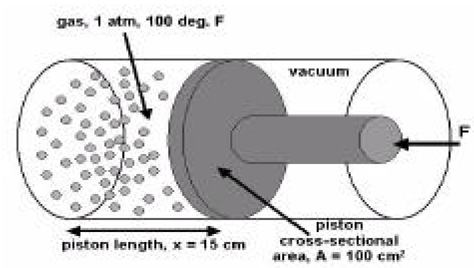
Spring 2002. 10.301. Introduction to Fluid Mechanics

Assignment 1. Solution

1. We have a piston, of cross-sectional area A = 100 cm^2, in which a dilute gas is constrained by a moving wall. Initially, the distance between the stationary and moving walls of the piston, x, is 15 cm, and the piston chamber is filled with gas at a pressure of 1 atmosphere and a temperature of 100 degrees F. Outside the piston is a vacuum.



We now compress the gas by decreasing the piston length x from 15 cm to 10 cm. If we move the piston very slowly, there will be enough time to add or remove heat from the surroundings after each small movement of the piston so that we may assume that the temperature remains constant at 100 degrees F. You may assume that the ideal gas law is a reasonable description of the PV behavior of the gas. Answer the following questions:

1.a. What is the inward force required to hold the piston at its original position at x = 15 cm?

1.b. After the compression, what is the density of the gas, reported in SI units of Kg/m^3? The gas in the piston is nitrogen, N_2.

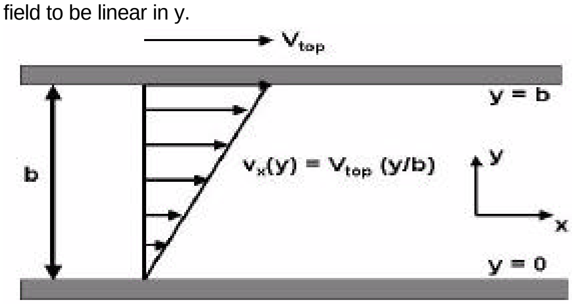
1.c. What is the inward force on the piston shaft required to maintain the piston length at 10 cm? Report your answer in SI units (force unit = 1 Newton).

1.d. What is the net work that must be performed to compress the piston length from 15 cm to 10 cm? Report your answer in SI units (work/energy unit = 1 Joule).

1.e. If we now assume that the outside of the piston is no longer a vacuum, but is rather a gas with a constant pressure of 1 atmosphere and a temperature of 100 degrees F, what is the new value of the work required to change the piston length from 15 cm to 10 cm?

2. Viscosity has the SI units of Pa*s, where 1 Pa (Pascal) = N/m^2, a measure of pressure. Unfortunately, one often encounters values of viscosity reported in different units, in particular the CGS (cm-g-s) system in which the unit of viscosity is defined as 1 Poise = 1 dyne-s/cm^2, a dyne being the CGS unit of force. For example, the viscosity of water is usually reported to be about 1 centipoise = 1 cP = 0.01 P. Perform the unit conversions necessary to calculate the viscosity of water in SI units.

3. In class, we said that if we had a fluid confined between two plates separated by a distance b, with the bottom plate stationary and the top plate moving at a constant velocity V_{top} in the x direction, then we could expect the steady state velocity field to be linear in y.



3.a. For water, using the viscosity calculated in question 2, what is the force per unit area, reported in units of pressure 1 Pa = N/m^2, that must act on the top plate to move it at a velocity of 1 cm/s if the plates are separated by a distance of 10 cm?

As long as we move the top plate very slowly, the flow will be steady and uniform (laminar), but if we move the top plate too fast, we will get an irregular, unsteady, turbulent flow. As discussed in class, the parameter that controls whether we will observe turbulence is the Reynolds number, defined for our experiment as

$$Re = \frac{\rho V_{top} D}{\mu}$$

When the Reynolds number is more than one or two orders of magnitude greater than one, the inertia of the moving fluid is too large for the viscous forces to "smooth" any irregularities in the velocity field, and the flow becomes irregular. In our calculation above, we have assumed that this is not the case.

3.b. Make sure that we are justified in making this approximation by calculating the Reynolds' number for this flow example.

3.c. How fast would we have to move the top plate, in m/s, in order to have a Reynolds' number of 100, where we would have to worry more about turbulence?

4.I have a balloon of volume 200 cm^3, filled with water with a density of 1 g/cm^3. I drop the balloon out my fifth floor window in building 66 that is, say, 60 feet off of the ground, right as a bunch of noisy students from the East Campus dorm are walking underneath.

4.a. What is the momentum of the balloon right before it hits the ground?

4.b. Based on your reasoning, if I have a "chunk" of fluid of density ρ, occupying a volume V, and moving with a velocity vector $\underline{\boldsymbol{v}}$, what is the linear momentum of this fluid "chunk"?

4.c. What is the momentum per unit volume carried by the fluid as it moves? Since mass per unit volume is the mass density, we could call this value the momentum density of the fluid.

Solutions

Solutions

1.a. To hold the piston at its original position, the sum of the forces must equal to zero.

$$\sum F = 0$$

In this case we have a force from the gas, Fg, acting outwards; a force from the surrounding atmosphere, Fa, acting inwards; and the the force pushing the piston inward, Fp. For convention purposes, we define inward forces to be positive and outward forces to be negative.

$$-Fg + Fa + Fp = 0$$

Pressure P is defined as:

$$P = \frac{F}{A}$$

Where F is the force imparted on the cross-sectional area A.

In this case, Fa = 0 since outside the piston is a vacuum.

Rearranging so we can substitute for Fg:

$$Fp = Fg$$

$$Fp = PgA$$

We have Pg and A from the problem statement, just be aware of the units. Generally, it is easiest to work in SI units.

$$Fp = (1\,\text{atm})\left(\frac{1.01 \text{ x } 10^5\, N/m^2}{1 atm}\right)(100\, cm^2)\left(\frac{1 m^2}{10000\, cm^2}\right)$$

$$\mathbf{Fp = 1.01 \text{ x } 10^3 \text{ N}}$$

1.b. Density:

$$r = \frac{\text{mass}}{\text{volume}} = \frac{Nm_w}{V}$$

Since the gas is enclosed, the number of moles of gas does not change after the compression. To find the number of moles, we can use the ideal gas law.

PV = NRT

T = 100^o F = 310.9 K

$$\text{N} = \frac{\text{PV}}{\text{RT}} = \frac{(1atm)\left(\dfrac{1.01x10^5\ N/m^2}{1atm}\right)(100\,cm^2)(15\,cm)\left(\dfrac{1m^3}{100^3 cm^3}\right)}{\left(8.314\dfrac{m^3Pa}{mol\,K}\right)(310.9\,K)}$$

N = 0.0588 moles

Mw of nitrogen is 28 g/mol.

$$r = \frac{Nm_w}{V} = \frac{(0.0588\,moles)(28\,g/mol)\left(\dfrac{1kg}{1000\,g}\right)}{(0.01m^2)(0.1m)}$$

ρ = 1.64 Kg/m^3

1.c. The solution method for this is similar to that of 1.a.

∑ F = 0

-Fg + Fa + Fp = 0

Fa again equals 0.

Fp = PgA

However, in this case we need to calculate the new pressure using the number of moles calculated from 1.b, again we can use the ideal gas law.

$$\text{Pg} = \frac{\text{NRT}}{\text{V}} = \frac{(0.0588\,moles)\left(8.314\dfrac{m^3Pa}{mol\,K}\right)(310.9\,K)}{(100\,cm^2)(10\,cm)\left(\dfrac{1m^3}{100^3\ cm^3}\right)}$$

Pg = 1.52 x 10^5 N

Substitute back to find Fp:

Fp = (1.52 x 10^5 N)(0.01 m^2)

Fp = 1520 N

1.d. Work is defined as:

$$W = \int_{x_1}^{x_2} F_p \, dx = \int_{x_1}^{x_2} (Fg - Fa)\, dx$$

Substituting in for our definition of force:

$$W = \int_{x_1}^{x_2} (PgA - 0)\, dx$$

Now we must note that pressure of the gas is a function of the enclosed volume, which is a function of the piston length. Thus, we must substitute for pressure using the ideal gas law before integrating.

$$W = \int_{x_1}^{x_2} \left(\left(\frac{NRT}{V} \right) A \right) dx = \int_{x_1}^{x_2} \left(\left(\frac{NRT}{Ax} \right) A \right) dx = \int_{x_1}^{x_2} \left(\left(\frac{NRT}{x} \right) \right) dx$$

$$W = N\mathrm{RT} \ln x \Big|_{x_1}^{x_2}$$

W = (0.0588 moles)(8.314 m^3 Pa/mol K)(310.9 K)[ln (0.15m/0.10 m)]

W = 61.6 N m = 61.6 J

1.e. Here we no longer have a vacuum. The atmosphere now contributes a force and Fa does not equal zero.

$$W = \int_{x_1}^{x_2} F_p \, dx = \int_{x_1}^{x_2} (Fg - Fa)\, dx$$

$$W = \int_{x_1}^{x_2} (PgA - PaA)\, dx = \int_{x_1}^{x_2} \left(\frac{NRT}{x} - PaA \right) dx$$

$$W = \left(N\mathrm{RT} \ln x - PaA\,x \right) \Big|_{x_1}^{x_2}$$

W = (0.0588 moles)(8.314 m^3 Pa/mol K)(310.9 K) ln (0.15 m /0.1 m) – (1 atm)
(1.01 x 10^5 Pa/1 atm)(0.01 m^3)(0.15 m – 0.10 m)

W = 11 J

Note that less work is required than the case in 1.d!

2. Here are the necessary unit conversions needed for this problem. Although not every unit conversion is listed in your textbook, it is not difficult to find the ones you need from other sources.

$$\grave{\imath} = (1\,\mathrm{cP}) \left(\frac{0.01\,\mathrm{P}}{1\,\mathrm{cP}} \right) \left(\frac{1\,\mathrm{dynes} \cdot \mathrm{s/cm}^2}{1\,\mathrm{P}} \right) \left(\frac{1\,\mathrm{N}}{10^5\,\mathrm{dynes}} \right) \left(\frac{10000\,\mathrm{cm}^2}{1\,\mathrm{m}^2} \right) \left(\frac{1\,\mathrm{Pa}}{1\,\mathrm{N/m}^2} \right)$$

μ = 0.001 Pa s

3.a. On p. 19 of the class notes from 2/7/02, we have the same situation as described in this problem.

$$\left(\frac{\mathrm{Fx}}{\mathrm{A}}\right) = \mu\left(\frac{\mathrm{Vtop}}{\mathrm{b}}\right) = \frac{(0.001\,\mathrm{Pa\cdot s})(1\,\mathrm{cm/s})\left(\dfrac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right)}{(10\,\mathrm{cm})\left(\dfrac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right)}$$

Fx = 1 x 10^{-4} Pa/unit area

3.b. Plug n' chug.

$$\mathrm{Re} = \frac{\rho \mathrm{VtopD}}{\mu} = \frac{(1\,\mathrm{g/cm}^3)(1\,\mathrm{kg}/1000\,\mathrm{g})(100^3\,\mathrm{cm}^3/1\,\mathrm{m}^3)(1 cm/s)(1\,\mathrm{m}/100\,\mathrm{cm})(10\,\mathrm{cm})(1\,\mathrm{m}/100\,\mathrm{cm})}{0.001\,\mathrm{Pa\cdot s}}$$

Re = 1000

This is more than 2 orders of magnitude greater than one, thus the flow is turbulent and we cannot use the approximation.

3.c. Given the equation for Reynold's number, we can rearrange it for Vtop and solve. Again, make sure the units are correct. It never hurts to do a dimensional analysis.

$$\mathrm{Vtop} = \frac{\mathrm{Re}\,\mu}{\rho \mathrm{D}} = \frac{(100)(0.001\,\mathrm{Pa\cdot s})}{(1 g/cm^3)(1\,\mathrm{kg}/1000\,\mathrm{g})(100^3\,\mathrm{cm}^3/1\,\mathrm{m}^3)(0.1\,\mathrm{m})}$$

Vtop = 0.001 m/s

4.a. Starting with a sketch of the problem:

Momentum is mass times velocity.

Next step is to determine the velocity of the balloon right before it hits the ground. We ignore any air resistance or drag and use the following equation to find the velocity for a constant accelerating body:

$$v^2 = v_o^2 + 2a(x - x_o) = 0 + 2(9.8\,\text{m/s}^2)(60\,\text{ft} - 0\,\text{ft})\left(\frac{1\,\text{m}}{3.28\,\text{ft}}\right)$$

$$v = 18.9\ \text{m/s}$$

We can now solve for momentum.

$M = mv = V\rho v$

$M = (200\ \text{cm}^3)(1\ \text{g/cm}^3)(1\ \text{kg}/1000\ \text{g})(18.9\ \text{m/s})$

M = 3.78 kg m/s

4.b.

$\mathbf{M = V\rho\underline{v}}$

4.c.

$M = V\rho\underline{v} / V$

$\mathbf{M = \rho\underline{v}}$