1.63J/2.26J Advanced Fluid Dynamics Spring 2014

Problem Set No. 4

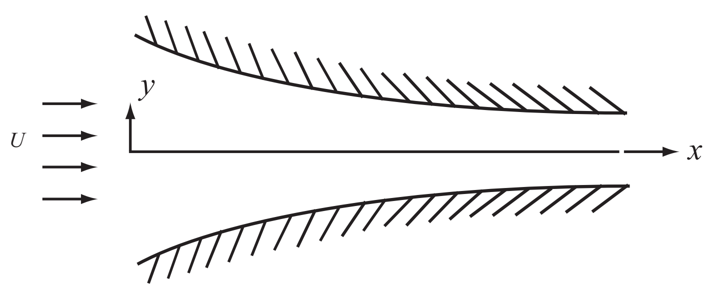
Out: Monday, April 7, 2014
Due: Monday, April 14, 2014 (in class, or before 5:00pm in Room 3-362)
Recitation: 4:00–5:00pm, Wednesday, March 9, 2014 in Room 1-150

Problem 1

Consider flow past a flat plate in the throat of a 2D channel, as sketched below. Suppose the free-stream velocity is given by $U = \lambda x$, where x is the distance from the leading edge and λ is a constant.

(a) Write down the equations for the boundary layer on the plate at steady state.

(b) Use the method of similarity to show that the flow in the boundary layer is governed by an ordinary differential equation. How does the boundary-layer thickness grow with x?

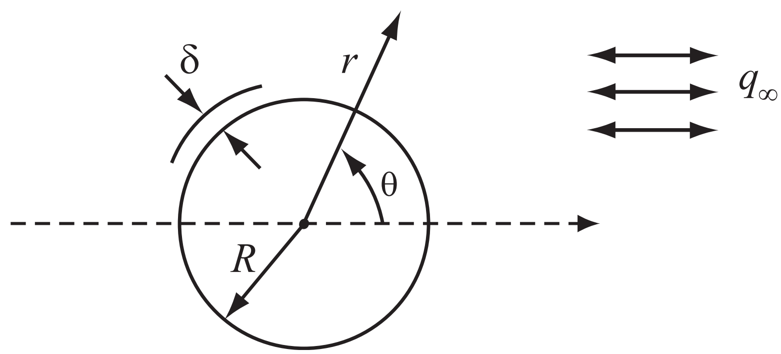


Problem 2

A circular cylinder of radius R is executing small-amplitude ($a/R \ll 1$) time-harmonic oscillations (frequency ω) normal to its symmetry axis in a viscous fluid at rest. To analyze the flow induced by these oscillations, it is convenient to work in the reference frame of the cylinder. We shall take the cylinder to be at rest with its center at the origin $r = 0$ and the flow far away to be oscillatory:

$$\mathbf{q}_\infty = \mathscr{R}\left\{(U_\infty, 0)e^{-i\omega t}\right\} \qquad (r \to \infty),$$

as sketched below.



Outside the thin Stokes boundary layer (thickness δ) next to the cylinder surface, the inviscid velocity potential is

$$\phi = \mathscr{R}\left\{U_\infty(r + \frac{R^2}{r})\cos\theta e^{-i\omega t}\right\},$$

and the tangential velocity along $r = R$ is given by

$$\left.\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right|_{r=R} = -\mathscr{R}\left\{2U_\infty \sin\theta e^{-i\omega t}\right\}.$$

Assuming that $R/\delta >> 1$, curvature effects are negligible. We can then analyze the Stokes boundary layer on the cylinder surface by the perturbation procedure discussed in class, taking the 'outer' oscillatory flow to be

$$U(x,t) = \mathscr{R}\left\{U_0(x)e^{-i\omega t}\right\}$$

with

$$U_0(x) = -2U_\infty \sin\theta, \quad x = R(\pi - \theta).$$

Use the results derived in class and work out the detailed distribution of induced streaming velocity field in the Stokes boundary layer, and sketch the streamline pattern.