Problem 1

Viscous fluid occupies the region above a plane rigid boundary \( z = 0 \) which is rotating with angular velocity \( \Omega \).

(a) Verify that there is a similarity solution to the Navier–Stokes equations (in the inertial frame) of the form:

\[
q_r = \Omega r f(\xi), \quad q_\theta = \Omega r g(\xi), \quad q_z = (\nu \Omega)^{1/2} h(\xi),
\]

where

\[
\xi = z(\Omega/\nu)^{1/2};
\]

if

\[
f^2 + hf' - g^2 = f'', \quad 2fg + hg' = g'', \quad 2f + h' = 0,
\]

with boundary conditions:

\[
f = 0, \quad g = 1, \quad h = 0 \quad (\xi = 0); \quad f \to 0, \quad g \to 0 \quad (\xi \to \infty).
\]

This classical problem was first investigated by von Kármán in 1921. The above boundary-value problem has to be solved numerically. (You are not asked to do this!)

(b) Do you expect that \( h \to 0 \) (\( \xi \to \infty \))? Explain your answer in physical terms.

Problem 2

Using the techniques discussed in class, calculate the effective diffusivity for a passive solvent in pressure-driven steady laminar flow in a 2D channel of width \( H \).