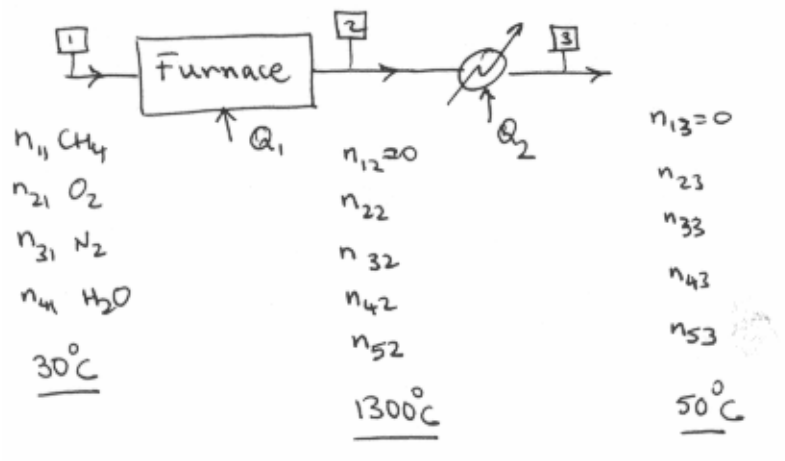
Problem 13 **10.213**



Component	#
CH_4	1
O_2	2
N_2	3
H_2O	4
CO_2	5

The reaction occurring in the furnace is

$$CH_4 + 2\ O_2 \quad CO_2 + 2\ H_2O$$

On the basis of 1 mole per unit time of methane in the feed stream, we calculate the molar flow rate of the components in the feed stream (stream 1)
n_{11} = 1 mole

The stoichiometric amount of O_2 is 2 mole. We use 20% excess. Therefore,
n_{21} = 2 * 1.2 = 2.4 mole

The ratio of N_2 to O_2 in air is 79:21. Therefore,
n_{31} = 2.4 * 79 / 21 = 9.03 mol

The feed stream is saturated with water vapor. Therefore the partial pressure of water in the feed stream equals the vapor (saturated) pressure of water.
$p_4 = p_4^{sat}$

We can get the saturation pressure of water from the steam tables at 30 °C (S+VN p. 669).
p_4^{sat} = 4.241 kPa = p_4

We know that the partial pressure equals the mole fraction multiplied by the total pressure.
$p_4 = x_{41}\ P$

Assuming that the furnace operates at 1 bar, we get
x_{41} = 4.241 / 100 = 0.04241

Now, can calculate the amount of water in stream 1.

$$x_{41} = \frac{n_{41}}{N_1} = \frac{n_{41}}{n_{11} + n_{21} + n_{31} + n_{41}}$$

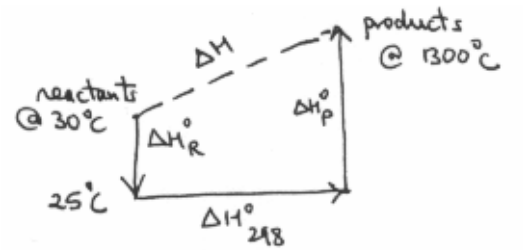
Subtracting the numerator from the denominator on both sides (math trick), we get

$$\frac{x_{41}}{1 - x_{41}} = \frac{n_{41}}{n_{11} + n_{21} + n_{31}}$$

Therefore,
n_{41} = 0.04241 * (1 + 2.4 + 9.03) / (1 – 0.04241) = 0.551 mole

Now, we calculate the molar flow rates of the product stream. Since the reaction goes to completion
$\varepsilon = 1$ mole
$n_{12} = n_{11} + \nu_1 \varepsilon = 1 + (-1)\ 1 = 0$ mole
$n_{22} = 2.4 + (-2)\ 1 = 0.4$ mole
$n_{32} = 9.03$ mole
$n_{42} = 0.551 + (2)\ 1 = 2.551$ mole
$n_{52} = 0 + (1)\ 1 = 1$ mole
$N_2 = 0.4 + 9.03 + 2.551 + 1 = 12.981$ mole

With the usual assumptions that the kinetic and potential energy changes are negligible and the fact that there is no shaft work, the overall energy balance for the process reduces to $\Delta H = Q$ (Q is heat transferred **to** the system). Since H is a state function, we use a path that simplifies the calculations as shown.



$\Delta H = \Delta H_R^o + \Delta H^o_{298} + \Delta H_P^o$

The heat of reaction can be calculated from the heats of formation. Heats of formations at standard conditions and 298 K
CH_4 : -74,520 J
CO_2 : -393,509 J
H_2O (g): -241,818

Since,

$$\Delta H^o \equiv \sum_i \nu_i \, \Delta H^o_{fi}$$

Therefore,
$\Delta H^o_{298} = -393{,}509 + (2)\ (-241{,}818) - (-74{,}520) = -802{,}625$ J

The enthalpy change of the reactants as they are cooled from 303 K (30 °C) to 298 K is given by

$$\Delta H^o_R = \left(\sum_i n_i \left\langle C^o_{Pi} \right\rangle_H \right) (298 - 303) = \left\langle C^o_P \right\rangle_H (298 - 303)$$

To get $\langle Cp^o \rangle_H$ of the reactant stream, we use equation (4.8) (S+VN p. 122)

$$\frac{\left\langle C_P \right\rangle_H}{R} = A + \frac{B}{2} T_o (\tau + 1) + \frac{C}{3} T_o^2 \left(\tau^2 + \tau + 1\right) + \frac{D}{\tau T_o^2}$$

to get

$$\left\langle C^o_P \right\rangle_H = R \left(\sum_i n_i A_i + \frac{\sum_i n_i B_i}{2} T_o (\tau + 1) + \frac{\sum_i n_i C_i}{3} T_o^2 \left(\tau^2 + \tau + 1\right) + \frac{\sum_i n_i D_i}{\tau T_o^2} \right)$$

From table C.1, we get

Component	A	10^3 B	106^6 C	10^{-5} D
1	1.702	9.081	-2.164	
2	3.639	0.506		-0.227
3	3.280	0.593		.040
4	3.470	1.450		0.121
5	5.457	1.045		-1.157

We have T_o = 303, and τ = 298/303 = 0.9835. Using the molar flow rate of the feed stream and the above equation, we get

$<Cp^o>_H$ = 387.300 J/K (for reactant stream)

Therefore,

ΔH_R^o = 387.3 * (298 – 303) = -1,936.5 J

Repeating the same procedure for the product stream, we get

$<Cp^o>_H$ = 456.409 J/mole K (for product stream)

Therefore,

ΔH_P^o = 456.409 * (1573 – 298) = 581,921.5 J

ΔH = -1,936.5 -802,625 + 581,921.5 = -222,640 J

Therefore

Q = ΔH = -222,640 J (negative because heat is transferred **from** the system)

Since there is no reaction occurring in the heat exchanger, we know that the molar flow rates in stream 2 equals those in stream 3. However, it is important to realize that at 50 °C some of the water might condense. To check for that, we refer to the vapor pressure of water at 50 °C from the stream tables (S+VN p. 669).

p_4^{sat} = 12.34 kPa

If there is no condensation, the partial pressure of water vapor in stream 3 will be

$p_4 = n_{43} / N_3 * P_t$ = 2.551 / 12.981 * 100 kPa = 19.65 kPa

Since the partial pressure of water vapor, can not exceed that of saturation, condensation occurs. Stream 3 becomes saturated with water. The mole fraction of water vapor is 0.1234. The total dry molar flow rate is (12.981 – 2.551 = 10.43 mole). To get the amount of water vapor, we use the following ratio

$n^v_{43} / N_3^{dry} = x^v_{43} / (1 - x^v_{43})$

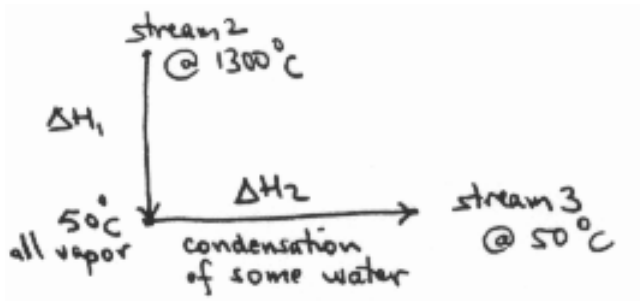
Therefore,

n^v_{43} = 0.1234 * 10.43 / (1 – 0.1234) = 1.468 mole

The amount of water condensed

n^l_{43} = 2.551 – 1.468 = 1.083 mole

To get the enthalpy change, we follow the following path.



$\Delta H_1 = \langle Cp^o \rangle_H (323 - 1573)$

We get $\langle Cp^o \rangle_H$ as before taking into consideration the different temperature range.

$\langle Cp^o \rangle_H = 457.559$ J

Therefore,
$\Delta H_1 = 457.559 * (323 - 1573) = -571{,}949$ J

$\Delta H_2 = n^l_{43} \{H^l(323 \text{ K}, 1 \text{ bar}) - H^v(323 \text{ K}, 1 \text{ bar})\}$

If we make the assumptions that the enthalpy of liquid is not a function of pressure (a good assumption) and that water vapor at 1 bar behaves as an ideal gas (i.e. its enthalpy is not a function of pressure either), we can use the heat of vaporization at 323 K and any pressure. Therefore, we use the saturated steam tables (S+VN p. 699) to get the heat of vaporization at 323 K = 2592.2 kJ/kg. Therefore,
$\Delta H_2 = 1.083 \text{ mole} * 18 \text{ g} / 1 \text{ mole} * (-2592.2 \text{ J/g}) = -50{,}532$ J

Therefore,
$Q_2 = -571{,}949 - 50{,}532 = -622{,}481$ J