Example: Simple Probabilistic Model of Rainfall

Consider a series of 365 consecutive days that may have rain or no rain. The conditional probabilities of rain today are:

\[ P(\text{rain today} \mid \text{rain yesterday}) = 0.6 \]

\[ P(\text{rain today} \mid \text{no rain yesterday}) = 0.2 \]

Assume that the annual series starts with a no rain day (Day 0).

The intensity of rainfall on rainy days is described by an exponentially distributed random variable (see MATLAB help `exprnd` documentation) with a parameter (mean value) of:

\[ \text{MU}=10 \text{ mm/day}. \]

This rainfall model accounts for the persistence of rainy or dry weather over time. It also presumes that low rainfall intensities are more likely than high intensities.

Write a program to generate many (e.g. 1000) replicates of the 365 probabilistic rainfall series described above. Carry out the following tasks:

1. Plot the rainfall intensity vs. day (1-365) obtained for 1 replicate. Use MATLAB’s `stairs` plot command.

2. Derive the probabilities of the following events:
   
   A: The rainfall intensity exceeds 50 mm/day for at least one day during the year
   
   B: The total annual rainfall (summed over all 365 days) exceeds 1500 mm.
   
   C: During the year there is at least 1 drought period with no rain for 30 or more days.
   
   Hint for C: Use the MATLAB `find` function to obtain a vector of rainy day numbers (scattered between 1-365). Then use the
MATLAB `diff` function to obtain a vector of differences between these rainy day numbers. These differences are the lengths of the dry periods between rainy days. Then determine how many of the dry period lengths are greater than or equal to 30 days.

3. Turn in your MATLAB program listing together with the plot from 1). Write the probabilities from 2) on your listing.

These probabilities are useful for assessing the risks of floods and droughts but are quite difficult to derive without carrying out a virtual simulation.