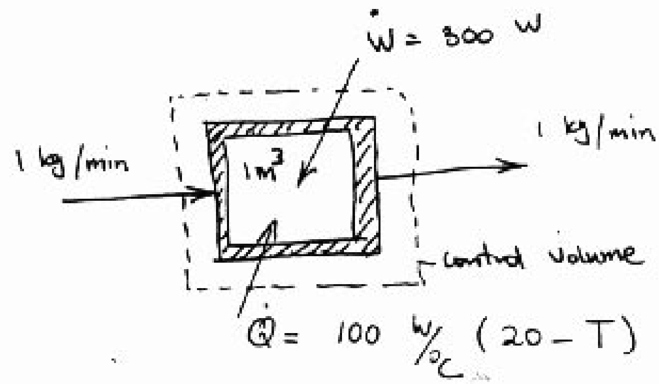
Problem 15



a) Steady-state temperature of outlet stream

$T_{in} = 40°C$

Apply energy balance on the control volume

<u>Energy in = Energy out</u> (no accumulation)

$$\dot{m} H_{in} + \dot{Q} + \dot{W} = \dot{m} H_{out}$$

$$\dot{m} (H_{out} - H_{in}) = \dot{Q} + \dot{W}$$

To get H, we can use the steam tables, C_p equation from table C.3 or, simply, constant $\langle C_p \rangle$. Using constant C_p is not the most accurate method, but for liquids at small temperature changes, it is a good approximation.

$$\dot{m} \langle C_p \rangle (T_{out} - T_{in}) = \dot{Q} + \dot{W}$$

Because the tank is well-mixed, the outlet water temperature is the same as the temperature inside the tank.

$T = T_{out}$

$$\therefore \quad \dot{m} \langle C_p \rangle (T - T_{in}) = 100 (20 - T) + 300$$

$$\dot{m} = 1 \frac{kg}{min} \frac{1 \, min}{60 \, s} \cdot \frac{1}{60} \frac{kg}{s}$$

$$\langle C_p \rangle = 4180 \ J/kg\,K$$

Note that 1 K = 1 °C. The temperature unit is the same. But the absolute value is different. Since we are using differences in temperature, we can use either K or °C.

$$\frac{4180}{60}(T-40) = 100(20-T) + 300$$

$$69.7\,T - 2788 = 2000 - 100\,T + 300$$

$$169.7\,T = 5088$$

$$\therefore \boxed{T = 30.0\ °C}$$

b) This is an unsteady state problem. We need to get the temperature of the water inside the tank as a function of time. We use the same energy balance equation but we add an accumulation term.

Energy in − Energy out = Accumulation

$$\dot{m}(H_{in} - H_{out}) + \dot{Q} + \dot{W} = \frac{d(mU)_{system}}{dt}$$

$$d(mU)_{system} = d(mU)_{water} + d(mU)_{tank}$$

For water $d(mU) = d(mC_vT)$

$C_v = C_p$ for any incompressible fluid (see Example 6.2 $_{SVN}$)

$d(mU)_{water} = mC_p\,dT$ (for constant C_p)

The heat capacity of the tank is 10% that of water.

Therefore $(mC_p)_{tank} = 0.1\ (mC_p)_{water}$

$$\therefore d(mU)_{system} = 1.1\ m_{water}\, C_{p_{water}}\, dT$$

$M_{water} = \rho V = 1000 \frac{kg}{m^3} \; 1\,m^3 = 1000\ kg$

Therefore, [substituting in the energy balance equation]

$$\frac{1}{60} \times 4180\,(60 - T) + 100\,(20 - T) + 300 = 1.1 \times 1000 \times 4180 \frac{dT}{dt}$$

$$4180 - 70T + 2000 - 100T + 300 = 4{,}598{,}000 \quad \frac{dT}{dt}$$

$$6480 - 170T = 4{,}598{,}000 \frac{dT}{dt}$$

Now, we need to solve this differential equation for $T(t)$ by separation of variables.

$$dt = 4{,}598{,}000 \frac{dT}{6480 - 170T} \qquad t \text{ in sec.}$$

Integrate

$$\int_{t_0}^{t} dt = 4{,}598{,}000 \int_{30}^{T} \frac{dT}{6480 - 170T}$$

result of part a is the temperature of the water inside the tank at $t = t_0$

$$t - t_0 = \frac{4{,}598{,}000}{(-170)} \ln\left[\frac{6480 - 170T}{6480 - 170 \times 30}\right]$$

$$t - t_0 = -27{,}047 \ln(4.7 - 0.123T)$$

$$\exp\left(\frac{-\Delta t}{27{,}047}\right) = 4.7 - 0.123\ T$$

$$\boxed{T = 38.15 - 8.13\ \exp\left[\frac{-(t - t_0)}{27{,}047}\right] \quad \begin{array}{l} t \text{ in sec} \\ T \text{ in } {}^\circ C \end{array}}$$

To check

at $t = t_0$, we should get the initial temperature

$$T = 38.15 - 8.13\ \underbrace{\exp(0)}_{1} = 30\ {}^\circ C \qquad \text{correct}$$

The steady state temperature can be calculated by putting $t = \infty$

$$T = 38.15 - 8.3\ \underbrace{\exp(-\infty)}_{0}$$

$$T_{\substack{\text{steady}\\\text{state}}} = 38.15\ {}^\circ C$$