10.301 Homework 4 (03/12/02)

Stresses

Due: Monday, March 18, 2002.

1. Measurements of force were made on $1\ \text{mm}^2$ test surfaces around a point (x_1, x_2, x_3) in a fluid. The unit vectors normal to these test surfaces (pointing from the "minus" to the "plus" sides of the respective surfaces) correspond to the coordinate directions $(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3)$. The measured force vectors on these surfaces were found to be as follows:

$f_1 = 1N$ in the $\hat{\mathbf{x}}_1$ direction;

$f_2 = 2N$ in the $-\hat{\mathbf{x}}_3$ direction;

$f_3 = 2N$ in the $-\hat{\mathbf{x}}_2$ direction.

(a) What is the stress tensor at the point (x_1, x_2, x_3)?

(b) What is the net force on $1\ \text{mm}^2$ surface whose normal lies in the direction $\hat{\mathbf{x}}_1 + \hat{\mathbf{x}}_2$ and which passes through the point (x_1, x_2, x_3)?

(c) What is the component of this force normal to the surface?

(d) Calculate the directions of the principal axes of stress and the principal stresses at the point.

2. (a) Determine the principal stresses for the stress tensor with rectangular Cartesian components given by the matrix below. Also, determine the directions (relative to the original Cartesian system) of the principal directions (i.e., axes) of stress. State whether the principal stresses are tensile or compressive.

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(b) Calculate the *eigenvalues and eigenvectors* of this (symmetric) matrix and confirm that these eigenvalues and eigenvectors are respectively identical to the principal stresses and principal directions calculated above in part (a). Explain why this is so!