

Lecture 16 Supply and demand, Groundwater Management  
Nov. 2, 2006

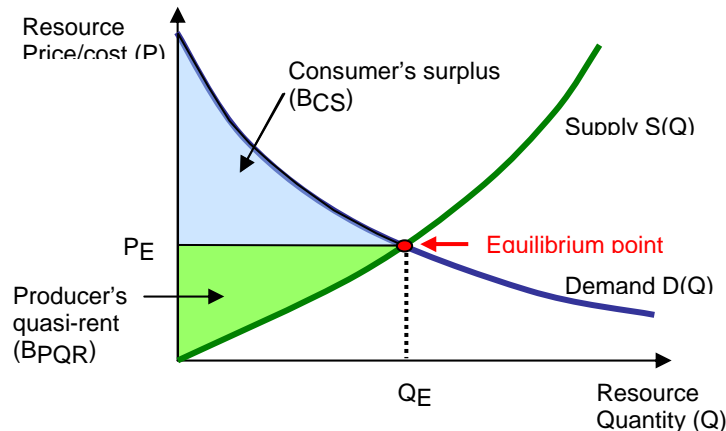
**Supply and demand**

Resource management problems often deal with the connection between the benefits of limited resources (e.g. water) and the cost of supply. As more of a resource becomes available **demand** for the resource typically decreases while **supply** costs increase. The optimum level of resource use depends on the relationship between supply and demand.

Demand: Price consumer is willing to pay for one more unit of resource (depends on value of products produced with resource, changes with amount of resource available).

Supply: Cost of providing one more unit of resource (changes with amount of resource available)

Optimum resource quantity and price/cost defined by intersection of demand and supply curves (**equilibrium point**).



To see this, consider net benefit:

$$B = \int_0^{Q} D(Q)dQ - \int_0^{Q} S(Q)dQ$$

Benefit is maximized when

$$\frac{dB}{dQ} = D(Q) - S(Q) = 0 \rightarrow D(Q) = S(Q)$$

Solution to this condition is **equilibrium quantity**  $Q = Q_E$

The corresponding **equilibrium price** is  $P_E = D(Q_E) = S(Q_E)$  .

The net benefit is sometimes divided into two parts  $B = B_{CS} + B_{PQR}$ :

Consumer's surplus: 
$$B_{CS} = \int_0^{Q_E} D(Q)dQ - P_E$$

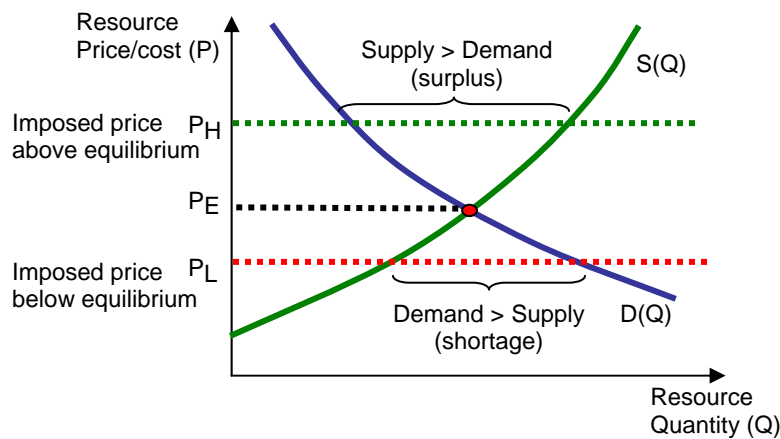
Producer's quasi rent: 
$$B_{PQR} = P_E - \int_0^{Q_E} S(Q)dQ$$

**Consumer's surplus** is the extra amount consumer would have paid for  $Q_E$  units if they were purchased in infinitesimally small increments from 0 to  $Q_E$ , rather than in a single batch of  $Q_E$ . It represents the **portion of net benefit accruing to the consumer**.

**Producer's quasi-rent** is extra amount producer obtains by selling  $Q_E$  units in a single batch rather than in infinitesimally small increments from 0 to  $Q_E$ . It represents the **portion of net benefit accruing to the producer**.

In the absence of constraints the quantity and price of the resource should converge to the equilibrium values, since there is always an incentive (for both consumer and producer) to move from non-equilibrium to equilibrium values.

Sometimes imposed limits on prices or quantities prevent movement to equilibrium:

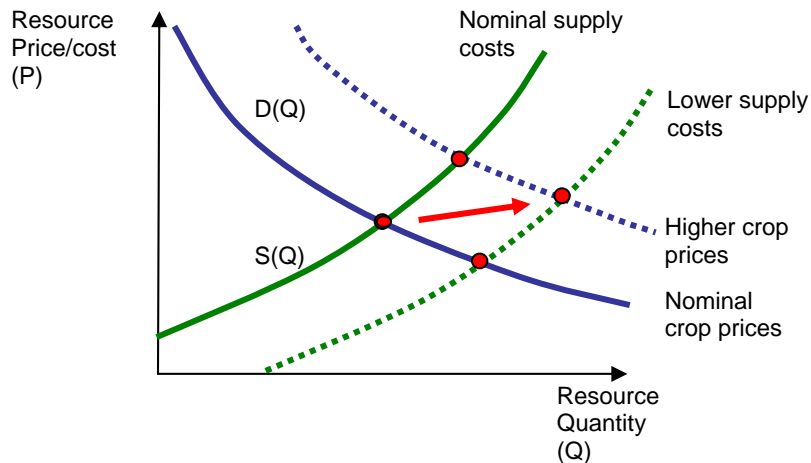


Imposed **prices above equilibrium** create **surpluses** since consumers are not willing to buy all that producers want to supply at high price..

Imposed **prices below equilibrium** create **shortages** since producers are not willing to supply all that consumers want to buy at low price.

If problem inputs are changed supply and demand curves can **shift**, changing equilibrium quantity/price.

For a problem where pumped water is used to irrigate crops lower supply costs and/or higher crop prices can shift equilibrium towards higher production.



### Example: Groundwater management

Consider the problem of pumping groundwater for crop production. Decision variables include well pumping rates and land allocated to various crops.

Supply and demand curves for this problem can be derived by considering the consumption and production of water in two related sub-problems.

- **Supply sub-problem:** Identify well pumping rates that minimize pumping cost, subject to constraint that specifies minimum water pumped. Provides a derived supply curve.
- **Demand sub-problem:** Allocate crops to maximize crop revenue, subject to constraint that specifies maximum available irrigation water. Provides a derived demand curve.

Illustrate this with an example with 3 crops and 5 wells that withdraw water from a confined aquifer below the farming area..

### Derived Supply

The supply problem minimizes the sum of the pumping costs  $\gamma L_j p_j$  at the 5 wells:

$L_j$  = pumping lift (m) at well  $j$  ;  $j = 1, \dots, 5$

$p_j$  = pumping rate ( $m^3$ /season) at well  $j$

$\gamma$  = coefficient that converts units and depends on the cost of energy.

Pumping lift depends on drawdown, which is related to pumping rate are related by a response matrix  $R_{jk}$  derived from a groundwater model.

Total pumping from all 5 wells must provide a specified quantity of water  $Q$  ( $m^3$ /season).

$$\text{Minimize } \sum_{j=1}^5 p_j$$

such that :

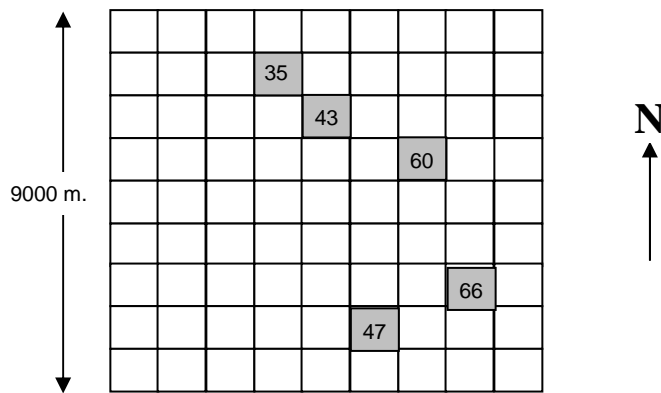
$$R_{jk} p_k = s_j \quad \text{Response matrix}$$

$$L_j = [h_g - h_{0j}] + s_j \quad \text{Lift-drawdown}$$

$$\sum_{j=1}^5 p_j \geq Q \quad \text{Supply requirement}$$

$$p_j \geq 0 \quad \text{Non-negativity}$$

Study area is a hypothetical 9 km. by 9 km. (8100 ha) farming region which is discretized, for simulation purposes, with a square grid of 9 by 9 cells, each 100 ha. in area:



The cells are numbered starting in the lower left corner of the grid, increasing upward to the upper left corner, continuing from bottom to top in each column, moving from left to right.

- Confined quifer is represented by a two-dimensional (vertically averaged) approximation
- Candidate water supply wells located in: Cells 35, 43, 47, 60, and 66.
- Northern and southern boundaries: No-flux
- Western boundary: Specified head = 0.0 m. (above mean sea level)
- Eastern boundary: Specified head = 20.0 m.
- Ground surface at cell  $i$ :  $h_g(i) = h_g = 30$  m.
- Transmissivities: Variable, with a conductive zone running generally from the northwest to the southeast.
- Pumping cost = 0.10 \$/Kwhr.
- Growing/pumping season = 150 days

In the absence of pumpage, flow moves generally from east to west.

Nominal (unpumped) water level elevations  $h_{0j}$  (in m. above sea level) at the 5 wells are obtained from a finite difference groundwater model:

	Well				
	35	43	47	60	66
$H_{0i}$ (m)	6.77	9.01	13.95	13.01	16.68

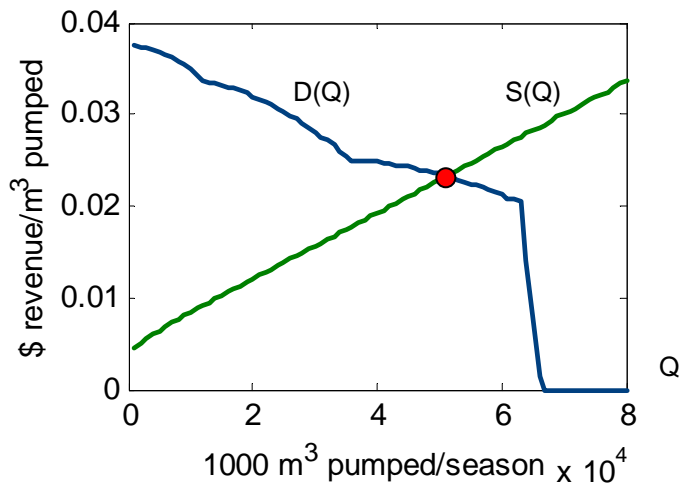
The response matrix for this problem, which is listed in the table below, is composed of the following partial derivatives, where  $s_j$  is drawdown at Well  $j$  and  $p_k$  is pumping at Well  $k$ :

$$R_{jk} = \frac{\partial s_j}{\partial p_k} \quad j = 35, 43, 47, 60, 66; \quad k = 35, 43, 47, 60, 66$$

These are obtained from a finite difference groundwater model, as described in Lecture 11 .

Response Matrix Coefficients (m/1000 m <sup>3</sup> /day)						
		Pumping Well				
		35	43	47	60	66
Response Well	35	.215	.123	.054	.071	.031
	43	.123	.202	.071	.097	.041
	47	.054	.071	.492	.078	.094
	60	.071	.097	.078	.212	.054
	66	.031	.041	.094	.054	.202

The **derived supply**  $S(Q)$  for this problem is obtained by plotting the **shadow price** for the supply constraint vs. this constraint's right-hand side value  $Q$  (a separate GAMS solution is required for each value of  $Q$ ). The shadow price is the increase in pumping cost required to generate an additional unit of water. This price increases linearly since the relationship between quantity pumped and lift is linear.



## Derived Demand

In this example the demand for water is generated by crop production. Crop revenue is maximized by allocating land to crops to make best use of available land and water. This is an extension of the crop allocation problem of Lecture 7:

Decision variables:  $x_1, x_2, \dots, x_n$  = land (ha) devoted to Crops 1, ...,  $n$

Objective: Maximize crop revenue (\$) for one growing season

$$\text{Maximize } c_j y_j(x_j) x_j \\ x_1, \dots, x_3$$

such that :

$$10^4 A_{1j} x_j \leq Q \quad \text{Water constraint}$$

$$A_{2j} x_j \leq 8100 \quad \text{Land constraint}$$

$$x_j \geq 0 \quad \text{Non-negativity}$$

Crop yield  $y_j(x)$  often decreases as more land is developed since the best land is typically used first.

- Crop  $j$  yield function:  $y_j(x_j) = y_{1j} - y_{2j} x_j^2$  (kg/ha)
- Crop  $j$  yield coefficients:  $y_{1j}$  (kg/ha),  $y_{2j}$  (kg/ha<sup>3</sup>)
- Crop  $j$  price:  $c_j$  (\$/kg)
- Crop  $j$  unit water requirement:  $A_{1j}$  (m)
- Crop  $j$  land requirement:  $A_{2j}$  (ha/kg) =  $1/y_j(x)$

To illustrate, use 3 crops with the following inputs:

	$c_j y_{1,j}$ (\$1000/ha)	$c_j y_{2,j}$ (\$1000/ha <sup>3</sup> )	Water Requirement (m/season)
Crop 1	0.3	4.5 E-9	0.8
Crop 2	0.2	3.05 E-9	0.6
Crop 3	0.25	3.8 E-9	1.0

The **derived demand**  $D(Q)$  for this problem is obtained by plotting the **shadow price** for the water constraint vs. this constraint's right-hand side value  $Q$  (a separate GAMS solution is required for each value of  $Q$ ). The shadow price is the increase in crop revenue obtained from an additional unit of irrigation water. This price decreases (in a piecewise nonlinear fashion) until it reaches 0.0 (water no longer limiting).

## Coupled Problem

The equilibrium point (but not the supply and demand curves) for this example can be obtained by solving a coupled problem that combines the supply and demand subproblems. In this case the objective is to maximize **net benefit** (crop revenue – pumping cost):

$$\text{Maximize}_{x_1, \dots, x_3, p_1, \dots, p_5} c_j y_j(x_j) x_j - \sum_j L_j p_j$$

such that :

$$10^4 A_{1j} x_j = \sum_{j=1}^5 p_j$$

$$A_{2j} x_j \leq 8100$$

$$R_{jk} p_k = s_j$$

$$L_j = [h_g - h_{0j}] + s_j$$

$$p_j \geq 0$$

$$x_j \geq 0$$