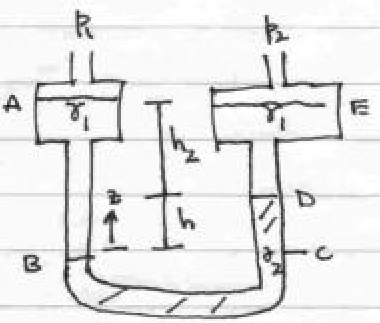
10-301 Problem Set #2 K. Dorthan

2-34



Since $A_e \ll A_r$, we assume that the fluid heights open to the air are at the same level, ie. $h_A = h_E = h + h_2$. Compute the pressure at the points A, B, C, D, E:

A: $p = p_1$

B: $p = p_1 + \gamma_1(h_2 + h)$ from the head of fluid 1

C: $p = p_1 + \gamma_1(h_2 + h)$ since the connecting fluid is homogeneous

D: $p = p_1 + \gamma_1(h_2 + h) - \gamma_2 h$ from the head of fluid 2

E: $p = p_1 + \gamma_1(h_2 + h) - \gamma_2 h - \gamma_1 h_2$ from the head of fluid 1

But point E is also at pressure p_2:

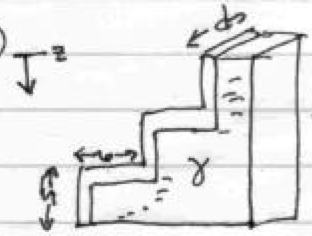
$$p_2 = p_1 + \gamma_1(h_2 + h) - \gamma_2 h - \gamma_1 h_2$$

Rearranging:

$$\boxed{h = \frac{p_1 - p_2}{\gamma_2 - \gamma_1}}$$

Note: h does not "blow up" if the entire fluid is homogeneous ($\gamma_2 = \gamma_1$). Rather, the assumption that the top fluid levels are equal becomes invalid.

2-48)



$d = 3\text{ ft}$

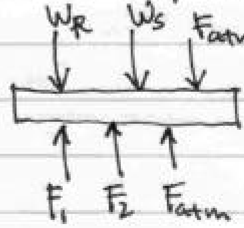
$W = 10\text{ in} = 5/6\text{ ft}$

$h = 8\text{ in} = 2/3\text{ ft}$

$\gamma = 150\ \text{lb/ft}^3$

$W_R = 85\text{ lb}$

Draw a simple free body diagram:



where F_1 = Force from cement under step 1

F_2 = Force from cement under step 2

F_{atm} = Force from the atmosphere

W_R = Weight of the riser

W_S = Weight of the sand

For the steps to be stationary:

$$W_R + W_S + F_{atm} = F_1 + F_2 + F_{atm}$$

$$W_S = F_1 + F_2 - W_R$$

Compute the force on each step from the hydrostatic eqn with z=0 at the open top step

$$F_1 = p_1 A = (2\gamma h)(Wd) \qquad F_2 = (\gamma h)(Wd)$$

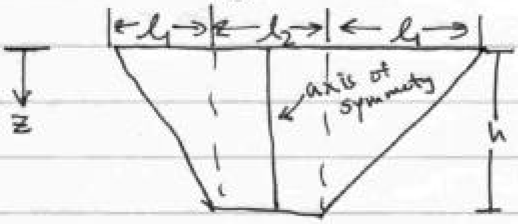
So

$$W_S = 3\gamma h W d - W_R$$

$$W_S = 3\left(\frac{150\text{ lb}}{\text{ft}^3}\right)\left(\frac{2}{3}\text{ ft}\right)\left(\frac{5}{6}\text{ ft}\right)(3\text{ ft}) - 85\text{ lb}$$

$$\boxed{W_S = 665\text{ lb}}$$

2.72) Since we are only concerned with the horizontal force on the dam, we can just compute the pressure force on the projection in (b). The subscript R is the rectangle and T is the triangle.



$h = 715$ ft $\qquad \gamma = 62.428$ lb/ft³

$l_1 = 295$ ft $\qquad$ (for water)

$l_2 = 290$ ft

Note: Atmospheric pressure cancels out on both sides of the dam.

Method 1: Use the centroids to compute the force:

$$c_T = h/3, \qquad c_R = h/2$$

The force on the triangle is the pressure @ the centroid times the area of the triangle:

$$F_T = \underbrace{\left(\frac{\gamma h}{3}\right)}_{\text{pressure at } c_T}\underbrace{\left(\frac{1}{2} l_1 h\right)}_{\text{Area}} = \frac{\gamma l_1 h^2}{6}$$

Likewise for the square rectangle

$$F_R = (\gamma h/2)(l_2 h) = \frac{\gamma l_2 h}{2}$$

The total force acting on the dam is then

$$F = 2F_T + F_R = \gamma h^2\left(\frac{l_1}{3} + \frac{l_2}{2}\right)$$

$$F = (62.428)(715)^2\left(295/3 + 290/2\right)$$

$$\boxed{F = 7.7 \times 10^9 \text{ lb}}$$

The point of action will be along the axis of symmetry. To compute the height on this axis, weight the centroids by the force on each component:

$$y_c = \frac{2C_T F_T + C_R F_R}{F_T + F_R}$$

$$y_c = h\left[\frac{4l_1 + 9l_2}{12l_1 + 18l_2}\right]$$

$$y_c = 0.432h$$

To get the answer in the book, we need to put the origin at the base so

$$\boxed{y_c = (1-0.432)h = 406 \text{ ft from the base.}}$$

Method 2: Instead of using the centroids, the force on a part of the dam can be computed by integrating the pressure over the area:

$$F_i = \int_0^h \underbrace{w_i(y)}_{\text{width of the section as a function of } y}\overbrace{[\gamma y]}^{\text{pressure}}\, dy$$

For the triangle,

$$w_T = l_1(1 - y/h)$$

$$F_T = \int_0^h l_1\left(1-\frac{y}{h}\right)\gamma y\, dy = \frac{l_1 \gamma h^2}{6}$$

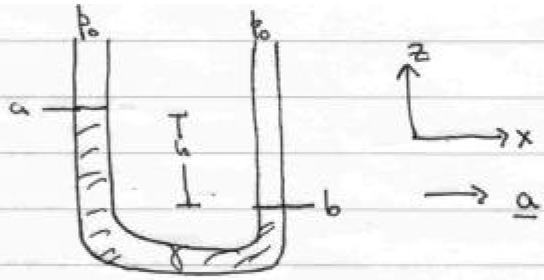
For the ~~square~~, rectangle

$$w_R = l_2$$

$$F_R = \int_0^h l_2 \gamma y\, dy = \frac{l_2 \gamma h^2}{2}$$

which of course agree with the results of the centroid method.

2.97)



$\underline{a} = a\hat{i}$

From the hydrostatic eqn

$$-\nabla p = \gamma \hat{k} + \rho a \hat{i}$$

we have the components

$$\frac{\partial p}{\partial z} = -\gamma \quad \text{and} \quad \frac{\partial p}{\partial x} = \rho a$$

Integrate both eqns

$$p(x,z) = -\gamma z + f(x) \qquad p(x,z) = -\rho a x + g(z)$$

These are satisfied by the choice

$$f(x) = -\rho a x + C \qquad g(z) = -\gamma z + C$$

So the pressure is

$$p(x,z) = -\gamma z - \rho a x + C$$

Since both a and b are open to the atmosphere

$$p_a = p_b$$

$$-\gamma z_a - \rho a x_a + C = -\gamma z_b - \rho a x_b + C$$

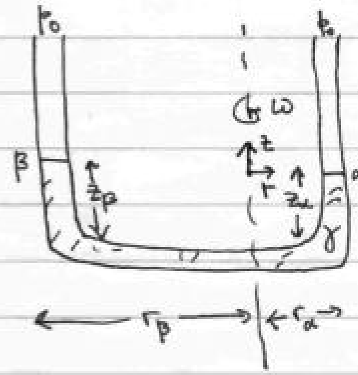
$$\gamma (z_a - z_b) + \rho a (x_a - x_b) = 0$$

$$\gamma h + \rho a(-\ell) = 0$$

So the relationship between a, ℓ, and h is

$$\boxed{\gamma = \frac{\rho a \ell}{h}} \quad \text{or} \quad \boxed{g = \frac{a\ell}{h}}$$

2-100)



We are given $r_\alpha = 90\text{mm}$; $r_\beta = 220\text{mm}$ and $|z_\beta - z_\alpha| = 75\text{ mm}$.

From eqn (2-33) in Munson et. al,

$$p(r,z) = \frac{\rho\omega^2 r^2}{2} - \gamma z + C$$

Since the position α and β are open to the air:

$$p_\alpha = p_0$$

Using the expression for the pressure

$$\frac{\rho\omega^2}{2}(r_\alpha^2 - r_\beta^2) - \gamma(z_\alpha - z_\beta) = 0$$

$$\omega = \frac{2\gamma(z_\alpha - z_\beta)}{\rho(r_\alpha^2 - r_\beta^2)}$$

Since $r_\alpha^2 - r_\beta^2 < 0$, then we know $z_\alpha - z_\beta < 0$ and the point β is higher than the point α. Noting that $\gamma = \rho g$

$$\omega = \frac{2g(z_\alpha - z_\beta)}{r_\alpha^2 - r_\beta^2}$$

$$\omega = \frac{2(9.8\,\text{m/s}^2)(-75\,\text{mm})(1\,\text{m}/1000\,\text{mm})}{\{(90\,\text{mm})^2 - (220\,\text{mm})^2\}(1\,\text{m}/1000\,\text{mm})^2}$$

$$\boxed{\omega = 6.03\text{ rad}}$$