

Problem Set No. 2

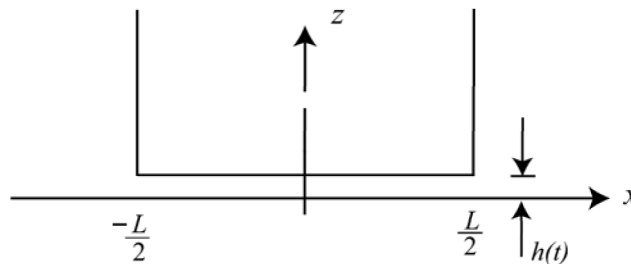
Out: Monday, February 24, 2014

Due: Monday, March 10, 2014 (in class, or before 5:00pm in Room 3-362)

Recitation: 4:00–5:00pm, Wednesday, March 5, 2014 in Room 1-150

Problem 1

It is a common experience that lifting an ice block from a wet plane surface or separating two pieces of glass from each other, is difficult. This is because of the slow flow in the very thin gap, that produces an adhesive force. Referring to the sketch below, consider a two-dimensional problem for simplicity. A plate of finite width L originally rests on top of a rigid and smooth plane $z = 0$ with a small and uniform gap h_0 . The entire system is surrounded by air. At time $t = 0$, a vertical force F is applied to pull the plate upward. Let the fluid viscosity be μ and density be ρ . Find the rate of separation as a function of F , L and the fluid properties. Instead of pulling a plate up by force, solve the problem if the plate drops downward by its own weight W (as in the dropping of a phonograph disk).



Problem 2

A finite mass of lava is initially released on a horizontal plane and spreads slowly in all radial directions. Invoking the lubrication approximation, the local radial velocity turns out to be

$$u(r, z, t) = -\frac{g}{2\nu} \frac{\partial h}{\partial r} z(2h - z).$$

(a) Use mass conservation,

$$\frac{\partial h}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \int_0^h u \, dz \right) = 0,$$

to show that the lava thickness $h(r, t)$ satisfies the evolution equation

$$\frac{\partial h}{\partial t} = \frac{g}{3\nu} \frac{1}{r} \frac{\partial}{\partial r} \left(r h^3 \frac{\partial h}{\partial r} \right).$$

(b) Argue that $h(r, t)$ must obey the constraint

$$2\pi \int_0^{R(t)} r h(r, t) dr = \text{constant} = V,$$

where $R(t)$ is the front of the spreading lava, as well as the boundary conditions

$$h(R(t), t) = 0, \quad \frac{\partial h(0, t)}{\partial r} = 0.$$

(c) Show that the evolution equation for $h(r, t)$ subject to the integral constraint above admits a similarity solution of the form

$$h(r, t) = \frac{A}{t^{1/4}} f(\eta) \quad \text{with} \quad \eta = \frac{Cr}{t^{1/8}}.$$

(d) Derive the governing equation and boundary conditions for $f(\eta)$ and adjust the constants A and C so that the governing equations look the simplest. Discuss the physics of axisymmetric gravitational spreading in relation to the analogous two-dimensional problem discussed in class.

Problem 3

A rigid sphere of radius a is rotating with constant angular velocity Ω in fluid which is at rest at infinity. Assuming that the Reynolds number $Re = a^2\Omega/\nu \ll 1$, calculate the total moment exerted on the sphere by the fluid. Verify that the rate of energy dissipation in the fluid (see Problem 3 of PS# 1) equals the power needed to maintain the rotation.

Hint: It is convenient to use spherical polar coordinates (see, for example, Appendix 2 in Batchelor). Based on symmetry and the assumption that inertia is negligible, argue that the flow velocity component in the azimuthal direction dominates and, moreover, the pressure gradient vanishes in this direction. The momentum equation for the azimuthal flow velocity can then be solved by separation of variables.