MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.731 Water Resource Systems

Problem Set 4 – Groundwater Management, Quadratic Programming
Due: Tuesday, Oct. 17, 2006

The problem is to determine the optimum (highest net benefit) amount of groundwater to pump from four wells screened in a confined groundwater aquifer. The decision variables are the pumping rates and pumping lifts (depths to the water table) at the four wells:

$x = [\, x_1 ,\ x_2 \ ,\ x_3 ,\ x_4 \,]$ Pumping rates (m^3/sec) at wells 1, 2, 3, and 4

$y = [\, y_1 ,\ y_2 \ ,\ y_3 ,\ y_4 \,]$ Pumping lifts (m.) at wells 1, 2, 3, and 4

Net benefit is measured in terms of the difference between the annual revenues obtained from irrigated crops and the annual costs associated with pumping and delivery:

Objective: $F = R - C$

Revenue: $R = \sum_{i=1}^{4} a x_i (b - x_i)$ Cost: $C = \sum_{i=1}^{4} e x_i y_i$

Note that the marginal revenue dR/dx_i for each well decreases to zero as the quantity pumped increases to $b/2$. This reflects the decreasing demand for increasing amounts of pumped water. The water cost is proportional to the electrical energy required to pump at a rate x_i over a depth y_i for a growing season of 4 months.

Pumping at each well affects the drawdown at the other wells. This effect may be quantified with a groundwater model or through pumping tests. In either case, there is a linear relationship between the vectors x and y if the aquifer is confined (as we assume here). If we also assume the groundwater system is in steady-state the pumping-lift relationship may be summarized with a symmetric response matrix A. Each element A_{ij} of this matrix relates the steady state lift y_i at well i (in m.) to the steady-state pumping rate x_j at well j (m^3/sec):

$$y_i = A_{ij} x_j \quad \text{where } A = \begin{bmatrix} 2000 & 600 & 300 & 200 \\ 600 & 3000 & 500 & 400 \\ 300 & 500 & 1500 & 500 \\ 200 & 400 & 500 & 2000 \end{bmatrix} \text{sec/m}^2$$

When the response matrix relationship is substituted in the objective function expressions above the result is a quadratic objective. For a GAMS solution to this problem the response matrix equation and the definitions of *R* and *C* may be included as a set of equality constraints. Then the objective function can be written in terms of the intermediate decision variables *R* and *C*.

Additional problem constraints are:

$$x_i \leq \frac{b}{2} \qquad y_i \leq 30 \qquad x_i \geq 0 \qquad y_i \geq 0 \qquad \text{for all } i$$

Assume that:

$$a = 4.0\ 10^8\ \$/\text{m}^6/\text{sec}^2 \qquad b = .01\ \text{m}^3/\text{sec}$$

In order to solve this problem, carry out the following tasks:

1). Derive the cost coefficient *e*, given that the price of electricity is $ 0.20 / kwhr.

2). Check to see if a local maximum for this problem is also global (i.e. is the objective function concave?). To check this you will need to construct a Hessian matrix and use MATLAB to test its eigenvalues for your particular set of problem inputs.

3). Solve the problem using GAMS. If the objective function is not concave try a few different initial feasible solutions to provide confidence that the GAMS solution is a global maximum.

4). Evaluate the shadow prices associated with any pumping lift (y_i) constraints that are active at the GAMS solution.

Please hand in only enough information to adequately document your solution.