MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.731 Water Resource Systems

Problem Set 4 – Groundwater Management, Quadratic Programming
Solution

See PS06_4.gms, PS06_4.lst

1). Derive the cost coefficient *e*, given that the price of electricity is $ 0.20 / kwhr.

$e = 5650.56$

2). Check to see if a local maximum for this problem is also global (i.e. is the objective function concave?). To check this you will need to construct a Hessian matrix and use MATLAB to test its eigenvalues for your particular set of problem inputs.

$$F(x) = ax_i(b - x_i) - ex_i y_i = abx_i - ax_i^2 - ex_i A_{ij} x_j$$

Use *i*, *j* for summation, *k*, *l* for derivatives

$$\frac{\partial F(x)}{\partial x_k} = ab - 2ax_k - eA_{kj} x_j - ex_i A_{ik} \quad H_{kl} = \frac{\partial F(x)}{\partial x_k \partial x_l} = -2a - eA_{kl} - eA_{lk} = -2a - 2eR_{kl}$$

This follows from symmetry of *R*.

Using MATLAB, `eig(H)` gives 4 negative eigenvalues, so *F*(*x*) is negative definite (strictly concave). Since feasible region is defined by linear constraints it is convex so local maximum is also a global maximum.

3). Solve the problem using GAMS. If the objective function is not concave try a few different initial feasible solutions to provide confidence that the GAMS solution is a global maximum.

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4). Evaluate the shadow prices associated with any pumping lift (y_i) constraints that are active at the GAMS solution.

Shadow prices are zero for nominal values but are non-zero if nominals are changed so that either flow or drawdown constraints limit net revenue. For example, if:

$x_i \le \frac{0.95b}{2}$ or $y_i \le 10$ then some shadow prices are non-zero.