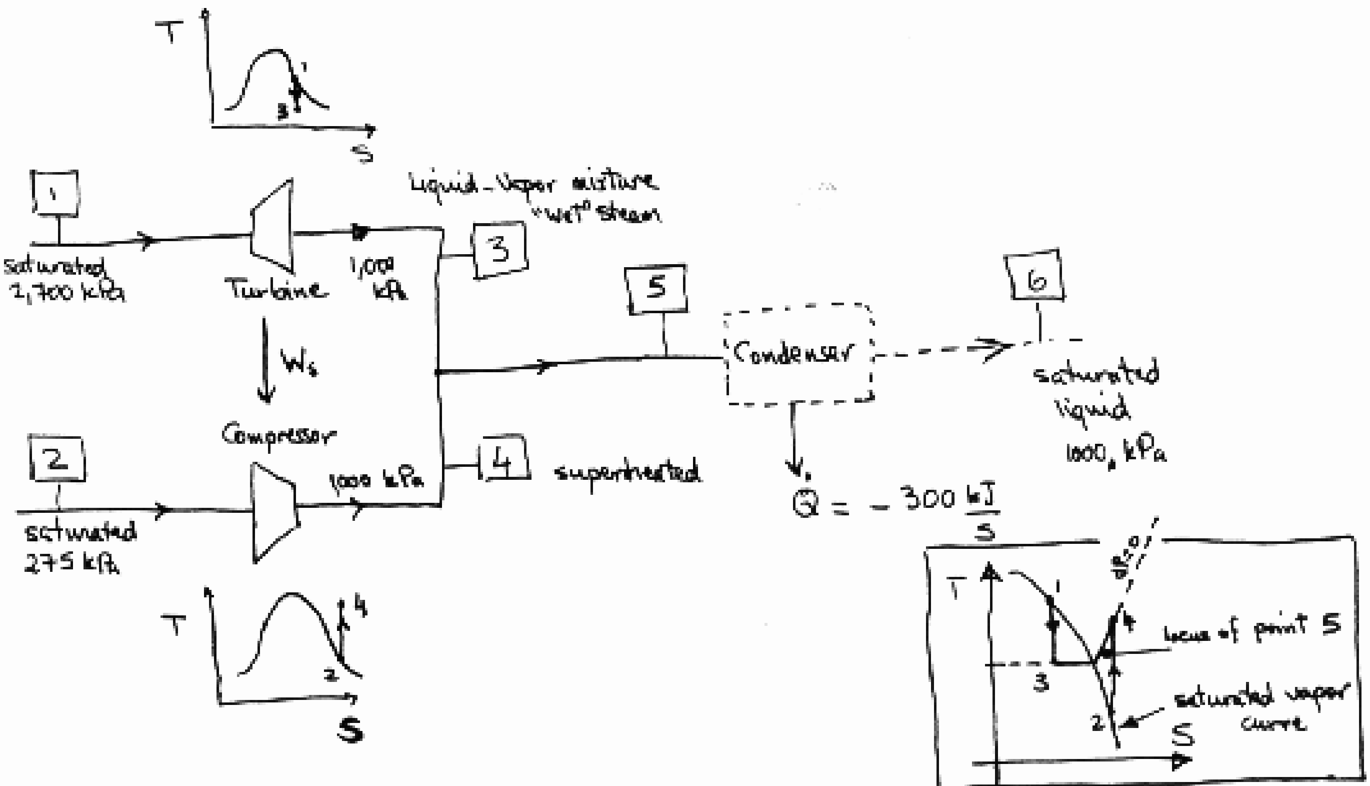
<u>Problem 17</u>



Stream 4 is a superheated steam and stream 3 is a wet steam (i.e. contains some liquid). Stream 5 can either be superheated, saturated or wet. That depends on the relative amounts of streams 3 and 4.

a) To solve for $\dot{m}_1$ and $\dot{m}_2$ we start with a mass balance about the overall system.

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_5 \qquad (1)$$

Energy balance

$$\dot{m}_1 H_1 + \dot{m}_2 H_2 = \dot{m}_5 H_5 \qquad (2)$$

Since the process is reversible, the entropy generation is zero.

$$\dot{m}_1 S_1 + \dot{m}_2 S_2 = \dot{m}_5 S_5 \qquad (3)$$

We can easily get the values of H_1, H_2, S_1 and S_2 from the steam tables. This leaves us with 5 unknowns $\dot{m}_1$, $\dot{m}_2$, $\dot{m}_3$, H_5 and S_5. So we need two more equations to solve the system. We can get these two equations from the fact that upon condensation of stream 5, 300 kJ/s is released. Therefore,

$$\dot{m}_5 (H_5 - H_6) = 300 \quad kJ/s \qquad \text{———} (4)$$

Assuming that point 5 is either a wet steam or saturated steam, condensation will occur at constant temperature (we will need to check that assumption later).

Therefore,

$$\Delta S_{condensation} = \frac{Q_{condensation}}{T_{saturation}}$$

$$S_5 - S_6 = \frac{H_5 - H_6}{T_{sat}} \qquad \text{———} (5)$$

Now we have the complete set of equations.

From (4), $$H_5 = H_6 + \frac{300}{\dot{m}_5} \qquad \text{———} (6)$$

From (5) & (4), $$S_5 - S_6 = \frac{300}{\dot{m}_5 T_{sat}} \Rightarrow S_5 = S_6 + \frac{300}{\dot{m}_5 T_{sat}} \qquad \text{———} (7)$$

Substituting (6) in (2),

$$\dot{m}_1 H_1 + \dot{m}_2 H_2 = \dot{m}_5 H_6 + 300 \qquad \text{———} (8)$$

Substituting (7) in (3)

$$\dot{m}_1 S_1 + \dot{m}_2 S_2 = \dot{m}_5 S_6 + \frac{300}{T_{sat}} \qquad \text{———} (9)$$

From (1)

$$\dot{m}_2 = \dot{m}_5 - \dot{m}_1 \qquad \text{———} (10)$$

Substituting (10) in (8)

$$\dot{m}_1 H_1 + (\dot{m}_5 - \dot{m}_1) H_2 = \dot{m}_5 H_6 + 300 \qquad \text{———} (11)$$

Same for entropy

$$\dot{m}_1 S_1 + (\dot{m}_5 - \dot{m}_1) S_2 = \dot{m}_5 S_6 + \frac{300}{T_{sat}} \qquad \text{———} (12)$$

Solving for $\dot{m}_1$ in both (11) and (12). From (11)

$$(H_1 - H_2)\,\dot{m}_1 = \dot{m}_5\,(H_6 - H_2) + 300$$

$$\dot{m}_1 = \dot{m}_5\left(\frac{H_6 - H_2}{H_1 - H_2}\right) + \frac{300}{H_1 - H_2} \qquad \text{—— (13)}$$

From (12)

$$(S_1 - S_2)\,\dot{m}_1 = \dot{m}_5\,(S_6 - S_2) + \frac{300}{T_{sat}} \qquad \text{—— (14)}$$

Substituting for $\dot{m}_1$ from (13),

$$\dot{m}_5\left(\frac{H_6 - H_2}{H_1 - H_2}\right) + \frac{300}{H_1 - H_2} = \dot{m}_5\left(\frac{S_6 - S_2}{S_1 - S_2}\right) + \frac{300}{T_{sat}(S_1 - S_2)}$$

Solving for $\dot{m}_5$,

$$\boxed{\dot{m}_5 = \frac{\left[\dfrac{300}{T_{sat}(S_1 - S_2)} - \dfrac{300}{H_1 - H_2}\right]}{\left[\left(\dfrac{H_6 - H_2}{H_1 - H_2}\right) - \left(\dfrac{S_6 - S_2}{S_1 - S_2}\right)\right]}} \qquad \text{—— (15)}$$

Getting the values from the steam tables (F.1).

<u>Stream 1</u> [saturated steam @ 2700 kPa] {use closest value from table}

$\boxed{H_1 = 2801.7 \text{ kJ/kg}}$ $\boxed{S_1 = 6.2249 \text{ kJ/kg K}}$

<u>Stream 2</u> [saturated steam @ 275 kPa]

$\boxed{H_2 = 2719.9 \text{ kJ/kg}}$ $\boxed{S_2 = 7.0261 \text{ kJ/kg K}}$

<u>Stream 6</u> [saturated liquid @ 1,000 kPa]

$\boxed{H_6 = 763.1 \text{ kJ/kg}}$ $\boxed{S_6 = 2.1393 \text{ kJ/kg K}}$ $\boxed{T_{sat} = 453.15 \text{ K}}$

Substituting in (15), $\dot{m}_5 = 0.1497$ kg/s

Substituting in (13)

Therefore,

$$\boxed{\begin{aligned} \dot{m}_1 &= 0.0864 \text{ kg/s} \\ \dot{m}_2 &= 0.0633 \text{ kg/s} \end{aligned}}$$

We still need to check if point 5 lies within the liquid-vapor dome.

Using eqs ⑥, we get

$$H_5 = 2767.1 \text{ kJ/kg}$$

less than the saturated vapor ethalpy @ 1,000 kPa of 2776.3 kJ/kg

Therefore, the above calculation is valid.

b) Since the process is now irreversible, we can not use equation ③. Equations ① and ② are still valid of course. We also know that the work generated by the turbine is used by the compressor. Therefore

$$-\dot{m}_1 (H_3 - H_1) = \dot{m}_2 (H_4 - H_2) \qquad ⑯$$

We can use the thermodynamic efficiencies given to get H_3 and H_4. For the turbine, $\eta = 0.78 = \left(\frac{W_{reversible}}{W_{actual}}\right)^{-1} = \frac{W_{actual}}{W_{reversible}} = \frac{H_3 - H_1}{H_{3'} - H_1}$

$H_{3'}$ is based on an isentropic process. $S_{3'} = S_1 = 6.2249$ kJ/kg K

Point 3' is in the liq-vap. dome

from steam table @ 1,000 kPa

mass fraction of vapor $\rightarrow X_{3'} = \frac{S_{3'} - S_\ell}{S_v - S_\ell} = \frac{6.2249 - 2.1393}{4.4426} = 0.9196$

$$H_{3'} = H_\ell + X_{3'}(H_v - H_\ell) = 763.1 + 0.9196\,(2013.1)$$
$$= 2614.4 \text{ kJ/kg}$$

Therefore, $H_3 = H_1 + \eta_{turbine}(H_{3'} - H_1)$

$$= \overset{2801.7}{} + 0.78\,(2614.4 - 2801.7) = \boxed{2655.6 \ \frac{kJ}{kg}}$$

H_3

We do the same thing for the compressor to get H_4.

$$\eta = 0.75 = \frac{W_{reversible}}{W_{actual}} = \frac{H_{4'} - H_1}{H_4 - H_1}$$

$H_{4'}$ is based on an isentropic process $S_{4'} = S_2 = 7.0261$ kJ/kg K

Point 4' is in the superheated regime. Using table F.2 @ 1000 kPa (p. 683)

$$H_{4'} = 2965.2 + \left(\frac{7.0261 - 6.968}{7.0485 - 6.968}\right)(3009 - 2965.2) = 2996.8 \ \frac{kJ}{kg}$$

Therefore, $H_4 = H_1 + \frac{H_4' - H_1}{\eta} = 2807.7 + \frac{2996.8 - 2801.7}{0.75}$

$$\boxed{H_4 = 3061.8 \text{ kJ/kg}}$$

Now the equations that we can use are

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_5 \qquad (1)$$

$$\dot{m}_1 H_1 + \dot{m}_2 H_2 = \dot{m}_5 H_5 \qquad (2)$$

$$\dot{m}_5 (H_5 - H_6) = 300 \qquad (4)$$

$$-\dot{m}_1 (H_3 - H_1) = \dot{m}_2 (H_4 - H_2) \qquad (16)$$

We need to solve these 4 equations to get 4 unknowns $\dot{m}_1$, $\dot{m}_2$, $\dot{m}_5$ and H_5.

$$H_5 = H_6 + \frac{300}{\dot{m}_5}$$

$$\dot{m}_1 H_1 + \dot{m}_2 H_2 = \dot{m}_5 H_6 + 300$$

From (16), $\dot{m}_2 = -\dot{m}_1 \frac{(H_3 - H_1)}{(H_4 - H_2)}$ (17)

From (1) $\dot{m}_1 + (-\dot{m}_1) \frac{H_3 - H_1}{H_4 - H_2} = \dot{m}_5$

Therefore,

$$\dot{m}_1 H_1 - \dot{m}_1 \left(\frac{H_3 - H_1}{H_4 - H_2}\right) H_2 = \dot{m}_1 H_6 \left[1 - \left(\frac{H_3 - H_1}{H_4 - H_2}\right)\right] + 300$$

Solving for $\dot{m}_1$,

$$\boxed{\dot{m}_1 = \frac{300}{\left[H_1 - H_6 + \left(\frac{H_3 - H_1}{H_4 - H_2}\right)(H_6 - H_2)\right]}}$$

Substituting, we get

and from (17)

$$\boxed{\begin{aligned} \dot{m}_1 &= 0.1043 \text{ kg/s} \\ \dot{m}_2 &= 0.0446 \text{ kg/s} \end{aligned}}$$

Thermodynamic Analysis

Assume $T_0 = 300$ K

Using Eqs 16.2,

$$\dot{W}_{ideal} = \cancel{\Delta(H\dot{m})_{fs}}^{\,0} - T_0\, \Delta(S\dot{m})_{fs}$$

$$\dot{W}_{ideal} = +T_0\left(\dot{m}_1 S_1 + \dot{m}_2 S_2 - \dot{m}_5 S_5\right)$$

To get S_5, we need H_5.

$$H_5 = H_6 + \frac{300}{\dot{m}_5} = 763.1 + \frac{300}{(0.1043 + 0.0446)} = \boxed{2777.9\ \frac{kJ}{kg}} \quad H_5$$

$$\boxed{\dot{m}_5 = 0.1489\ \frac{kg}{s}}$$

Point 5 is slightly superheated ($H_5 > H_{sat.vap}$ @ 1000 kPa)

By interpolation, (page 683) $\qquad$ 2776.3 kJ/kg

$$S_5 = 6.5828 + \frac{2777.9 - 2776.2}{2826.8 - 2776.2}(6.6922 - 6.5828) = \boxed{6.5864\ \frac{kJ}{kg\,K}} \quad S_5$$

$$\therefore\ \dot{W}_{ideal} = +300\left[0.1043 \times 6.2249 + 0.0446 \times 7.0261 - 0.1489 \times 6.5864\right]$$

$$\boxed{\dot{W}_{ideal} = -5.428\ \frac{kJ}{s}}$$

That means that we could have extracted 5.428 $\frac{kJ}{s}$ from the system, if the process was ideal. Now we need to analyze each unit to calculate the work lost.

Turbine $\qquad \dot{W}_{lost} = +\dot{m}_1 T_\sigma (S_3 - S_1)$

To get S_3, we use the quality x_3. $\qquad x_3 = \frac{H_3 - H_l}{H_v - H_l} = \frac{2655.6 - 763.1}{2013.1} = 0.94$

$$S_3 = S_l + x_3 (S_v - S_l) = 2.1393 + 0.94\,(4.4426) = \boxed{6.3153\ kJ/kg\,K = S_3}$$

$$\boxed{\dot{W}_{lost,\,turbine} = +0.1043 \times 300\,(6.3153 - 6.2249) = +2.83\ \frac{kJ}{s}}$$

Compressor

$$\dot{W}_{lost} = +\dot{m}_2 T_0 (S_4 - S_2)$$

To get S_4, we interpolate based on the value of $H_4 = 3061.8$ kJ/kg

$$S_4 = 7.1009 + \left(\frac{3061.8 - 3050.8}{3104.4 - 3050.8}\right)(7.1924 - 7.1009) = 7.1197 \ \frac{kJ}{kg K}$$

$$\boxed{\dot{W}_{lost,\ compressor} = +0.0446 \times 300 (7.1197 - 7.0261) = +1.252 \ \frac{kJ}{s}}$$

Mixing Point

$$\dot{W}_{lost} = +T_0 (\dot{m}_5 S_5 - \dot{m}_1 S_3 - \dot{m}_2 S_4)$$

$$= +300 (0.1489 \times 6.5864 - 0.1043 \times 6.3153 - 0.0446 \times 7.1197)$$

$$\boxed{\dot{W}_{lost,\ mixing} = +1.347 \ \frac{kJ}{s}}$$

	kW	Percent of $\lvert \dot{W}_{ideal} \rvert$
$\dot{W}_{act}$ (whole system)	0	0
$\dot{W}_{lost}$ (Turbine)	+ 2.83	52
$\dot{W}_{lost}$ (compressor)	+1.252	23
$\dot{W}_{lost}$ (mixing)	+ 1.347	25
$\dot{W}_{rev} = \dot{W}_{act} - \dot{W}_{lost}$	− 5.43	100