1.63J/2.26J Advanced Fluid Dynamics

Take-Home Exam

Wednesday, May 14, 2014

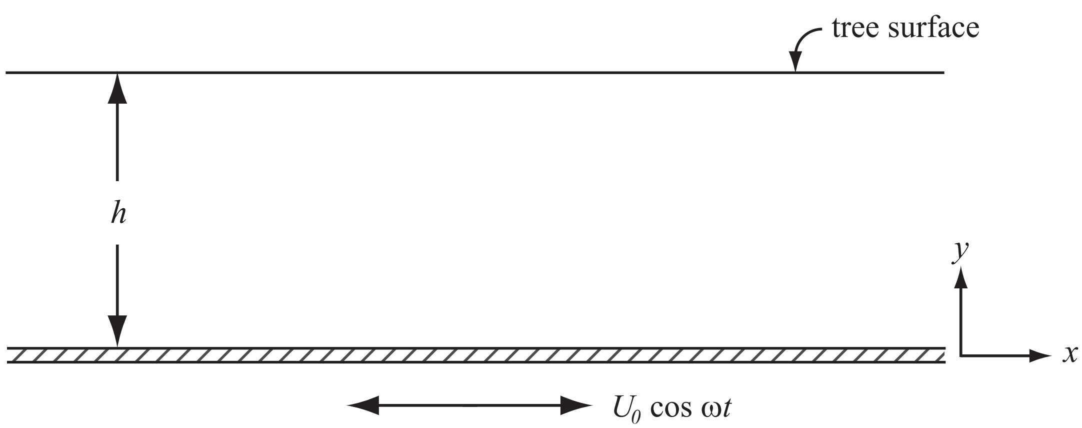
This is a *closed-book* exam. You may use your own class notes, problem sets and the lecture notes posted on the 1.63J/2.26J website only. You are not allowed to discuss this exam with anyone else. If you need clarification regarding the problems 1, 2 or 3, please ask Professor Akylas (trakylas@mit.edu); for problem 4 (Biolocomotion), please ask Professor Hosoi (peko@mit.edu). The exam is due on Friday, May 16th, 2014 before 5:00pm, in Professor Akylas's office (Room 3-362).

Problem 1. (10 points)

An infinite rigid plate is covered by a layer of viscous fluid of kinematic viscosity ν and thickness h, with the upper surface being free, as sketched below.

(i) Determine the flow induced, at steady state, when the plate oscillates in its own plane with speed $U_0 \cos \omega t$.

(ii) Plot the induced velocity profile at $\omega t = 0, \pi/2, \pi, 3\pi/2$ and discuss the nature of the flow for small and large values of $\omega h^2/\nu$.

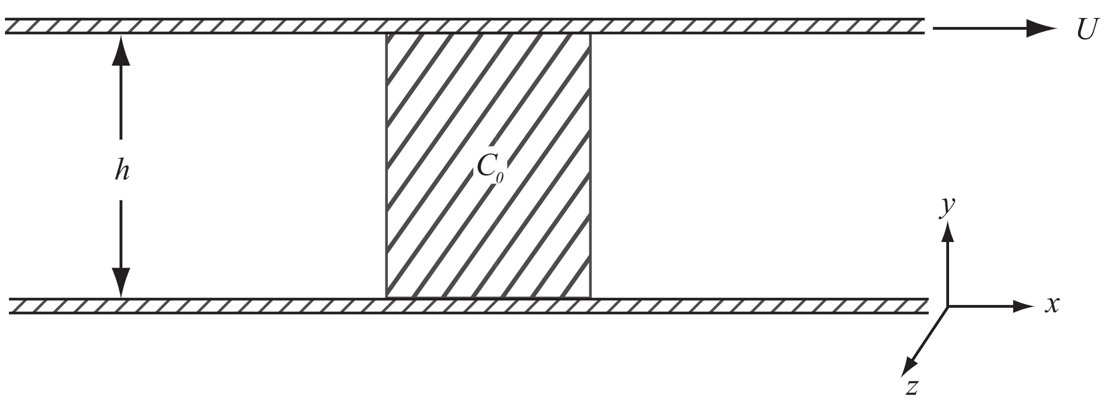


Problem 2. (10 points)

Water is contained between two infinite parallel plates separated by a distance h. The bottom plate is held stationary, and the top plate is moved at a constant velocity U so that a simple shear flow is generated between the plates. A band of a dye is injected between and perpendicular to the plates extending fully across the gap, as sketched below. (The band depth in z is very deep and may be supposed to be infinite.) The initial dye concentration is C_0, and its molecular diffusivity in water is D.

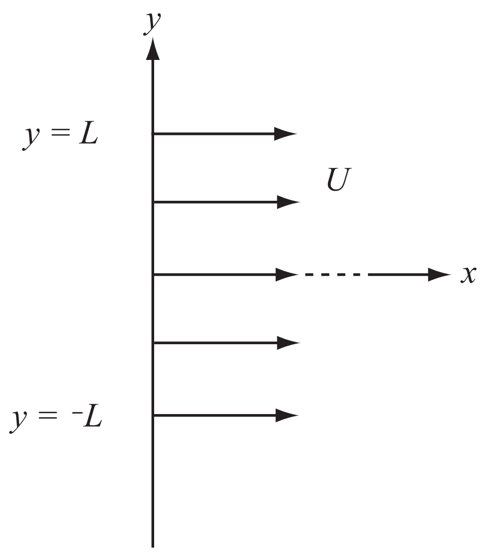
(i) Calculate the effective diffusivity for the injected dye in this shear flow. State explicitly the assumptions and approximations that you made.

(ii) Discuss qualitatively the long-time evolution of the dye concentration.



Problem 3. Hydrodynamic stability (10 points)

Consider the simple 'top-hat' model for a plane jet of homogenous fluid, sketched below.



Discuss the inviscid stability of this flow profile to infinitesimal two-dimensional perturbations.

Problem 4. Biolocomotion(10 points)

Consider the swimmer depicted below. Throughout this problem you may assume that all flows are low Reynolds number flows and that $\alpha_0 \ll 1$. The swimmer consists of two rigid, slender bars of length L_0 connected by a central hinge. The two bars are actuated such that:

$$\alpha_1(t) = \alpha_2(t) = \alpha_0 \cos(\omega t).$$

1. Show that the swimmer (without the spheres S_1 and S_2) is incapable of net motion by computing the time-averaged swimming velocity over one period. (Please include a sketch in your solution that indicates your chosen direction for the unit normal and unit tangent vectors, $\hat{\mathbf{n}}$ and $\hat{\mathbf{t}}$).

2. Now consider the same swimmer with spheres S_1 and S_2 attached. Both spheres have constant radius R. The spheres are actuated to move along the bars symmetrically such that
$$x_1(t) = x_2(t) = \frac{L_0}{2}[1 + \sin(\omega t)].$$
   (a) Write down the kinematic constraint that relates the velocity of the spheres to the (unknown) instantaneous swimming velocity $V(t)$ and to other known quantities.

   (b) Compute the average swimming velocity of this swimmer (to lowest non-zero order in α_0).

   (c) Compute the swimming efficiency of this swimmer (to lowest non-zero order in α_0).

3. Select kinematics $\alpha_1(t)$, $\alpha_2(t)$, $x_1(t)$, and $x_2(t)$ that allow the swimmer to turn a 90° corner. (You do *not* need to calculate velocities or efficiencies for your chosen kinematics as long as you provide a convincing physical argument for why the swimmer will turn.)