

1.017/1.010 Class 4

Joint Probability, Independence, Repeated Trials

Joint Probabilities and Independence

Joint probability of 2 events A and B defined in the same sample space (probability that outcome lies in A **and** B):

$$P(AB) = P(C) \ ; \ \text{where event } C = A \cap B = AB$$

If A and B are **independent** then:

$$P(AB) = P(A)P(B)$$

Note that **mutually exclusive events are not independent** since if one occurs we know the other has not.

Example:

Consider the following events A and B defined from a die toss experiment with outcomes $\{1, 2, 3, 4, 5, 6\}$

$$A = \{2, 4, 6\} \quad B = \{1, 2, 3, 4\}$$

Then:

$$P(A) = 1/2, P(B) = 2/3, P(AB) = 2/6 = P(A)P(B)$$

So A and B are independent.

Composite experiments

Related experiments are often conducted in a sequence.

For example, suppose we toss a fair coin (with 2 equally likely outcomes $\{H, T\}$) and then throw a fair die (with 6 equally likely outcomes $\{1, 2, 3, 4, 5, 6\}$). This process can be viewed as two separate experiments E_1 and E_2 with different sample spaces.

Or ... it can be viewed as a single composite experiment E (with 12 ordered equally likely outcomes $\{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$).

Events defined in E_1 and E_2 have equivalent events in E .

Example:

$$A_2 = \{2, 3\} \text{ in } E_2 \text{ corresponds to } A = \{H2, H3, T2, T3\} \text{ in } E.$$

A particular ordered sequence of events from E_1 and E_2 also has an equivalent event in E :

Example:

$$A_1 = \{H\} \text{ in } E_1 \text{ then } A_2 = \{2, 3\} \text{ in } E_2 \text{ corresponds to } A = \{H2, H3\} \text{ in } E.$$

Suppose that A is the composite experiment event that corresponds to event A_1 from experiment E_1 and then event A_2 from experiment in E_2 .

A_1 and A_2 are **independent** if:

$$P_E(A) = P_{E_1}(A_1)P_{E_2}(A_2)$$

The subscript on each probability identifies the corresponding experiment and sample space.

The events A_1 and A_2 defined in the above coin toss/die roll example satisfy the independence requirement.

Repeated trials

Repeated identical experiments are called **repeated trials**.

Example:

Consider a composite experiment composed of 3 successive fair coin tosses.

This experiment can yield $2^3 = 8$ equally likely ordered outcomes:

$$\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

The probability of the event $A = \{\text{exactly 2 heads in 3 tosses}\}$ is the fraction of total number of outcomes that yield exactly 2 heads:

$$P(A) = 3/8$$

Now consider a particular composite experiment event:

$$A_1 = \{H\} \text{ then } A_2 = \{H\} \text{ then } A_3 = \{T\}$$

Suppose the repeated trials are independent. Then the probability of this composite event is:

$$P(A_1 \text{ then } A_2 \text{ then } A_3) = P(A_1)P(A_2)P(A_3) = (1/2)(1/2)(1/2) = 1/8.$$

This is one of 3 mutually exclusive repeated trial event sequences that yield exactly 2 heads. It follows that the probability of exactly 2 heads is $3(1/8) = 3/8$. Since this is equal to the probability obtained from the composite experiment the independence assumption is confirmed.



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