1.138J/2.062J/18.376J, WAVE PROPAGATION

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Homework no. 2

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In all exercises, please describe the physical meaning of your mathematical results. Use graphics if it can help the explanation. If you do any numerical computations, feel free to use Matlab.

1. Reflection from a semi-infinite rod. Consider the longitudinal waves in a semei-infinite elastic rod of uniform cross section. The end at $x = 0$ is stress-free. There is no external stress along the rod. The initial displacement and velocity are :

$$u(x,0) = f(x), \quad \frac{\partial u(x,0)}{\partial t} = g(x), \quad x > 0.$$

Find the deflection in the rod for all time $t > 0$ by using the method of images.

2. Read §1. Chapter one, Notes.

Consider an infinitely long string taut with tension T, $-\infty < x < \infty$ free from any lateral support. A concentrated mass M is attached to the string at the origin. Show first that Newton's law for the mass requires that

$$M\frac{\partial^2}{\partial t^2}V_0(t) = -T\frac{\partial}{\partial x}V_-(0-,t) + T\frac{\partial}{\partial x}V_+(0+,t), \quad t > 0. \tag{H.2.1}$$

where $V_0(t)$ is the displacement of the mass, V_- the string displacement on the left side ($x < 0$) and V_+ the string displacement on the right ($x > 0$).

An incident pulse with finite extent $V_I(x,t)$ arrives from $x \sim -\infty$. Its front arrives at $x = 0$ when $t = 0$, i.e., $V_I(0,0) = 0$. Find the reflected and the transmitted waves and the motion of the mass for all $t > 0$.

Suggestions:

Take as the solution:

$$V_-(x,t) = V_I\left(t - \frac{x}{c}\right) + V_R\left(t + \frac{x}{c}\right), \quad x < 0$$

$$V_+(x,t) = V_T\left(t - \frac{x}{c}\right), \quad x > 0.$$

here the subscripts mean: $I=$ incident, $R=$ reflected and $T=$ transmitted. From the boundary condition at $x = 0$ find a differential equation for $V_0(t)$. State proper initial conditions and solve for $V_0(t)$, hence get $V_R\left(t + \frac{x}{c}\right)$ and $V_T\left(t - \frac{x}{c}\right)$.

To see the physics more explicitly , you may specify the pulse, e.g., half of a sine curve and carry out the necessary integration.

3. Two semi-infinite cylindrical rods of different materials but the same uniform cross section S are butted toegether at $x = 0$. The elastic constant is E_1 in $x < 0$ and E_2 in $x > 0$. At $t = 0$ the rod on the left has a nonuniform displacement but no velocity

$$u(x,0) = f(x), \quad \frac{\partial u}{\partial t}(x,0) = 0, \quad x < 0$$

where $f(x)$ is nonzero only in a finite domain. The rod on the right is free of initial deformation and velocity

$$u(x,0) = 0, \quad \frac{\partial u}{\partial t}(x,0) = 0, \quad x > 0$$

. Find the solution in both rods for all $t > 0$. Note that in the left rod there will be a left-going (reflected) wave after some time. In the right rod there is only a right-going wave for all time.

3. Consider forced waves governed by

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = h(x,t) \tag{H.2.2}$$

where the forcing is limited in range and duration so that h is constant h_o in a rectangular region in the $x \sim t$ plane and zero elsewhere, i.e.,

$$h(x,t) = \begin{cases} h_0, & -L < x < L, 0 < t < T \\ 0, & \text{otherwise} \end{cases}$$

where $L = cT$.

Find the wave for all $t > T$ and $|x| < \infty$.