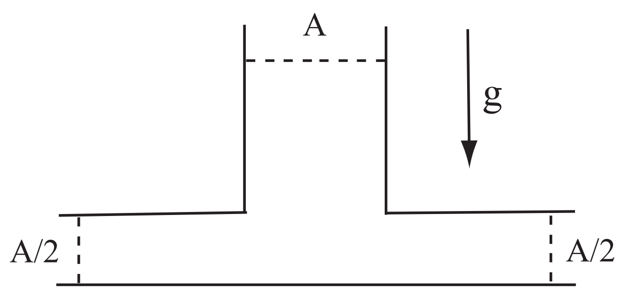
1.63J/2.26J Advanced Fluid Dynamics Spring 2014

Problem Set No. 1

Out: Monday, February 10, 2014
Due: Monday, February 24, 2014 (in class, or before 5:00pm in Room 3-362)
Recitation: 4:00–5:00pm, Wednesday, February 19, 2014 in Room 1-150

Problem 1

A vertical tube of constant cross-sectional area A branches off at the lower end into two horizontal tubes, as sketched below. The cross-sectional areas of the branches are both $A/2$. At the points of branching there are two valves. With the valves closed, the vertical tube is filled with water to the height H. At $t = 0$ the valves are suddenly open. Find the motion in the vertical and horizontal tubes as a result of gravity. Ignore viscosity and treat the flow as one dimensional and transient.



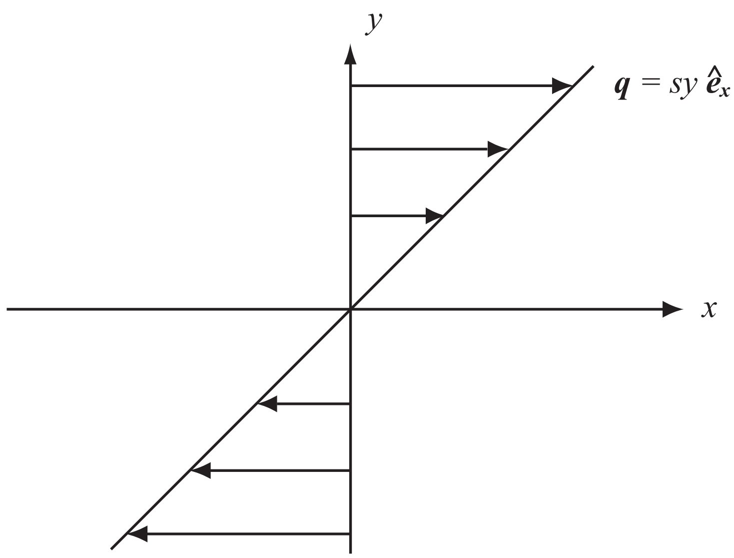
Problem 2

Consider the parallel shear flow

$$\mathbf{q} = (sy, 0, 0)$$

(a) Compute the associated vorticity $\boldsymbol{\zeta}$ and determine the principal directions of the rate-of-strain tensor e_{ij}

(b) Decompose the instantaneous rate of deformation of a fluid element into translation, rigid-body rotation and pure rate of strain



Problem 3

Starting from the governing equations for an incompressible fluid,

(a) Show that

$$\rho \frac{D}{Dt}\left(\frac{q_i q_i}{2}\right) = \rho f_i q_i + \frac{\partial(\sigma_{ij} q_i)}{\partial x_j} - \tau_{ij}\frac{\partial q_i}{\partial x_j}$$

where τ_{ij} is the viscous stress tensor. (Repeated subscripts are automatically summed throughout.)

(b) What is the physical meaning of each term above?

(c) Derive an explicit expression for the last term

$$\Phi = \tau_{ij}\frac{\partial q_i}{\partial x_j}$$

for two-dimensional flow, in terms of u, v and x, y, and comment on the sign of Φ.

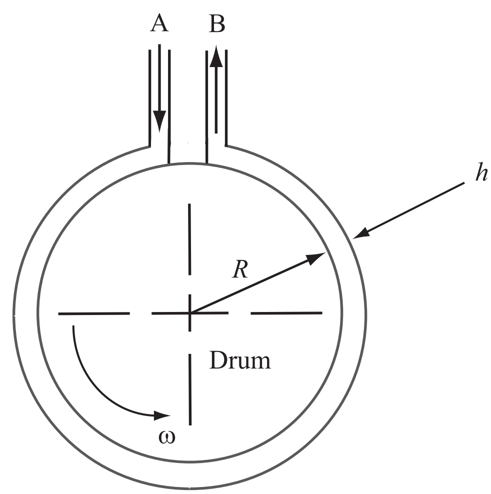
(d) If there is no body force, show that within a fixed container of volume V filled with viscous fluid,

$$\rho \frac{\partial}{\partial t}\int_V \frac{1}{2} q_i q_i dV = -\int_V \Phi dV$$

What is the physical meaning of Φ?

Problem 4

The device shown below is known as a viscosity motor. It consists of a stationary case inside which is a rotating drum. The case and the drum are concentric. Incompressible fluid enters at A, flows through the annulus between the case and the drum, and leaves at B. The pressure at A is higher than at B, the difference being $\triangle P$. The length of the annulus is ℓ (in the circumferential direction). The width of the annulus h is very small compared to the diameter of the drum, so that the flow in the annulus is equivalent to the flow between two flat plates. Let the density of the fluid be ρ and the viscosity be μ.



(a) Find the power delivered to the drum by the fluid and the flow rate, as functions of the pressure drop.

(b) Calculate the viscous dissipation in the fluid film explicitly. Make a complete energy balance and verify that the difference between the power in and power out is equal to the total viscous dissipation rate.