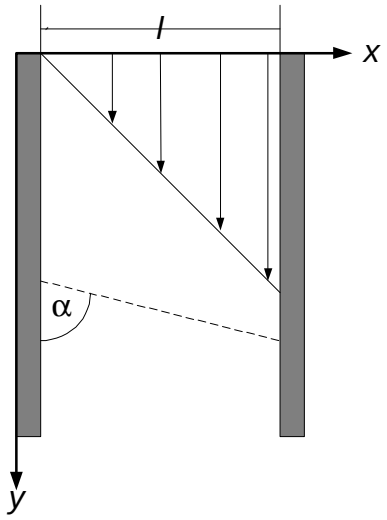
Practice Problems for Exam 2
Kevin D. Dorfman

1. Consider the simple shear flow between two plates separated by a distance l in the x-direction.



The velocity profile for this flow is given by

$$\mathbf{v} = v_0 \frac{x}{l} \mathbf{i}_y$$

where v_0 is a characteristic velocity.

(a) Is this flow incompressible?

(b) What is the volumetric flow rate across the dashed surface oriented at an angle a with respect to the y-axis? Consider the case $0 < a < 90°$.

(c) Compute the deviatoric stress in the fluid from the constitutive relationship

$$t = 2h\left[\frac{1}{2}\left[\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right] - \frac{1}{3}\mathbf{I}(\nabla \bullet \mathbf{v})\right].$$

(d) What is the force exerted by this stress on the dashed surface?

2. For a small sinusoidal traveling wave of amplitude A and wavelength k ($Ak << 1$), the velocity profile in the wave is given by

$$\mathbf{v} = u\mathbf{i}_x + v\mathbf{i}_y$$

with the scalar velocity components

$$u = Aw\exp(ky)\cos(kx - wt)$$
$$v = Aw\exp(ky)\sin(kx - wt)$$

where ω is the frequency of the traveling wave.

(a) What is the acceleration of the fluid?
(b) What are the streamlines at $t = 0$ that pass through (i) (0,0) and (ii) (0,-2π/k)? (This question was posed by Professor Ken Smith in 10.52.)