

Notes on  
**1.63 Advanced Environmental Fluid Mechanics**  
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 3-4press-grad.tex

### 3.4 The effects of pressure gradient

Let us follow the incoming flow from infinity A towards the stagnation point O, and then along the body. The flow first decelerates from A to O, accelerates from O to E, then decelerates again from E to F. In the inviscid flow outside the boundary layer, the pressure variation can be estimated by Bernoulli's theorem. The pressure must increase from A to O, decrease from O to E and decrease from E to F again as shown in Figure 3.4.1. Inside

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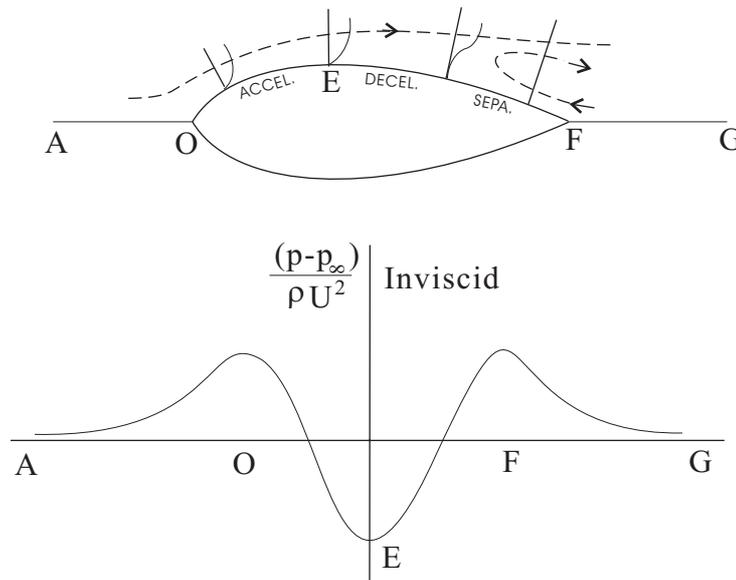


Figure 3.4.1: Pressure variation along the surface of a blunt body

the boundary layer the pressure variation must be the same.

Let us denote the tangential coordinate by  $x$  and the transverse coordinate by  $y$ , and recall the boundary layer equation

$$uu_x + vu_y = UU_x + \nu u_{yy} \quad (3.4.1)$$

On the body surface, the no slip condition leads to

$$UU_x + \nu u_{yy} = 0 \quad (3.4.2)$$

which implies that

$$u_{yy} > 0, \quad \text{if } U_x < 0, \quad \text{deceleration} \quad (3.4.3)$$

$$u_{yy} = 0, \quad \text{if } U_x = 0, \quad \text{no acceleration} \quad (3.4.4)$$

$$u_{yy} < 0, \quad \text{if } U_x > 0, \quad \text{acceleration} \quad (3.4.5)$$

Let us differentiate (3.4.1),

$$u_y u_x + uu_{xy} + vu_{yy} + v_y u_y = \nu u_{yyy}$$

On the body surface the no slip condition and continuity require that

$$u_{yyy} = 0;$$

therefore the curvature of the velocity profile  $u_{yy}$  is an extremum. Now let us examine the implied profiles of shear and velocity by integration, sketched in Figure 3.4.2. In the stage of acceleration,  $u_{yy} < 0$ , hence  $u_y$  (shear) decreases in  $y$ . At the point of no acceleration,  $u_{yy} = 0$ , hence  $u_y$  (shear) must have a negative maximum at some height  $y > 0$ . In the stage of deceleration,  $u_{yy} > 0$ , there must be a point where  $u_{yy} = 0$  at some height  $y = 0$  where shear is the greatest,  $u$  has a point of inflection.

If the flow external flow further decreases, the shear stress at the boundary vanishes, the velocity profile becomes tangential to the  $y$  axis. Still further deceleration cause flow reversal. The flow separates!!

**Remark: Pressure gradient and vorticity:** As another physical insight, let us consider the same flow around the blunt body, see Figure 3.4.3, Since the vorticity in the boundary layer is dominated by  $\zeta = u_y$ , the total vorticity in the boundary layer is

$$\int_0^\delta \zeta dy = \int_0^\delta u_y dy = U(y = \delta)$$

which is the inviscid velocity at the outer edge of the boundary layer. Now the total rate of flux of vorticity is

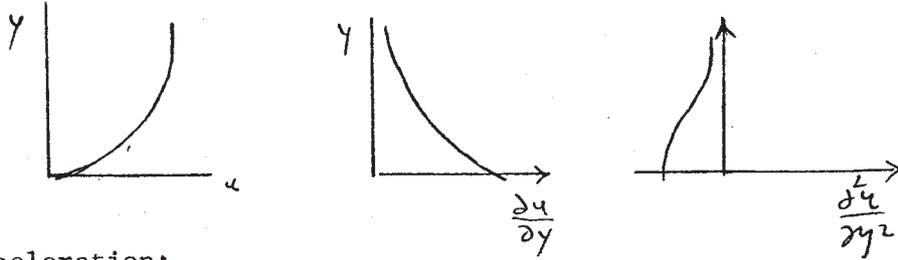
$$\int_0^\delta u \zeta dy = \int_0^\delta uu_y dy = \frac{U^2}{2}$$

hence the mean speed of vorticity transport is  $U/2$ . The spatial variation of vorticity transport is

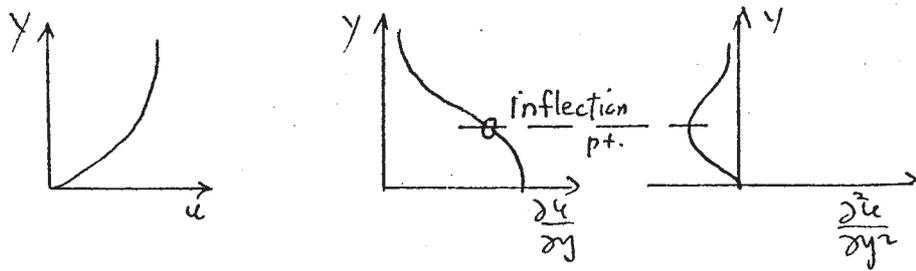
$$\frac{d}{dx} \left( \frac{U^2}{2} \right) = UU_x = -\frac{1}{\rho} p_x$$

This vorticity increase or decrease, which is forced by the external pressure gradient, must come from the body surface. Thus  $UU_x$  is the vorticity source strength per unit length of the surface. From O to A,  $U_x > 0$  vorticity is generated. From A to B,  $U_x < 0$ , vorticity is destroyed.

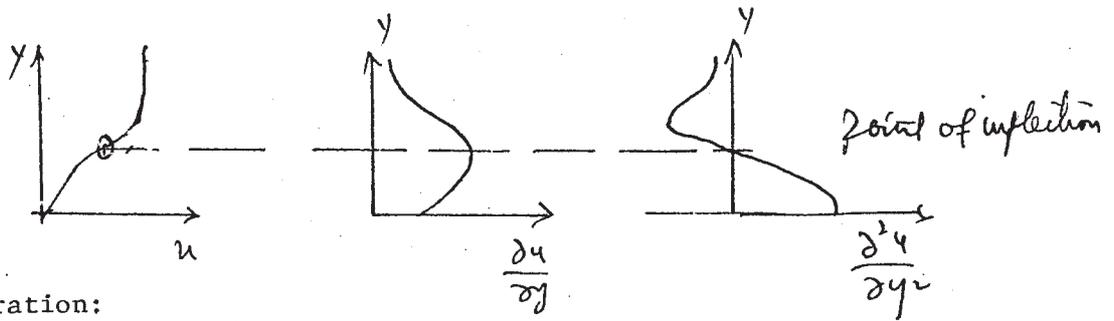
Accelerated flow:



No acceleration:



Decelerated flow:



Separation:

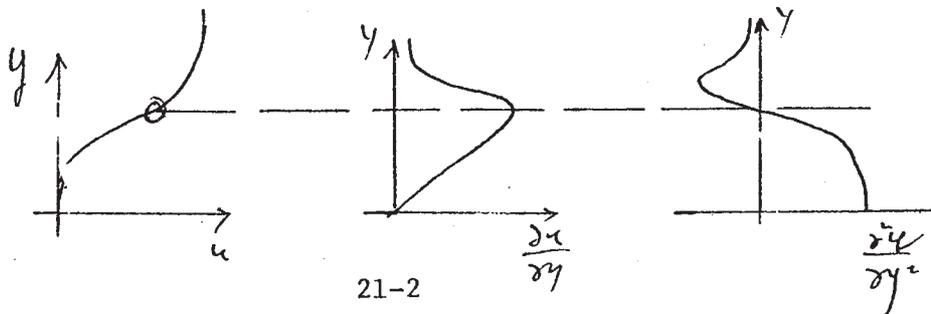


Figure 3.4.2: Effects of pressure gradient on velocity shear.

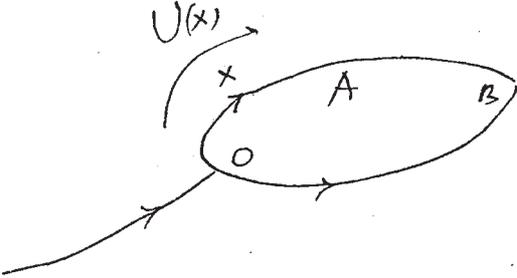


Figure 3.4.3: Vorticity and pressure gradient