Quiz 4 Solution

10.213

① a) Using modified Raoult's law

$$P_A = x_A \gamma_A P_A^{sat}$$

For component 1 & 2 :

$$P = x_1 \gamma_1 P_1^{sat} + x_2 \gamma_2 P_2^{sat}$$

$$x_1 = x_2 = 0.5$$

Since G^E is symmetric with respect to x_1 and x_2. Therefore, for equimolar quantities of two liquids $\gamma_1 = \gamma_2$

$$55 = 0.5\gamma_1 * 50 + 0.5\gamma_1 60$$

That means that $\gamma_1 = \gamma_2 = 1$

Therefore the solution is ideal and $\underline{B = 0}$.

Method 2 We can also prove this by getting the equation for γ from G^E

$$\ln \gamma_1 = \left[\frac{\partial\left(\frac{nG^E}{RT}\right)}{\partial n_1}\right]_{n_2,T,P} \qquad \frac{nG^E}{RT} = \frac{nB}{RT}\frac{n_1}{n}\frac{n_2}{n} = \frac{B}{RT}\frac{n_1 n_2}{n}$$

$$\left[\frac{\partial\left(\frac{nG^E}{RT}\right)}{\partial n_1}\right]_{n_2,T,P} = \frac{Bn_2}{RT}\left[\frac{\partial(n_1/n)}{\partial n_1}\right]_{n_2,T,P} = \frac{Bn_2}{RT}\left[\frac{-n_1}{n^2} + \frac{1}{n}\right]$$

$$= \frac{Bn_2}{RT}\left[\frac{-n_1 + n}{n^2}\right] = \frac{Bn_2 n_2}{RTn^2} = \frac{B}{RT}x_2^2$$

Similarly for γ_2, $\ln \gamma_2 = \frac{B}{RT} x_1^2$

Substituting in the total pressure equation

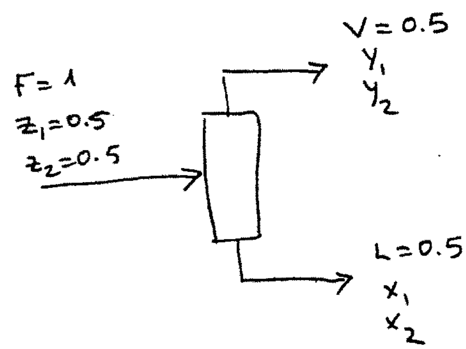
$$55 = 0.5 \exp[B(0.5)^2]\, 50 + 0.5 \exp[B(0.5)^2]\, 60$$

$$55 = 55 \exp\left[B(0.5)^2\right] \rightarrow B(0.5)^2 = 0 \rightarrow \boxed{B = 0}$$

(b) Using Raoult's law (since γ's $= 1$)

$$\boxed{y_1 = \frac{x_1 P_1^{sat}}{P} = \frac{0.5 \times 50}{55} = 0.455}$$

(c) The 50%-composition liquid is allowed to evaporate to equal amounts of liquid and vapor. This is a flash problem as follows:



Raoult's law

$$x_1 P_1^{sat} = y_1 P \qquad 50\, x_1 = y_1 P \quad — (1)$$

$$(1-x_1) P_2^{sat} = (1-y_1) P \qquad 60(1-x_1) = (1-y_1) P \quad — (2)$$

MB

$$F z_1 = V y_1 + L x_1 \qquad 0.5 = 0.5 y_1 + 0.5 x_1$$

$$x_1 + y_1 = 1 \quad — (3)$$

Three equations in three unknowns (x_1, y_1, P)

Using (3) in (1) & (2)

$$50\, x_1 = (1-x_1) P$$

$$60(1-x_1) = x_1 P$$

dividing

$$\frac{50\, x_1}{60(1-x_1)} = \frac{(1-x_1)}{x_1}$$

$$50\, x_1^2 = 60(1-x_1)^2$$

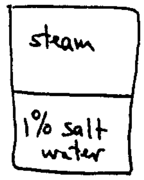
$$5 x_1^2 = 6(1 - 2x_1 + x_1^2)$$

$$x_1^2 - 12 x_1 + 6 = 0$$

$$\boxed{\begin{array}{ll} x_1 = \dfrac{12 - \sqrt{12^2 - 4\times 6}}{2} = 0.523 & x_2 = 0.477 \\ y_1 = 0.477 & y_2 = 0.523 \\ P = \dfrac{50 \times 0.523}{0.477} = 54.8\text{ kPa} & \end{array}}$$

② a) At equilibrium

$$f^L = f^V$$



300°C

For a pure component, $f^V = \phi P$

Using the generalized correlation

$$\phi = \phi^0 (\phi')^{\omega}$$

For water $T_c = 647.1\ K$ $P_c = 220.55\ bar$ $\omega = 0.345$

$$T_r = \frac{T}{T_c} = \frac{300+273}{647.1} = 0.885 \sim 0.9$$

$$P_r = \frac{P}{P_c} = \frac{8592.7}{22{,}055} = 0.39 \sim 0.4$$

From the table $\phi^0 = 0.8204$ $\phi' = 0.9141$

$$\phi = 0.8204 \times (0.9141)^{0.345} = 0.795$$

$$f^V = f^L = 0.795 \times 8592.7 = 6{,}834\ kPa$$

b) For the mixture $\hat{f}_i^L = x_i \gamma_i f_i^L$

$\gamma_w \sim 1$ because $x \sim 1$

$$\therefore \hat{f}_i^L = 0.99 \times 6834 = 6766\ kPa$$

c) $\hat{f}_i^V = \hat{f}_i^L = 6766\ kPa$

$\hat{f}_i^V = \phi P$

$\phi \sim 0.795$ [not exact because P is diff. than for part a]

$$P = \frac{6766}{0.795} = 8510\ kPa$$

<u>Another method</u> $P = x_A P_A^{sat} = 0.99 \times 8592.7 = 8507\ kPa$

this comes from

$$f_A^V = \hat{f}_A^L$$

$$\phi_A P = x_A \underbrace{\cancel{\gamma_A}}_{\to 1} P_A^{sat} \hat{\phi}_A \underbrace{\cancel{\mathcal{P}}}_{\to 1 \text{ poynting factor}}$$

$\because P_A^{sat} \sim P \therefore \phi_A \sim \hat{\phi}_A$ and can be canceled out

$$\Rightarrow P = x_A P_A^{sat}$$