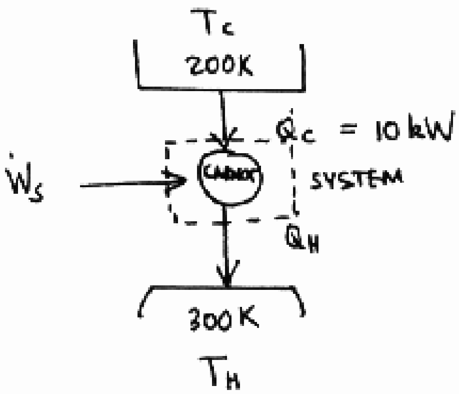
Problem Set #14 — Solution

Carnot Cycle



a) ◇ In this configuration, the Carnot cycle represents a HEAT PUMP

◇ From S+VN, pg 296 :

$$\omega_{CARNOT} = \frac{T_C}{T_H - T_C} \qquad (9.3)$$

$$= \frac{(200)}{(300)-(200)}$$

$$\boxed{\omega = 2}$$

b) ◇ From 1st Law, $\Delta U = 0 = \dot{Q}_c + \dot{Q}_H + \dot{W}_s$

(with Carnot heat pump as the system)

LEAVE AS +VE, EQUATION SHOULD TAKE CARE OF THE SIGN.

◇ thus,

$$\dot{Q}_H = -\dot{W}_s - \dot{Q}_c$$

$$\dot{Q}_H = -\dot{W}_s - (10\,kW) \qquad (1)$$

◇ From the definition of COEFFICIENT OF PERFORMANCE, ω :
(S+VN pg 296)

$$\omega = \frac{\text{heat absorbed at } T_C}{\text{net work}}$$

$$= \frac{|\dot{Q}_c|}{|\dot{W}_s|} \qquad (9.2)$$

thus, $|\dot{W}_s| = |\dot{Q}_c| / \omega = (10\,kW)/2 = 5\,kW$

$$\boxed{\dot{W}_s = 5\,kW}$$

$\dot{W}_s > 0$ since work being done ON the system.

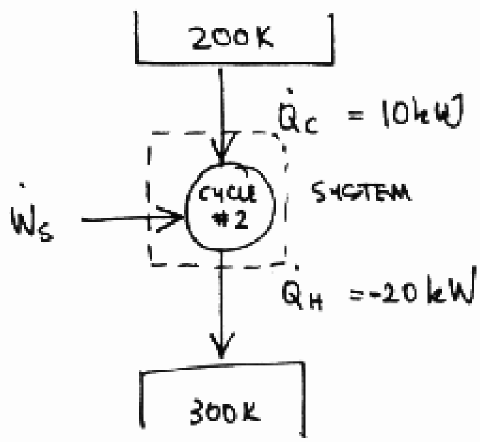
c) ◇ From eqn. (1) in part (c),

$$(1): \quad \dot{Q}_H = -\dot{W}_s - \dot{Q}_c$$
$$= -(5) - (10)$$
$$= -15 \text{ kW}$$

◇ Since $\dot{Q}_H$ leaves system, $\dot{Q}_H < 0$

$$\boxed{\dot{Q}_H = -15 \text{ kW}}$$

Cycle # 2



d) ◇ 1st Law energy balance:

$$\Delta U = 0$$
$$= \dot{Q}_c + \dot{Q}_H + \dot{W}_s$$

◇ thus,

$$\dot{W}_s = -\dot{Q}_c - \dot{Q}_H$$
$$= -(10) - (-20)$$
$$\boxed{\dot{W}_s = 10 \text{ kW}}$$

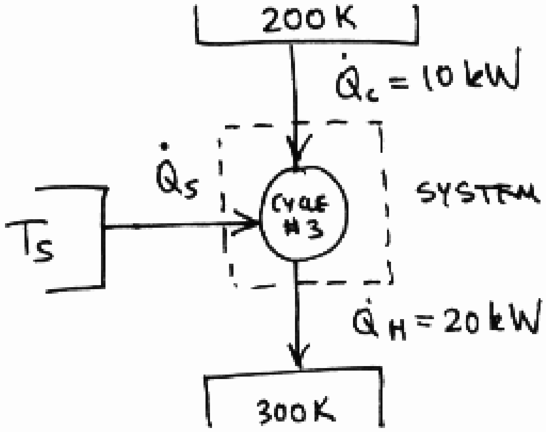
(i.e. requires 2x as much as Carnot Cycle)

e) ◇ Coefficient of performance:

$$\omega = \frac{|\dot{Q}_c|}{\dot{W}_s} = \frac{(10 \text{ kW})}{(10 \text{ kW})} = 1$$

$$\boxed{\omega_{cycle\#2} = 1}$$ (lower than Carnot)

Cycle # 3



f) ◦ 1st law energy balance

$$\Delta U = 0$$
$$= \dot{Q}_c + \dot{Q}_H + \dot{Q}_S$$

so, $\dot{Q}_S = -\dot{Q}_c - \dot{Q}_H$

$$= -(10) - (-20)$$

$$\boxed{\dot{Q}_S = 10 \text{ kW}}$$

>0 so added to system

g) ◦ From second law, entropy balance

$$\Delta \dot{S} = 0 \quad (\text{state function like } U)$$

$$= \frac{\dot{Q}_c}{T_c} + \frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}_S}{T_S}$$

so, $\left(\frac{10 \text{ kW}}{200 \text{ K}}\right) + \left(\frac{-20 \text{ kW}}{300 \text{ K}}\right) + \left(\frac{10 \text{ kW}}{T_S}\right) = 0$

$$\frac{10 \text{ kW}}{T_S} = -(0.05) + (0.06667) \text{ [kW]}$$

$$= 0.016667$$

so, $T_S = \frac{(10)}{(0.016667)}$

$$\boxed{T_S = 600 \text{K}}$$