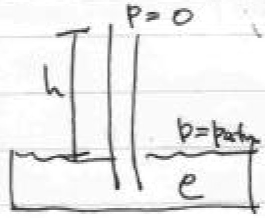
Problems for Exam 1 Kevin Dorfman

1)



If the top pressure is at a vacuum, then the hydrostatics says

$$p_{atm} - \cancel{p_{top}}^{0} = \rho g h$$

$$h = \frac{p_{atm}}{\rho g} = \frac{10^5\ N/m^2}{\rho (kg/m^3) \cdot 9.8\, m/s^2} = \frac{10204}{\rho}\ m$$

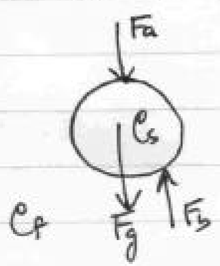
a) water: $\rho = 999\ kg/m^3$;

$h = 10.2m \approx 33.4\ ft$

b) Hg $\rho = 13800\ kg/m^3$

$h = 0.75m = 750\ mm$

2) Force on a sphere of radius a



$F_{net} = F_a + F_g - F_b$

weight $F_g = \rho_s g V_s$; $V_s = \frac{4}{3}\pi a^3$

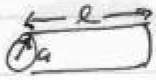
$F_b = \rho_f g V_s$

Since the ~~fluid~~ particle is stationary

$F_{net} = 0 = F_a + \rho_s g V_s - \rho_f g V_s$

$F_a = (\rho_f - \rho_s) g V_s$

For the cylinder



$F_{net} = F_a + F_g - F_b = 0$

$F_g = \rho_s g V_c$; $V_c = \pi a^2 l$

$F_b = \rho_f g V_c$

$$0 = (\rho_f - \rho_s) g V_s + \rho_s g V_c - \rho_f g V_s$$

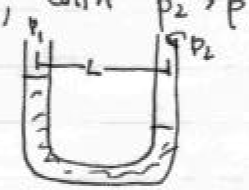
So $V_s = V_c$

$$\frac{4}{3}\pi a^3 = \pi a^2 \ell$$

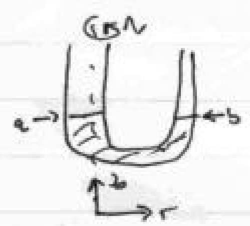
$$\boxed{\ell = \frac{4}{3} a}$$

The volumes are the same because the buoyant force only depends upon the volume of displaced fluid.

3) Initially, with $p_2 > p_1$



To get the fluid heights equal, rotate around the left leg



$$\rho \underline{a} = -\nabla p + \rho \underline{g}$$

$$\underline{a} = -\rho \Omega^2 r \, \underline{i}_r \qquad \underline{g} = -\underline{i}_z g$$

$$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} dz$$

$$dp = \rho \Omega^2 r dr - \rho g dz$$

Between pts. a and b

$$\int_a^b dp = p_b - p_a = \int_{r_a}^{r_b} \rho \Omega^2 r \, dr + \int_{z_a}^{z_b} -\rho g dz \quad (\to 0)$$

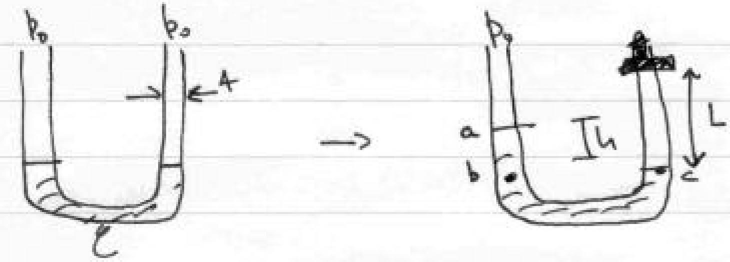
$$p_b - p_a = \rho \Omega^2 L^2/2$$

But we know p_a and p_b

$$p_2 - p_1 = \rho \Omega^2 L^2/2$$

$$\Omega = [2 (p_2 - p_1)/\rho L^2]^{1/2}$$

4)



initially

with time

As pointed out by the Cambridge group in the review session, a better way to pose this question is to say that the distance L between the piston and the fluid is decreasing linearly in time.

From the hydrostatics

$$dp = -\rho g \, dz$$

So we have

$$p_a = p_0$$

$$p_b = p_a + \rho g h = p_0 + \rho g h$$

$$p_c = p_b = p_0 + \rho g h.$$

To determine how p_c changes with time, we use Boyle's law

$$p_c(t) V(t) = p_0 V_0 = p_0 A L$$

The change in volume with time is

$$V(t) = A L(t)$$

$$\frac{dV}{dt} = A \frac{dL}{dt} = -A \frac{dz}{dt} = \text{const}$$

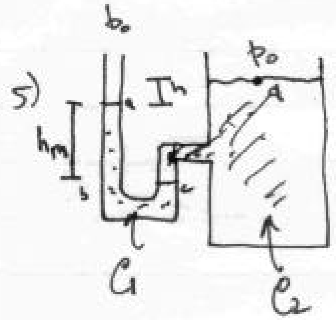
Integrate and use the initial $V(t=0) = V_0$

$$V(t) = V_0 - A \, {}^{dz}/_{dt} \, t$$

So $$p_c(t) = \frac{p_0 V_0}{V_0 - A(dz/dt)t} = p_0 + \rho g h$$

$$h = \frac{p_0}{\rho g}\left[V_0/[V_0 - A(dz/dt)t] - 1\right]$$

5)



from hydrostatics

$$dp = -\rho g\, dz$$

$$p_a = p_0$$
$$p_b = p_a + \rho_1 g h_m$$
$$p_c = p_b \quad \text{(constant x-plane)}$$
$$p_d = p_c - \rho_2 g (h_m + h) = p_0$$
$$p_0 + \rho_1 g h_m - \rho_2 g (h_m + h) = p_0$$

$$\rho_2 = \rho_1 \left(\frac{h_m}{h_m + h} \right)$$