Macroscopic traffic models

Traffic Flow Theory

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1.041/1.200 Transportation: Foundations and Methods
Readings

Outline

1. Basic assumptions of traffic flow theory
2. Fundamental diagrams (FDs)
3. Highway delay problem
Unit 1: Traffic flow fundamentals

LAB 1: Build your own traffic jam
LAB 2: Build a queuing model for Seattle transit
LAB 3: Build an AI agent to optimize traffic
LAB 4: Solve the traveling salesman problem

Unit 1: Modeling
Deterministic
Outline

1. Basic assumptions of traffic flow theory
   a. Key variables
   b. Time vs space means

2. Fundamental diagrams (FDs)

3. Highway delay problem
Traffic flow theory

- Today: From traffic flow (traffic streams) to traffic flow theory

- **Traffic flow theory:**
  - Models and hypotheses for explaining traffic flow
  - i.e., what would happen to traffic streams if they were to flow on roads under different conditions, potentially not yet observed

- Models vs data

\[ a_{IDM}(s, v, \Delta v) \]
Basic assumptions

1. Study of a single traffic stream, flowing on a facility with a single entrance and a single exit

2. Uninterrupted traffic
   • Traffic regulated by interactions between vehicles, as opposed to being regulated by external means
   • E.g. on a highway or at unsignalized intersections, as opposed to traffic lights, stop signs.

3. Stationary traffic conditions (vs. time and space-varying dynamics)
Stationary vs non-stationary traffic

Stationary traffic conditions (vs. time- or space-varying dynamics):
- Traffic is stationary if it is a superposition of families of trajectories that are each parallel and equidistant.

Examples of non-stationary traffic
Traffic stream variables

- Main variables
  - Flow
  - Time headway
  - Density
  - Spacing
  - Speed (space-mean, time-mean)

- Aim: Obtain relationships that hold “on average”; i.e. for large stationary time-space regions containing many vehicles
Formulas for traffic characteristics

**Table 4.1.** Generalized formulas for various traffic characteristics using two observation methods. Boxed expressions correspond to the original definitions introduced in Chapter 1:

<table>
<thead>
<tr>
<th>Method of Observation</th>
<th>Instantaneous photograph at time $t_0$ (section length, $L$)</th>
<th>Observation from a fixed location $x_0$ (duration, $T$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $k(A)$</td>
<td>$\frac{n}{L}$</td>
<td>$\frac{1}{T} \sum_{j=1}^{m} p_j = \frac{1}{T} \sum_{j=1}^{m} \frac{1}{u_j}$</td>
</tr>
<tr>
<td>Flow, $q(A)$</td>
<td>$\frac{1}{L} \sum_{i=1}^{n} v_i$</td>
<td>$\left[ \frac{1}{m} \sum_{j=1}^{m} p_j \right]^{-1} = \left[ \frac{1}{m} \sum_{j=1}^{m} \frac{1}{u_j} \right]^{-1}$</td>
</tr>
<tr>
<td>Space-mean speed, $v(A)$</td>
<td>$\frac{1}{n} \sum_{i=1}^{n} v_i$</td>
<td>$\frac{1}{m} \sum_{j=1}^{m} p_j$</td>
</tr>
<tr>
<td>Average pace, $p(A)$</td>
<td>$\left[ \frac{1}{n} \sum_{i=1}^{n} v_i \right]^{-1}$</td>
<td>$\frac{1}{m} \sum_{j=1}^{m} p_j$</td>
</tr>
<tr>
<td>$t(A)$</td>
<td>ndt</td>
<td>$\frac{m}{dx} \sum_{j=1}^{m} p_j$</td>
</tr>
<tr>
<td>$d(A)$</td>
<td>$dt \sum_{i=1}^{n} v_i$</td>
<td>$mdx$</td>
</tr>
</tbody>
</table>

- **Notation**
  - $v_i$ = velocity (mi/hr) of vehicle $i$ in section $L$ at time $t_0$
  - $p_j$ = pace (hr/mi) of vehicle $j$ during duration $T$ at location $x_0$
  - $u_j$ = velocity (mi/hr) of vehicle $j$, $u_j = \frac{1}{p_j}$
  - $t(A)$ = total time spent in $A$ by all vehicles
  - $d(A)$ = total distance traveled by vehicles in $A$

- If traffic is stationary, then the two columns coincide, i.e. same values.
  - Proof: $v = v_i = \frac{1}{p_j}, \forall i, j$

- Then, for instance, density can be measured by counts at a fixed location.

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Outline

1. Basic assumptions of traffic flow theory
   a. Key variables
   b. Time vs space means

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Time and space means

- Space-mean: averages taken at an instant over a space interval
- Time-mean: averages taken at a specific location (with time-varying over an interval)
- Speed:
  - $\bar{v}_s$: space-mean speed
  - $\bar{v}_t$: time-mean speed

Other vehicle characteristics can be averaged across space or time. E.g., occupancies (number of persons per vehicle), energy consumption, emissions, etc.

- There is no a priori reason to expect averages taken across space or time to be the same.
- Example: You own two cars, they are both driven an equal distance of 100 miles. One gets 20 miles per gallon (mpg), the other 50 mpg. Is the average mpg 35 (i.e. $\frac{50+20}{2}$)?
Time and space means

Table 4.1. Generalized formulas for various traffic characteristics using two observation methods. Boxed expressions correspond to the original definitions introduced in Chapter 1:

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**Notation:**
- $v_i$ = velocity (mi/hr) of vehicle $i$
- $p_j$ = pace (hr/mi) of vehicle $j$
- $u_j$ = velocity (mi/hr) of vehicle $j$, $u_j = \frac{1}{p_j}$

**If traffic is stationary, then time-mean speed = space-mean speed.**
- Proof: $v = v_i = \frac{1}{p_j}$, $\forall i, j$

Time and space means in practice

- **Time-mean speeds:** Often how dual inductance loop detectors in traffic management systems are configured
  - Ex. arithmetic average of vehicle speeds over 20-second intervals

- **Space-mean speeds:** In nearly all cases of traffic analysis, space-mean speeds should be used
  - Statistically more stable in short segments/durations
  - Weighs slower vehicles’ speeds more heavily

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Time and space means in practice

- In practice, time-mean and space-mean speeds differ by 1-5%
- Differences are greater when there is more variability in speed (more congestion)

<table>
<thead>
<tr>
<th>Data Items</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
<th>Sum</th>
<th>Average</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time (sec)</td>
<td>153</td>
<td>103</td>
<td>166</td>
<td>137</td>
<td>127</td>
<td>686</td>
<td>137.2</td>
<td></td>
</tr>
<tr>
<td>Running Time (sec)</td>
<td>142</td>
<td>103</td>
<td>141</td>
<td>137</td>
<td>127</td>
<td>650</td>
<td>130.0</td>
<td></td>
</tr>
<tr>
<td>Stopped Delay Time (sec)</td>
<td>11</td>
<td>0</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>36</td>
<td>7.2</td>
<td></td>
</tr>
<tr>
<td>Average Travel Speed (km/h)</td>
<td>44.7</td>
<td>66.4</td>
<td>41.2</td>
<td>49.9</td>
<td>53.9</td>
<td>256</td>
<td>51.2</td>
<td>95</td>
</tr>
<tr>
<td>Average Running Speed (km/h)</td>
<td>48.1</td>
<td>66.4</td>
<td>48.4</td>
<td>49.9</td>
<td>53.9</td>
<td>n.a.</td>
<td>52.6</td>
<td></td>
</tr>
</tbody>
</table>

Section Length = 1.9 km

Difference between Time-Mean Speed and Space-Mean Speed
- Time-Mean Speed = \( \frac{\sum \text{speeds}}{\text{no. of runs}} \) = \( \frac{256}{5} = 51.2 \text{ km/h} \)
- Space-Mean Speed = \( \frac{\text{no. of runs} \times \text{distance}}{\sum \text{(travel times)}} \) = \( \frac{5 \times 1.9 \times 686}{49.8} = 49.8 \text{ km/h} \)
- Therefore, difference = \( \frac{1.4}{51.2} \text{ km/h} \)

Check Equation 1-5: Time-Mean Speed = 49.8 + 95 / 49.8 = 51.7 km/h = 51.2 km/h
Outline

1. Basic assumptions of traffic flow theory

2. Fundamental diagrams (FDs)
   a. Basic relationship
   b. FDs versus time-space diagrams

3. Highway delay problem
Basic relationship of traffic flow

\[ q = v_s k \]

where:
- \( q \): flow [veh/h]
- \( v_s \): speed (space-mean speed) [mi/h] or [km/h]
- \( k \): density [veh/mi] or [veh/km]

These are the three fundamental variables of traffic flow.
Traffic stream models

(1) Speed-density model

- Greenshields (1935), seminal work, assumes a linear relationship between speed and density
- From experimental data:
  - Light traffic $\rightarrow$ high speed
  - Heavy traffic $\rightarrow$ low speed (near zero).
- $v_f$: free flow speed
- $k_{cap} = k_j$: jam density
Traffic stream models
Traffic stream models are interrelated

(1) Speed-density model

- Greenshields (1935), seminal work, assumes a linear relationship between speed and density
- \( v_f \) : free flow speed
- \( k_{cap} = k_j \) : jam density
- What is the corresponding relationship between flow and density?
Interpretation of traffic stream models

- Each diagram relates the three fundamental variables, and are therefore called fundamental diagrams.
- For a given road, a fundamental diagram is fitted based on measurements
  - Points on the diagram describe possible traffic conditions
  - These relationships are postulated to be true "on average"
Fundamental Diagram in practice

- Paris: boulevard périphérique
- Three locations (detectors)
- Measurements of car passages (traffic counts) and occupancies for each detector over a chosen time interval
- Source: Papageorgiou et al. (1990) Modelling and real-time control of traffic flow on the southern part of Boulevard Peripherique in Paris: Part I: Modelling, Transportation Research: Part A
Fundamental Diagram in practice

- Flow-density model
- Measurements of car passages (traffic counts) and occupancies for each detector over time interval: 6am-10am of Nov. 27 1987
- Flow-density (volume-occupancy) diagrams taken over one-minute-intervals for a given location
- Traffic Occupancy (in %) (Def.: % of loop occupancy in a given time period)
- Conditions: heavily congested traffic after 7:40am, rainy
Triangle model

(3) Flow-density model
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Fundamental diagram + (t,x)-diagram

Source: C. Daganzo (1997)
Recall: traffic waves

Vehicle trajectories (Sugiyama et al. 2008)
Traffic waves: fundamental diagram characterization

Source: C. Daganzo (1997)
Outline

1. Basic assumptions of traffic flow theory
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Highway delay problem

A freeway exhibits a triangular flow-density relation with parameters: $v_f$ (free flow speed), $q_{max}$ (capacity) and $k_j$ (jam density)

1. Plot the fundamental diagram and derive an expression for the function that gives the (space-mean) speed as a function of density inside a queue.
   • Don’t forget to specify the range of $k$ for which the equation holds.

2. If $k_j = 600$ veh/mile, $v_f = 1$ mile/min and $q_{max} = 100$ veh/min. Determine the delay experienced by a vehicle that joins a 2 mile queue caused by a bottleneck that flows at $q = 50$ veh/min.

Source: C. Daganzo
Highway delay problem

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\( v_f \) (free flow speed), \( q_{\text{max}} \) (capacity) and \( k_j \) (jam density)

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References


2. Prof. Nikolas Geroliminis’ lecture Fundamentals of Traffic Operations and Control, Spring 2010 EPFL


5. Fred Hall (1997) Traffic stream characteristics. (available online)

6. Many slides adapted from Carolina Osorio