1.63J/2.26J Advanced Fluid Dynamics Spring 2014

Problem Set No. 3

Out: Monday, March 10, 2014
Due: Monday, March 31, 2014 (in class, or before 5:00pm in Room 3-362)
Recitation: 4:00–5:00pm, Wednesday, March 19, 2014 in Room 1-150

Problem 1

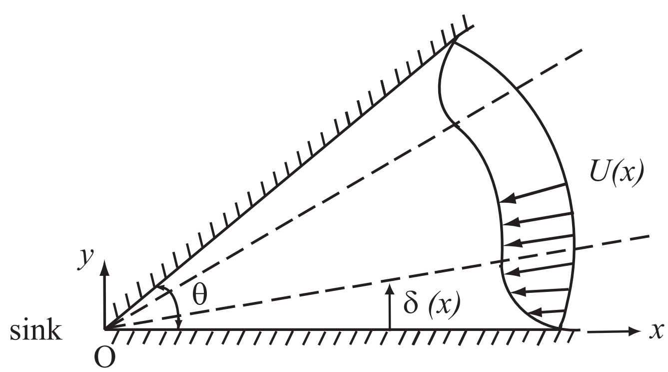
A fluid of low viscosity flows steadily into a line sink at the corner of two intersecting walls. The inviscid potential flow has the radial velocity $Q/\theta r$ where Q is the disharge and θ the corner angle. (See the sketch below.)

Consider the steady two-dimensional boundary layer along either of the two plane walls. Denote by $x > 0$ the distance from the sink along the wall. Outside the boundary layer the flow is in the negative x direction toward the origin and is given by

$$U(x) = -\frac{Q}{\theta}\frac{1}{x}, \quad Q, \theta > 0.$$

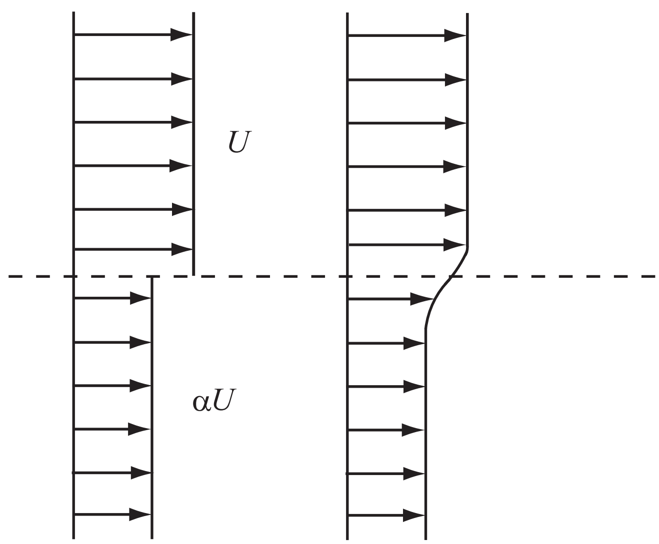
(a) Write down the boundary-layer equations along with the associated boundary and matching conditions.

(b) Find the velocity field in the boundary layer for all $x > 0$ by the method of similarity. Discuss the physics of the solution.



Problem 2

Two semi-infinite streams of the same fluid, but with different uniform parallel velocities, U and αU, are brought into contact. Owing to the action of viscosity, a boundary layer will develop over which the initial discontinuity in velocity will be smoothed out, as sketched below.



(a) Write down the boundary-layer equations and associated boundary conditions for this shear layer, assuming that the streamline dividing the upper and lower fluid streams remains flat and parallel to the flow direction upstream of the contact point.

(b) A similarity solution is possible. Derive the equation and boundary conditions to be satisfied by the similarity function.

Problem 3

Solve the problem of a round jet emitted from the origin $r = z = 0$. Assuming that the boundary layer approximation is valid, start with

$$\frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0,$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right),$$

u being the radial and w the axial velocity components. The momentum flux rate at the origin is M.

(a) State the boundary conditions.

(b) Introduce a stream function and solve the problem explicitly by the similarity method.

Hint: After suitable choice of coefficients, arrange the differential equation into the form:

$$\left(\frac{FF'}{\eta}\right)' = \left(\eta\left(\frac{F'}{\eta}\right)'\right)'$$

with the boundary conditions:

$$F' = F = 0 \text{ on } \eta = 0.$$

You can integrate this differential equation explicitly.

(c) Sketch the streamlines and discuss the physics.

(d) Use your result to assess the validity of the boundary layer approximation, by comparing the omitted term

$$\nu \frac{\partial^2 w}{\partial z^2}$$

with the term kept

$$\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right).$$

Find the necessary condition under which the boundary layer approximation is valid.

(e) Suppose the omitted term $\nu\, \partial^2 w/\partial z^2$ were kept; would the solution then be exact?