MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.017/1.010 Computing and Data Analysis for Environmental Applications/Uncertainty in Engineering

---

Problem Set 2: Combinatorial Methods for Deriving Probabilities (Solutions provided at end of each problem)
Due: Thursday, September 25, 2003

Problem 1

This is a classic combinatorial probability problem that can be solved by applying the counting and combinatorial principles from Lecture 4. Suppose that three balls are drawn successively and randomly from a box containing 6 red balls, 4 white balls, and 5 blue balls that are identical except for their colors. Determine the probability that these balls are drawn **in the order** red, white, blue if 1) each ball **is replaced** after it is drawn 2) each ball **is not replaced** after it is drawn. Note that since order is important you need to consider permutations rather than combinations when evaluating the number of outcomes for each event.

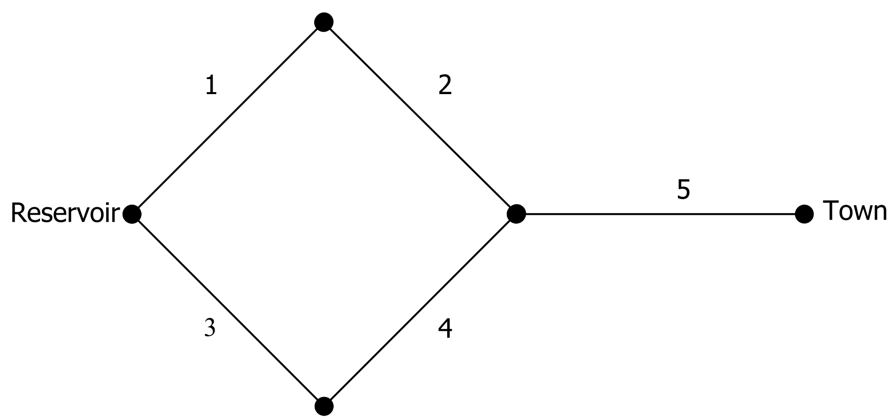
Problem 1 Solution:

```
% 1.017 Problem Set 2 -- Problem 1
% Combinatorially:
% with replacement: P(r,w,b)=
%(P 1,6 * P 1,4 *P 1,5)/ 15^3 = .0356
% without replacement: P(r,w,b)=
% (P 1,6 * P 1,4 *P 1,5)/ P 3,15 = .044
% Monte Carlo with replacement:
clear all
%generate an array of random numbers
totalnum=10000;
picks=rand(totalnum,3);
for i=1:totalnum
     rwb(i)=picks(i,1)<.4&picks(i,2)>= …
     .4&picks(i,2)<(2/3)&picks(i,3)>=(2/3);
end
prob1_rep=sum(rwb)/totalnum
% without replacement
for i=1:totalnum
    balls=[ones(1,6),2*ones(1,4),3*ones(1,5)];
    for j=1:3
        number=unidrnd(length(balls));
```

```
        draw(i,j)=balls(number);
        balls(number)=[];
    end
    rwb2(i)=draw(i,1)==1&draw(i,2)==2&draw(i,3)==3;
end
prob1_norep=sum(rwb2)/totalnum
```

Problem 2

The figure provided below shows a simple (five pipe) water distribution network designed to deliver water from a reservoir to a town. Suppose that the probability of any single pipe failing during an 7.0 Richter earthquake is *p*. What is the probability that some water will still get through after such an earthquake? Evaluate for *p*=0.3.



Problem 2 Solution:

```
% 1.017 Problem Set 2 -- Problem 2
% combinatorially:
% P(get water)=P(1,2,5 good)+P(3,4,5 good)-P(all good)
% (don't double count)
% probwater=(0.7)^3+(0.7)^3-(0.7)^5
% simulating:
clear all
totalnum=10000;
pfails=0.3;
pnofail=1-pfails;
randoms=rand(totalnum,5);
nofail=randoms<=pnofail;
% using vectors
a=nofail(:,1)==1&nofail(:,2)==1&nofail(:,5)==1;
b=nofail(:,3)==1&nofail(:,4)==1&nofail(:,5)==1;
```

```
c=nofail(:,1)==1&nofail(:,2)==1&nofail(:,5)==1&nofail(:,3)=
=1&nofail(:,4)==1;
probwater1=(sum(a)+sum(b)-sum(c))/totalnum
% using ifs
for i=1:totalnum
    if nofail(i,1)==1&nofail(i,2)==1&nofail(i,5)==1
        count(i)=1;
    elseif nofail(i,3)==1&nofail(i,4)==1&nofail(i,5)==1
        count(i)=1;
   else
        count(i)=0;
    end
end
probwater2=sum(count)/totalnum
```

Problem 3

In the game of poker five cards are drawn from a pack of 52 well shuffled cards. Find the probability of drawing 3 tens and 2 jacks.

Problem 3 Solution:

```
% 1.017 Problem Set 2 -- Problem 3
clear all
% combinatorics:
% total outcomes = C 5,52
% Tens: C 3,5 =10; Jacks: C 2,5 =10
prob1=10*10/(factorial(52)/factorial(5)/factorial(52-5))
totnum=100000;
pick=ceil(52*rand(totnum,5));
% don't pick the same card twice (w/o replacement)
% tens are 1-4, Jacks are 5-8 (random assignment)
% let's give 10's a point value of 1 and jacks a point
value of 10.
cards=[ones(1,4),10*ones(1,4),zeros(1,44)];
for i=1:totnum
    addemup(i)=sum(cards(pick(i,:)));
end
howmany=addemup==23;
prob2=sum(howmany)/totnum
```

Problem 4

Suppose that cracks are present in 8 out of 20 bridges in a particular city. If a sample of 5 bridges is inspected at random what is the probability that exactly 4 will these will have cracks? At least 4?

Problem 4 Solution:

```
% 1.017 Problem Set 2 -- Problem 4
% Analytically:
% P(4)= (C4,8*C 1,12)/ C 5,20 = .0542
% P(4+)=P(4)+P(5)=.0542+ (C 5,8 / C 5,20)=.0578
clear all
totnum=10000;
% initialize a matrix of picks for each replicate
cracked=zeros(totnum,5);
for i=1:totnum
    bridgeset=[ones(1,8),2*ones(1,12)];
    for k=1:5
        pick=unidrnd(length(bridgeset));
        bridge=bridgeset(pick);
        cracked(i,k)=bridge==1;
        bridgeset(pick)=[];
    end
     fourcracks(i)=(sum(cracked(i,:)))==4;
     fourorfive(i)=(sum(cracked(i,:)))== …
     4|(sum(cracked(i,:)))==5;
end
prob4cracked=sum(fourcracks)/totnum
prob4morecracked=sum(fourorfive)/totnum
```

Problem 5 (another classic)

What is the probability that at least two students in a class of 30 students (e.g. 1.017/1.010) will have the same birthday? This problem requires the computation of very large factorials, which is best done on MATLAB or a calculator.

Problem 5 Solution:

```
% 1.017 Problem Set 2 -- Problem 5
clear all
close all
% first do it with combinatorial methods:
% P=1-(365/365)(364/365)(363/365)...(336/365) = .706
prob=1;
for i=1:30
    temp=(365-(i-1))/365;
    prob=prob*temp;
end
prob1=1-prob
% now let's simulate it:
nrep=1000;
for i=1:nrep
```

```
    bdays=unidrnd(365,1,30);
    for j=1:365
        howmany(j)=sum(bdays==j);
    end
        yes(i)=sum(howmany>1);
end
count=yes>0;
prob2=sum(count)/nrep
```

Problem 6

Write a MATLAB program that duplicates one of the 5 experiments described in the above problems (your choice). Use a virtual experiment/relative frequency approach to compute all probabilities. Use at least 10,000 replicates to obtain a reasonable estimate of the exact probabilities. Please contact us if you need help with the required MATLAB programming. Also, feel free to do more than one program if you like (it's optional but we will check your codes for you).

Problem 6 Solution:

See above MATLAB programs