Examination, March 15, 2007

1.561 Motion-Based Design

Problem #1 (30%)

A machine represented by the mass $m$ is to be supported by the spring and dashpot as shown. The machine is sensitive to total acceleration and therefore needs to be isolated from the ground motion.

Consider the ground acceleration to consist of two dominant components:

$$\frac{a_g}{g} = 0.2 \sin(\pi t + \delta_1) + 0.1 \sin(2\pi t + \delta_2)$$

where

$g$ is the gravitational acceleration (9.81 m/s$^2$)

and $\delta_1, \delta_2$ are random phase angles that can range from 0 to $2\pi$

A reasonable approximation for the peak acceleration of the combined response is

$$a_t = \sqrt{(a_{t,1})^2 + (a_{t,2})^2}$$

where $a_{t,1}$ and $a_{t,2}$ are the total accelerations due to the individual harmonic excitations with random phasing.

Suppose the desired maximum total acceleration is 0.05$g$. Describe how you would establish design values for $k$ and $c$. 

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$m = 2000$ kg
Problem #2 (20%)

a) Can one achieve uniform deformation in all the members?

b) Determine the member properties so that the vertical displacement of A is equal to $v^*$. 

Problem #3 (50%)

a) Consider a cantilever beam having the mass distribution indicated, no rotational inertia, and an outrigger system. Assuming the beam acts as a bending beam, determine the rigidity distribution required in order for the fundamental mode shape to have the following form:

$$
\phi(x) = \left( \frac{x}{H} \right)^2
$$

b) Find the modal mass, $\tilde{m}$, that would be used in calibrating the rigidity of the system.