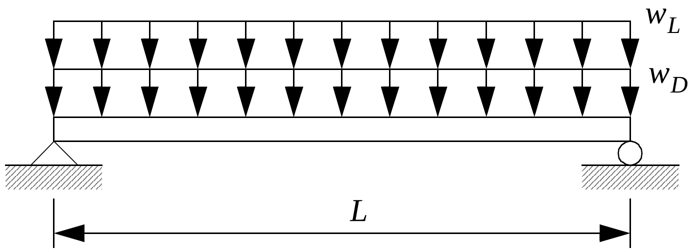
Examination, October 19, 2004

1.561 Motion-Based Design

Solutions

Problem #1




Consider a simply supported beam subjected to 2 uniform loadings, $w_D$ and $w_L$. Strength based design works with the total loading and selects the cross-sectional moment of inertia such that the maximum stress is less than an allowable value, $\sigma*$. Motion based design selects the moment of inertia such that the maximum displacement is less than the allowable value, $u*$. The design code for bridges works with the live load, $w_L$, and requires

$$u* \le \frac{L}{800} \qquad \text{for} \qquad w = w_L$$

a) Derive expression for $I$ corresponding to the 2 approaches. Note that the mid-span displacement for a simply supported beam subjected to a uniform loading is

$$u = \frac{wL^4}{128EI}$$

b) Determine the ratio of $I|_{motion}$ to $I|_{strength}$.

c) Take

$$(L/d) = 12$$

$$E = 3 \times 10^4 \text{ ksi}$$

and $\quad w_L = w/2$

Suppose one wants to use a very high-strength steel, such as $\sigma_y* = 100$ ksi. Is this choice appropriate? Discuss.

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a)

Displacement criteria yields

$$u = \frac{w_L L^4}{128EI} \le u* = \frac{L}{800}$$

giving

$$\boxed{I_{motion} \ge \frac{800}{128}\frac{w_L L^3}{E}}$$

For Strength criteria, need to find $M_{max}$ which occurs at $L/2$

$$M_{max} = \frac{wL}{2}\frac{L}{2} - w\frac{L}{2}\frac{L}{4} = \frac{wL^2}{8}$$

Taking $d$ as the beam depth and $w = w_L + w_D$

$$\sigma^* \geq \frac{M_{max}d}{2I} = \frac{(w_L + w_D)L^2 d}{16I}$$

giving

$$\boxed{I_{strength} \geq \frac{(w_L + w_D)L^2 d}{16\sigma^*}}$$

b)

$$\frac{I_{motion}}{I_{strength}} = \frac{\frac{800}{128}\frac{w_L L^3}{E}}{\frac{(w_L + w_D)L^2 d}{16\sigma^*}} = \boxed{\frac{100\sigma^* L}{Ed}\frac{w_L}{(w_L + w_D)}}$$

c)

Subbing $(L/d) = 12$, $E = 3\times10^4$ ksi, and $w_L = w/2$ into the result from part b), we get

$$\boxed{\frac{I_{motion}}{I_{strength}} = 0.02\sigma^*}$$

Therefore, $I_{motion}$ will dominate when $0.02\sigma^* > 1$ or, equivalently, $\sigma^* > 50$ ksi.

Using $\sigma_y^* = 100$ ksi is not appropriate since the motion constraint will dominate and determine the required $I$. Hence, one could use 50 ksi steel instead of 100 ksi steel. The same $I$ would be required, but the structure would be less expensive.

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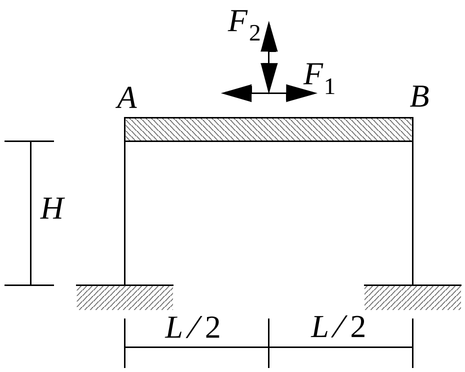
Problem #2




Assume member AB is infinitely stiff. A rotating machine located at mid-span generates horizontal and vertical periodic forces $F_1$ and $F_2$.

$$F_1 = p\sin\Omega t$$

$$F_2 = p\cos\Omega t$$

Describe how you would select the cross-sectional properties of the columns using a motion based design approach. Assume the limits on the horizontal and vertical displacement of point A and B are equal

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The first step is to relate the horizontal and vertical stiffnesses to the material properties.

$$k_{horizontal} = 2\cdot\frac{12EI}{H^3} = \frac{24EI}{H^3}$$

$$k_{vertical} = 2\cdot\frac{AE}{H} = \frac{4AE}{H}$$

To determine the required stiffnesses, we need to determine the required natural frequency using the $H_2$ function

$$H_2{**} = \frac{\Omega^2 mu*}{\hat{p}}$$

Use

$$\rho_{1,2} = \sqrt{\frac{1}{1 \pm \frac{1}{H_2{**}}}}$$

to determine possible limits for $\rho$, where $\rho = \frac{\Omega}{\omega}$. Use $\rho$'s to select an appropriate $\omega$.

Solve for $k = \omega^2 m$

Find column properties $A$ and $I$ as

$$\boxed{A = \frac{\omega^2 mH}{2E}}$$

and $$\boxed{I = \frac{\omega^2 mH^3}{24E}}$$

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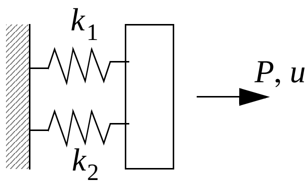
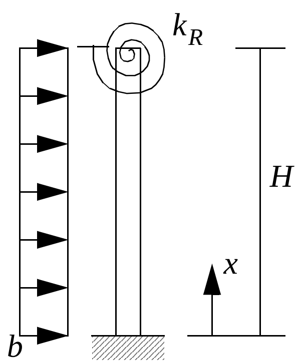
Problem #3

a)




Suppose the design objective is $u < u^*$ when $P$ is applied. Can one select $k_1$ and $k_2$ independently?

b)




Consider a cantilever beam with a rotational spring at the top. Determine expressions for the shear and bending rigidity distributions corresponding to constant shear and bending deformations.

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a)

No. One can solve for the total required $k$, and know that $k = k_1 + k_2$, but there are no other constraints which allow $k_1$ and $k_2$ to be found explicitly.

b)

Find shear and moment distribution

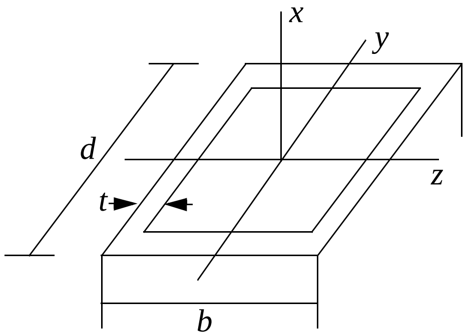
$$V(x) = b(H - x)$$

$$M(x) = -bHx + \frac{bx^2}{2} + \frac{bH^2}{2} - k_R\beta(H) = \frac{bx^2}{2} + \frac{bH^2}{2} - bHx - k_R\chi^*H$$

$$D_T(x) = \frac{V(x)}{\gamma^*} = \boxed{\frac{b}{\gamma^*}(H - x)}$$

$$D_B(x) = \frac{M(x)}{\chi^*} = \boxed{\frac{1}{\chi^*}\left(\frac{bx^2}{2} + \frac{bH^2}{2} - bHx\right) - k_RH}$$

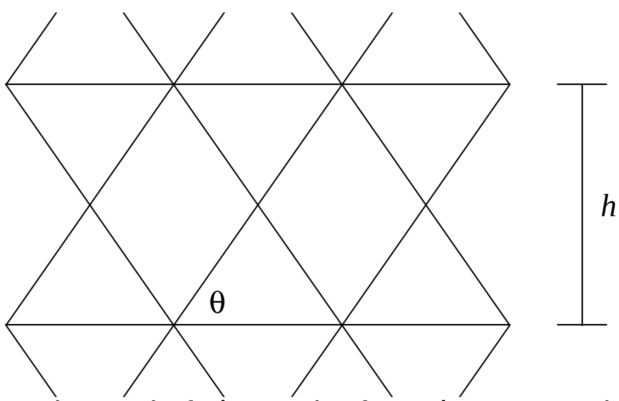
Problem #4




Consider a beam with a closed rectangular cross-section of constant thickness, $t$.

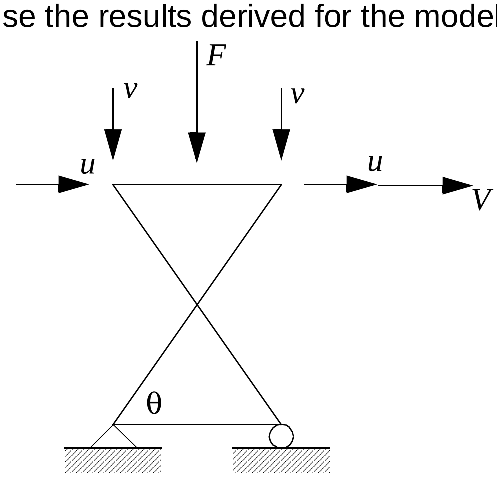
a) Derive expressions for the transverse shear and bending rigidities for displacement in the $x$-$y$ plane.

b)




Suppose, instead of plates, the four planes contain diagrids having the geometry shown in the sketch. Estimate the equivalent shear and bending rigidities provided by this bracing scheme.

Suggestion: Use the results derived for the model shown below



a)

$$D_B(x) = EI = E\left(\frac{1}{12}(bd^3 - (b-2t)(d-2t)^3)\right)$$

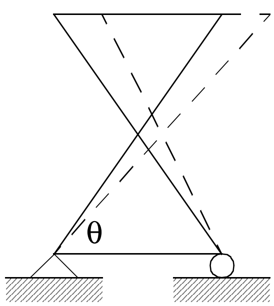
$$D_T(x) = GA = G(bd - (b-2t)(d-2t))$$

or estimate using flanges for bending and web for shear

$$D_B(x) = E\left(\frac{1}{12}(bd^3 - b(d-2t)^3)\right)$$

$$D_T(x) = G(2dt)$$

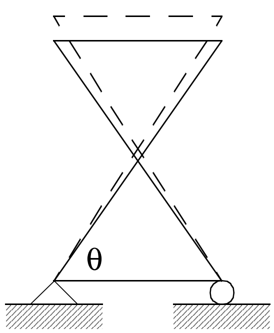
b)




From Example 2.2

$$D_T = A^d E^d \sin 2\theta \cos\theta$$

$$k_{hor} = \frac{A^d E^d \sin 2\theta \cos\theta}{h}$$




Similarly

$$e = v\sin\theta$$

$$F_i = \frac{A^d E^d v \sin\theta}{h/\sin\theta}$$

$$F = 2F_i \sin\theta = \frac{2A^d E^d \sin^3\theta}{h} v$$

$$k_{ver} = \frac{2A^d E^d \sin^3\theta}{h}$$

This leads to

$$\boxed{D_T = ndA^d E^d \sin 2\theta \cos\theta}$$

$$\boxed{D_B = nb2A^d E^d \sin^3\theta \frac{d^2}{2} = nbd^2 A^d E^d \sin^3\theta}$$

where $n$ = number of diagrid elements per unit length