1.63J/2.26J Advanced Fluid Dynamics Spring 2014

Problem Set No. 5

Out: Monday, April 14, 2014
Due: Monday, April 28, 2014 (in class, or before 5:00pm in Room 3-362)
Recitation: 4:00–5:00pm, Wednesday, April 23, 2014 in Room 1-150

Problem 1

Viscous fluid occupies the region above a plane rigid boundary $z = 0$ which is rotating with angular velocity Ω.

(a) Verify that there is a similarity solution to the Navier–Stokes equations (in the inertial frame) of the form:

$$q_r = \Omega r f(\xi), \;\; q_\theta = \Omega r g(\xi), \;\; q_z = (\nu\Omega)^{1/2} \; h(\xi),$$

where
$$\xi = z(\Omega/\nu)^{1/2},$$

if
$$f^2 + hf' - g^2 = f'', \;\; 2fg + hg' = g'', \;\; 2f + h' = 0,$$

with boundary conditions:

$$f = 0, \;\; g = 1, \;\; h = 0 \quad (\xi = 0); \quad f \to 0, \;\; g \to 0 \quad (\xi \to \infty).$$

This classical problem was first investigated by von Kármán in 1921. The above boundary-value problem has to be solved numerically. (You are not asked to do this!)

(b) Do you expect that $h \to 0$ $(\xi \to \infty)$? Explain your answer in physical terms.

Problem 2

Using the techniques discussed in class, calculate the effective diffusivity for a passive solvent in pressure-driven steady laminar flow in a 2D channel of width H.