
<td>Operations</td>
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<tr>
<td>Insurance</td>
</tr>
<tr>
<td>Taxes</td>
</tr>
<tr>
<td>Utilities</td>
</tr>
<tr>
<td>Efficiency of working space</td>
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<tr>
<td>Access time</td>
</tr>
<tr>
<td>Amenities</td>
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<tr>
<td>Safety</td>
</tr>
<tr>
<td>Land Use</td>
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<tr>
<td>Air quality</td>
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<tr>
<td>Noise</td>
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<tr>
<td>Water quality</td>
</tr>
<tr>
<td>Aesthetics</td>
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<tr>
<td>Risks</td>
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</tbody>
</table>

Summary: Comparison of Costing Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Main Use</th>
<th>Strengths</th>
<th>Weaknesses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accounting</td>
<td>Internal costing systems, Planning</td>
<td>Actual data, Consistent with MIS</td>
<td>Limited to historical experience and technologies, Limited by structure of MIS</td>
</tr>
<tr>
<td>Engineering</td>
<td>Investment planning, Technology assessment, Service design, Strategic planning</td>
<td>Can deal with new technologies, operating practice, or networks</td>
<td>May not match history, Analysis may be &quot;idealized&quot;</td>
</tr>
<tr>
<td>Statistical</td>
<td>Public policy, Research, Pricing, Strategic planning</td>
<td>Can estimate economic parameters, Minor data requirements</td>
<td>Limited to historical conditions, not meaningful to managers</td>
</tr>
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