1.63J/2.26J Advanced Fluid Dynamics

Take-Home Exam

Wednesday, April 2, 2014

This is a *closed-book* exam. *You may use your own class notes, problem sets and the lecture notes posted on the 1.63J/2.26J website only.* You are not allowed to discuss this exam with anyone else. If you need clarification regarding the problems, please ask me. The exam is due on Friday, April 4th, 2014 before 5:00pm, in my office (Room 3-362).

Problem 1 (10 points)

A steady uniform stream $U\hat{\boldsymbol{e}}_{\boldsymbol{x}}$ of incompressible fluid is perturbed by a vertical gauze lying on the y-axis. The velocity perturbation, $\boldsymbol{q'} = \boldsymbol{q} - U\hat{\boldsymbol{e}}_{\boldsymbol{x}}$, has the following form at $x = 0$:

$$\boldsymbol{q'} = \epsilon U \cos\left(\frac{y}{L}\right) \hat{\boldsymbol{e}}_{\boldsymbol{x}},$$

where $\boldsymbol{q}$ is the actual velocity, $\lambda = 2\pi L$ is the wavelength of the perturbation in the y-direction and $\epsilon \ll 1$.

(a) Explain in physical terms why, when $Re = U\frac{L}{\nu} >> 1$, the vorticity generated at the gauze extends upstream $(x < 0)$ a very short distance compared to its extent downstream $(x > 0)$.

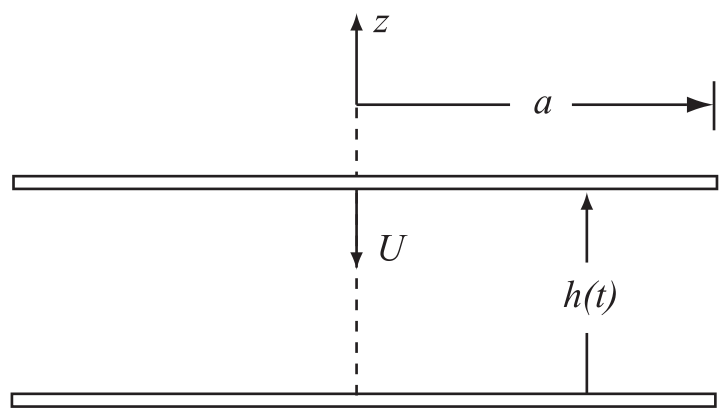
(b) Using scaling arguments, obtain order-of-magnitude estimates for the upstream, x_{up}, and the downstream, x_{down}, horizontal extents of the vorticity generated at the gauze. (Note: you are not asked to obtain exact expressions, just order-of-magnitude estimates.)

Problem 2 (10 points)

Two parallel plane circular disks of radius a lie one above the other with a viscous incompressible fluid between them. Consider one disk to be fixed and the other to approach it at a constant velocity U (see sketch below). Note that the flow is axisymmetric.

(a) Assuming that the fluid layer is thin, $h/a \ll 1$, find an approximate expression for the radial velocity as a function of the radial pressure gradient.

(b) Compute the drag on the moving disk.



Problem 3 (10 points)

Consider the flow downstream of a two-dimensional streamlined body at high Reynolds number, so that there is a thin wake in which $\boldsymbol{q}$ varies much more rapidly with y than with downstream distance x (see sketch below). Suppose also that we are sufficiently far from the body that the flow velocity has nearly returned to its far-upstream constant value $U\hat{\boldsymbol{e}}_{\boldsymbol{x}}$, so that $\boldsymbol{q} \approx (U - u_1)\boldsymbol{e}_{\boldsymbol{x}}$, where $\frac{u_1}{U} \ll 1$.

(a) Write down an approximate form of the momentum equation governing u_1 in these circumstances.

(b) Deduce that $\int_{-\infty}^{\infty} u_1 dy = \text{const.}$

(c) Obtain a similarity solution for $u_1(x, y)$ and sketch the wake velocity profiles at two different downstream distances x.