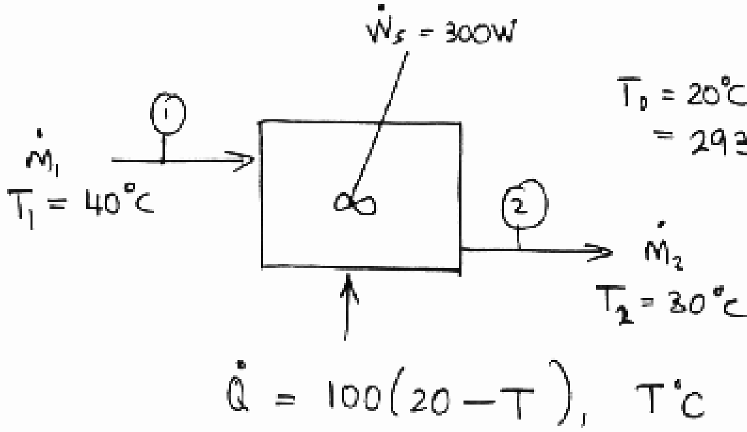
10.213

Problem 16 Solution



$T_0 = 20°C + 273.15$
$= 293.15 K$

* from MB:
$\dot{m}_1 = \dot{m}_2 = \dot{m}$

◇ Now,

$$\boxed{\dot{W}_{lost} = \dot{W}_{actual} - \dot{W}_{rev}}$$

and

$$\dot{W}_{rev} = \sum_{out} \dot{m}_j B_j - \sum_{in} \dot{m}_i B_i$$

$$= \dot{m}_2 B_2 - \dot{m}_1 B_1$$

$$= \dot{m}(B_2 - B_1) \qquad (\text{since } \dot{m}_1 = \dot{m}_2 = \dot{m})$$

$$= \dot{m}\left[(H_2 - T_0 S_2) - (H_1 - T_0 S_1)\right]$$

$$= \dot{m}(H_2 - H_1) - \dot{m} T_0 (S_2 - S_1)$$

$$= \dot{m}\Delta H - \dot{m} T_0 \Delta S \quad \text{———} \; 1$$

◇ from S+VN, p. 183 & 184:

$$dH = C_p\, dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right] dP \qquad (6.20)$$

$$dS = \frac{C_P}{T}\, dT - \left(\frac{\partial V}{\partial T}\right)_T dP \qquad (6.21)$$

but for a liquid, in this situation $H \simeq f(T)$, $S \simeq f(T)$ } since P is ~ constant assuming no ΔP

◦ Thus,

$$\Delta H \cong \langle C_p \rangle \Delta T = \langle C_p \rangle (T_2 - T_1)$$
$$\Delta S \simeq \langle C_p \rangle \ln(T_2/T_1)$$

— since C_p is roughly constant for liquids over small changes of temperature,

$$\langle C_p \rangle \simeq 4{,}180 \text{ J/kg}\cdot\text{K}$$

◇ hence,

$$\Delta H = (4{,}180 \text{ J/kg}\cdot\text{K})(30 - 40 \text{ K})$$
$$= -41{,}800 \text{ J/kg}$$
$$\Delta S = (4{,}180 \text{ J/kg}\cdot\text{K}) \ln\left(\frac{30+273}{40+273}\right)$$
$$= -135.73 \text{ J/kg}\cdot\text{K}$$

(*Note: could also get these from the steam tables for sat'd liquid at T_1 & T_2. You are then assuming (as above) that $H \neq f(P)$ and $S \neq f(P)$. Results are comparable.)

◇ going back to ①:

$$\dot{W}_{rev} = \underbrace{\left(\frac{1 \text{ kg/min}}{60 \text{ s/min}}\right)}_{\dot{m}}\underbrace{(-41{,}800 \text{ J/kg})}_{\Delta H} - \underbrace{\left(\frac{1 \text{ kg/min}}{60 \text{ s/min}}\right)}_{\dot{m}}\underbrace{(293 \text{ K})}_{T_0}\underbrace{(-135.73 \text{ J/kg}\cdot\text{K})}_{\Delta S}$$

$$= -696.67 \text{ W} - (-662.80 \text{ W})$$

$$\boxed{\dot{W}_{rev} = -33.9 \text{ W}}$$ is work obtained from a REVERSIBLE process

◇ Now,

$$\dot{W}_{lost} = \underset{\dot{W}_s}{\underset{\shortparallel}{\dot{W}_{actual}}} - \dot{W}_{rev}$$

$$= \underbrace{(+300\,W)}_{\text{work done}} - (-33.9\,W)$$

$$\boxed{\dot{W}_{lost} = 333.9\ W}$$

◇ Thus, if the process described — namely, cooling a stream of water from 40°C to 30°C — were done reversibly, you could obtain 33.9 W of work from it. Our process is very inefficient, however, and requires that we put 300 W of work into it!

◇ Sources of irreversibility:

(~90%) * Stirrer — mechanical irreversibility, work of stirrer generally dissipated as heat.

(~10%) * Heat loss to surroundings — heat transfer occurs over finite ΔT.

(small since $\underset{10°C}{\Delta T}$ is small)
