

Solving Cubic Equations

First, write your equation as a polynomial:

$$A V^3 + B V^2 + C V + D = 0$$

Method 1: Iteration

- 1.) Write the equation as $V=f(V)$

$$V = -(1/C) (A V^3 + B V^2 + D)$$
- 2.) Pick an initial guess for V_0 (eg -0 , V^{ig} , etc.) and evaluate:

$$V_1 = -(1/C) (A V_0^3 + B V_0^2 + D)$$
- 3.) Use V_1 as your next guess:

$$V_2 = -(1/C) (A V_1^3 + B V_1^2 + D)$$
- 4.) Repeat with:

$$V_{i+1} = -(1/C) (A V_i^3 + B V_i^2 + D)$$

 Until $V_{i+1} = V_i$ (within some tolerance, such as $V_{i+1}/V_i \sim .99-1.01$)

Method 2: Excel

For this example, let the polynomial be:

$$f(V) = V^3 - 8 V^2 + 17 V - 10 = 0$$

- 1.) In an Excel spreadsheet, set up the cells as follows:

	A	B
1	<u>V</u>	<u>f(V)=0</u>
2	10	360

Initial guess for V

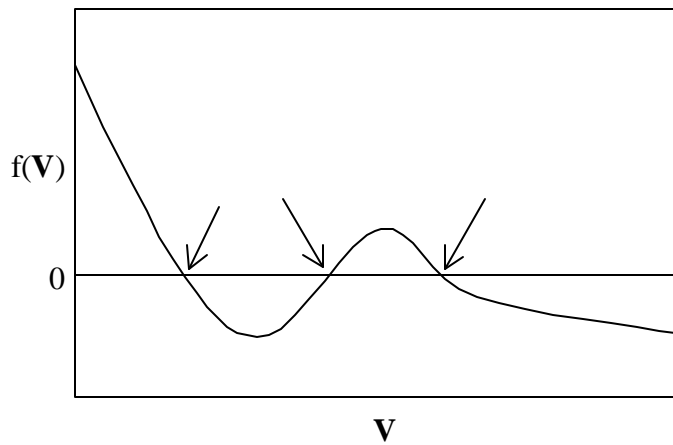
Typed in as: **=A2^3-8*A2^2+17*A2-10**

*Note that by typing **A2** in an equation in a cell, it acts like a variable, replacing that variable with the value in cell **A2**. When the value in cell **A2** is a root of $f(V)$, then cell **B2** will be 0.*

- 2.) Click on cell **B2** and go to **Tools → Goal Seek**. In the window that opens, enter the following:
 Set Cell: B2
 To Value: 0
 By Changing Cell: A2
- 3.) Click **OK**. Excel will then numerically solve for a value of **A2** that will cause **B2** to equal 0. Since it solves numerically, it will reach 0 to within some very small value (10^{-4}) and display the value of **A2** that gave this near-0 value → this is one of the roots of the equation.
- 4.) To get all 3 roots, try plotting the function and using approximate roots as your initial guesses (Excel will usually find the root closest to your initial guess) or use extreme values as your guesses (eg -0 and 100000) to find the largest and smallest roots.

Method 3: Graphical

For those of you with graphing calculators (or who like to plot in Excel), plot the polynomial and estimate where the roots are by zooming in or using the trace option:



Method 4: Maple

I know that not many of you feel comfortable with Maple, but it is the quickest and most accurate way – you might want to give it a try at least once...

Enter the following at the prompt ($>$) in the Maple window and hit Enter at the end of a line – it will return the italicized text. Note that once you define a variable (here, c), you can use it to describe other variable later in the worksheet.

The function **solve** is written as **solve(function, variable)**, where the numbers it returns are the values of the variable that are roots of the function.

```
> a:=1; b:=-8; c:=2*9-1; d:=-1*c+7;  
      a := 1  
      b := -8  
      c := 17  
      d := -10  
> solve(a*x^3+b*x^2+c*x+d,x);  
      5, 2, 1
```