10.302 Fall 1999
Exam 1, Wednesday, October 6, 1999
Solutions

TurboChef Oven

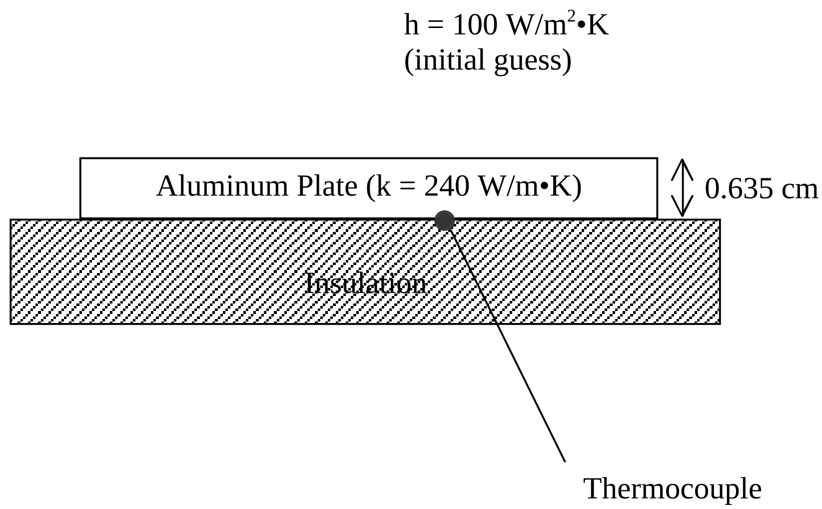
1 [25 points] You are using a heat transfer sensor to measure the average heat transfer coefficient to a horizontal surface in the TurboChef oven (Figure 1). It consists of an aluminum plate (diameter 15 cm, thickness 0.635 cm), which sits on a layer of insulation. Assume the properties of aluminum are $k = 240$ W/m•K, $\rho = 2700$ kg/m^3, and $c = 900$ J/kg•K. A thermocouple is taped to the bottom of the plate. Assume that the plate is initially at 25˚C. It is placed in the oven and, at $t = 0$, a fan that blows hot air (275ºC) at the plate is turned on. After 100 seconds, the thermocouple reads 160˚C. From data in the literature, you guess that the order of magnitude of h is about 100 W/m^2•K.

a) What criterion do you use to determine the method for analyzing the transient behavior of the sensor?

b) Estimate the average heat transfer coefficient to the plate during the time of the measurement.

c) Does the value of h you calculated satisfy the criterion in part (a)?

Figure 1. Heat Transfer Sensor



10.302 Fall 1999

Exam I Solutions

Problem 1: [25 points]

a) [5 points]

$$\text{Biot \#} = \frac{hL}{k} = \frac{100 \times 0.635 \times 10^{-2}}{240} = 26 \times 10^{-3} \ll 1 \ (\text{or} < 0.1)$$

The criteria is:

If $Bi\# \ll 1$ (or < 0.1), the temperature over the thickness of the plate is approximately uniform, we can use lump capacity model to analyze the transient behavior of the sensor.

NOTE:

You were asked to measure the average heat transfer coefficient to a horizontal surface. Therefore, L should be the thickness of the surface rather than diameter/radius of the plate. [#5 points]

b) Energy balance

[15 points]

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$\dot{E}_{in} = hA_s(T_\infty - T)$$

$$\dot{E}_{out} = 0$$

$$\dot{E}_{st} = \rho C_p V \frac{dT}{dt}$$

$$\rho C_p V \frac{dT}{dt} + hA_s(T - T_\infty) = 0$$ [# 3 points]

Converting variable

$$\theta \equiv T - T_\infty$$

$$\Rightarrow \quad \frac{d\theta}{dt} = -\frac{hA_s}{\rho C_p V}\theta = -\frac{h}{\rho C_p L}\theta$$

$$\ln\frac{\theta}{\theta_i} = -\frac{hA_s}{\rho C_p V}t$$

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{h}{\rho C_p L}\cdot t\right)$$

$$\theta = T - T_\infty = 160 - 275\,^\circ C = -115\,^\circ C$$

$$\theta_i = T_i - T_\infty = 25 - 275\,^\circ C = -250\,^\circ C$$ [# 7 points]

$$\Rightarrow \quad h = \frac{-\ln\frac{\theta}{\theta_i}\cdot \rho C_p L}{t} = \frac{-\ln\left(\frac{-115}{-250}\right)\times 2700\times 900\times 0.635\times 10^{-2}}{100}$$

$$= 120 \ W/m^2\cdot K$$ [# 5 points]

c) [5 points]

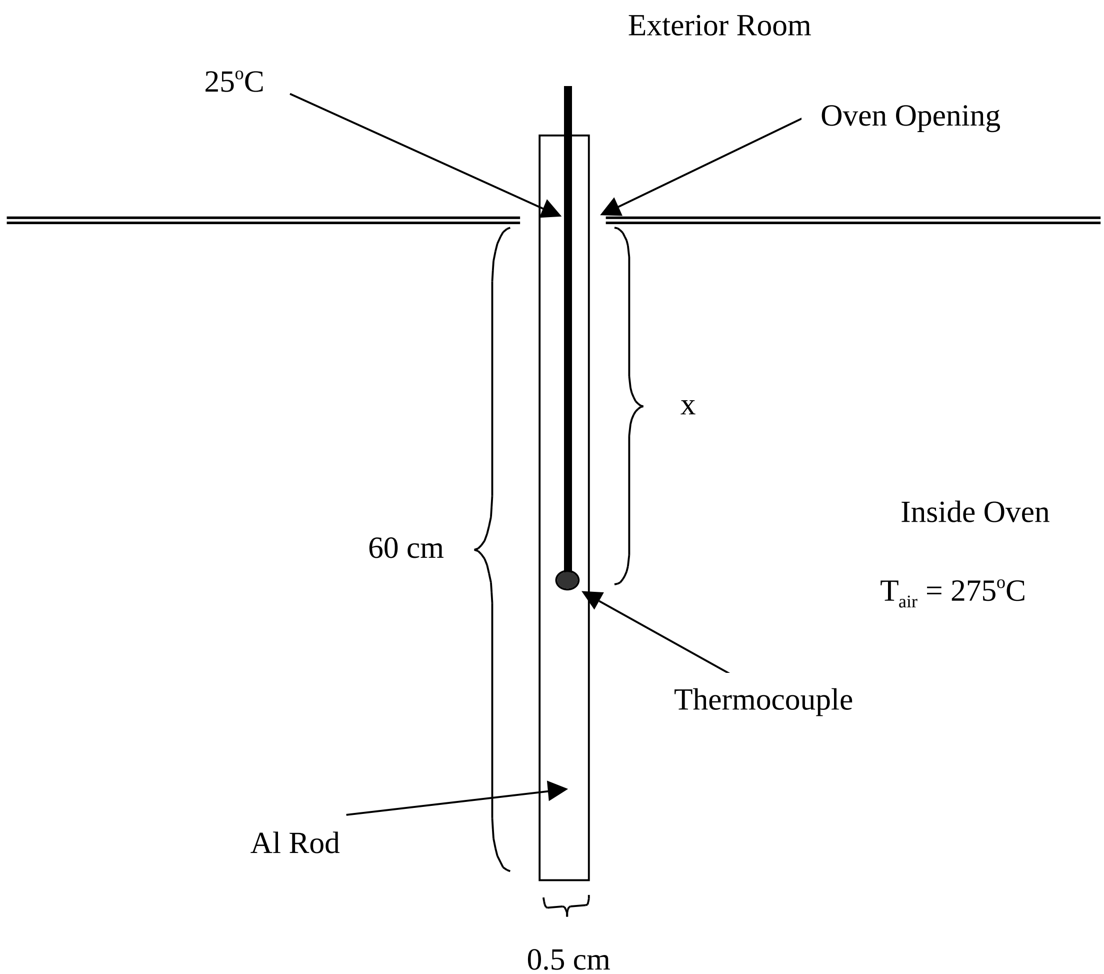
Check the criteria

$$\text{Biot \#} = \frac{hL}{k} = \frac{120\times 0.635\times 10^{-2}}{240} = 3.18\times 10^{-3} \ll 1 \ (\text{or} < 0.1)$$

– satisfy!

2 [25 points] An aluminum rod, 0.5 cm in diameter, passes through an opening in the oven from an exterior room that is at 25˚C. The rod protrudes 60 cm from the oven wall. The temperature of the rod at the entrance to the oven is 25ºC. A thermocouple is imbedded at some point in the rod. What is the minimum distance from the opening for placing the thermocouple (the value of x in Figure 2) so that it reads to within 2.5˚C of the oven air temperature (275ºC) during your experiment to measure the heat transfer coefficient? Assume that steady state conditions prevail in the rod, and that the heat transfer coefficient and physical properties are the same as specified in problem 1.

Figure 2. Temperature Measurement in Oven



Problem 2: [25 points]

- steady-state (known)
- Assume one-dimensional fin approximation
  - check the Biot # in radial direction

$$Bi_{radial} = \frac{h \cdot \frac{r}{2}}{k} = \frac{100 \times \frac{5\times10^{-3}}{2}}{240} = 0.001 \ll 1 \; (\text{or } < 0.1)$$

(valid!)

[# 2 points]

- Assume infinite fin
  - check the value of mL

$$m = \sqrt{\frac{hP}{kA_c}} \qquad P = 2\pi r \qquad A_c = \pi r^2$$

$$\therefore \; m = \sqrt{\frac{100 \times 2\pi r}{240 \times \pi r^2}} = \sqrt{\frac{100\times 2}{240 \times 2.5\times10^{-3}}} = 18.26 \quad \text{[\# 3 points]}$$

$$mL = m \times 0.6 = 18.26 \times 0.6 = 10.95 > 2.65$$

(valid!) [# 2 points]

Heat Diffusion Equation:

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

$$T(0) = T_b = 25^\circ C$$

$$T(\infty) = 275^\circ C$$

Variable transfer

$$\theta(x) \equiv T(x) - T(\infty)$$

$$\Rightarrow \quad \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(0) = T_b - T_\infty = 25^\circ C - 275^\circ C = -250^\circ C$$

$$\theta(\infty) = T_\infty - T_\infty = 0$$

[# 5 points]

$$\Rightarrow \quad \theta = \theta_0 \cdot e^{-mx}$$ [# 5 points]

when $\theta(x) = T(x) - T_\infty = (275 - 2.5) - 275^\circ C = -2.5^\circ C$

$$\frac{\theta(x)}{\theta(0)} = \frac{-2.5}{-250} = 0.01$$ [# 3 points]

$$\Rightarrow \quad x = \frac{-\ln\frac{\theta(x)}{\theta(0)}}{m} = \frac{-\ln 0.01}{18.26} = 0.252\,m$$ [# 5 points]

Note :

You can also choose other fin models with other boundary conditions. Whatever model you choose, you need to state the ~~boundary~~ tip boundary condition that you consider resonable and justify it.

In fact, in this problem you can use any one of the fin models in Table 3.4 (I&D) and get the same answer.

3 [50 points] The TurboChef oven, which we previously discussed, cooks food rapidly by delivering a high level of power to the food through use of several heat transfer modes. As indicated in Figure 3, heated air enters from the top, flows around the food, and exits from the bottom. The hot air also heats the oven walls to the same air temperature. Microwave energy is emitted from the bottom of the oven. Some of it reflects off surfaces and enters the food from the top. The microwave energy is absorbed by the food and is converted into heat. This process may be thought of as heat generation in the food. You plan to use the oven to cook a Porterhouse steak (assumed to be one-dimensional).

a) Write the appropriate form of the heat diffusion equation and appropriate boundary conditions that describe cooking of a steak when all modes of heat transfer are operating. For the steak, the local volumetric heat generation rate $\dot{q}$ (W/m^3) resulting from microwave radiation can be approximated by

$$\dot{q} = \dot{q}_{01}\, e^{-(H-x)/\delta} + \dot{q}_{02}\, e^{-x/\delta}$$

where the two terms represent, respectively, the generation rate from microwave radiation entering from the top and bottom of the steak. H is thickness of the steak, and x is distance from the bottom. Ignore conduction from the screen, vaporization of water, chemical reactions, and thermal radiation from the oven walls. Assume the heat transfer coefficient is the same on both sides of the steak.

b) You cook the steak with convection only (no microwaves). The heat transfer coefficient is h =150 W/m^2•K, assumed to be the same on both sides. The steak is 3.5 cm thick (with length and width of 20 cm and 10 cm), and its initial temperature is 25^{o}C. Its properties are ρ = 1200 kg/m^3, c = 4000 J/kg•K, and k = 1.2 W/m•K. You cook the steak for 100 s with an air temperature of 275˚C.

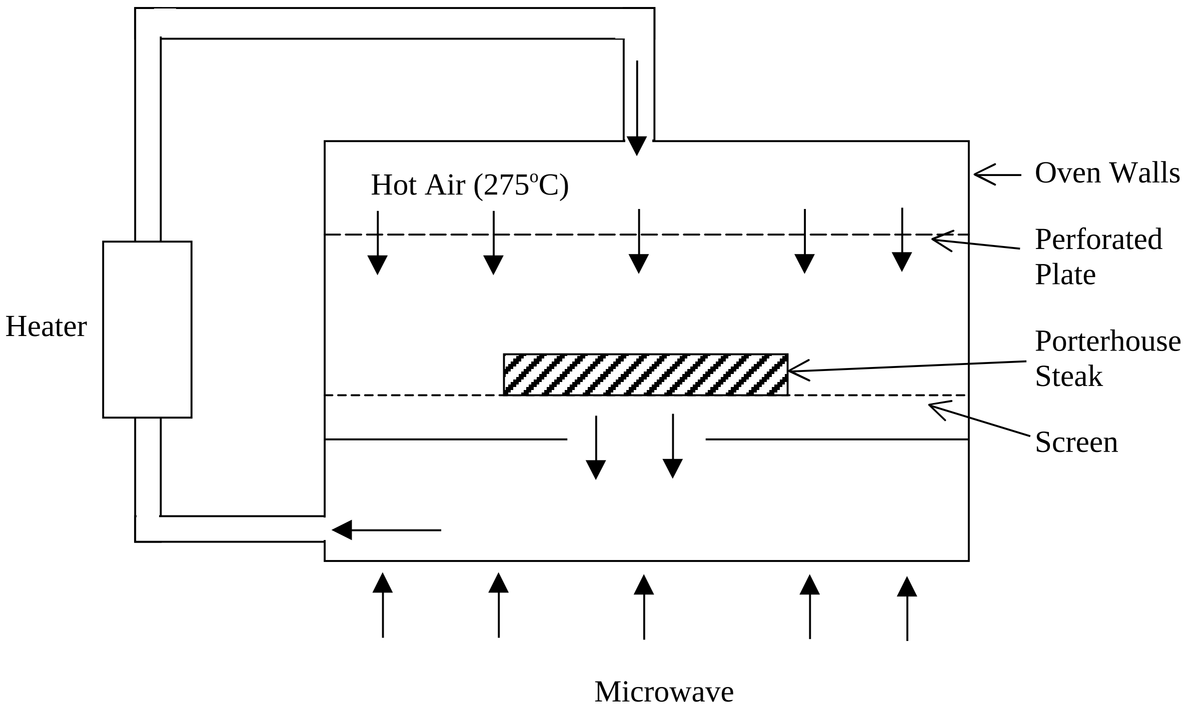
   What would you expect to be the temperature at the midplane of the steak?

c) The Department of Agriculture recommends cooking a “rare” steak to a minimum center temperature of 60˚C. How long should you cook the steak to achieve that condition?

d) Assume that you cook the steak by adding microwave radiation to the convective heating described in part b. The microwave energy absorbed adds a total heat generation rate of 1500 W to the steak. If all of that energy were uniformly distributed throughout the steak, and there were no other changes, how much would the temperature of the steak increase in 100 seconds of cooking time as a result of this heat generation rate?

e) Your friend suggests cooking your steak until steady state is reached because the oven cooks so rapidly. Is that a good idea? Why?

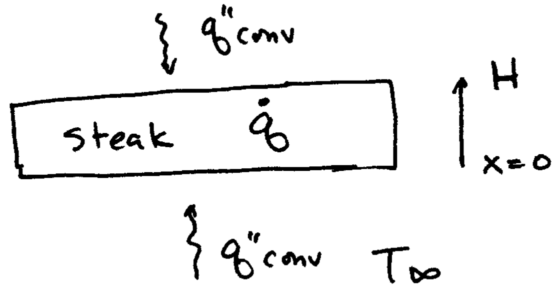
Figure 3. TurboChef Oven



10.302 Fall 99
Exam 1 Solutions

3 [50 points]

$\frac{10}{10}$ a) 1-D in x



Heat Diffusion Equation:

(+4) $$k\frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

where $\dot{q} = \dot{q}_{01} e^{-(H-x)/\delta} + \dot{q}_{02} e^{-x/\delta}$

2 Boundary Conditions:

(+2) @ $x=0$ $\quad -k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h(T_\infty - T(0,t))$

(+2) @ $x=H$ $\quad -k\left.\frac{\partial T}{\partial x}\right|_{x=H} = h(T(H,t) - T_\infty)$

Initial Condition:

(+2) @ $t=0$ $\quad T(x,0) = T_i$

note: since heat generation in the steak is not uniform, can not apply symmetry condition at center of steak (e.g. H/2)

③ continued

$\frac{10}{10}$ b) $Bi = \frac{hL}{k}$ $(L = H/2)$

(+2) $= \frac{(150\ W/m^2K)\left(\frac{0.035m}{2}\right)}{1.2\ W/mK} = 2.1875$

$\Rightarrow$ spatial effects important $(Bi \sim 1)$

$$\alpha = k/\rho c_p = \frac{1.2 W/mK}{(1200\ kg/m^3)(4000\ J/kgK)} = 2.5\times10^{-7}\ \frac{m^2}{s}$$

$$Fo = \frac{\alpha t}{L^2} = \frac{(2.5\times10^{-7}\ m^2/s)(100s)}{(0.035/2m)^2} = 8.16\times10^{-2}$$

(+2) $\Rightarrow$ penetration near surface $(Fo < 0.1)$

(+4) Approximate Semi-Infinite Slab

$\therefore$ mid-plane temperature should be near T_i

(+2) so $\boxed{T_{mid} \cong 25°C}$

or can calculate using analytical solution with surface convection boundary condition (I+D 5.60 or Figure 5.8)

$X = 1.75$ cm

$$\eta = \frac{X}{2\sqrt{\alpha t}} = \frac{0.0175m}{2\sqrt{2.5\times10^{-7}\frac{m^2}{s}\cdot 100s}} = 1.75$$

$erf(\eta) \cong 1$ from plot in handout

$$\frac{h\sqrt{\alpha t}}{k} = \frac{100\ W/m^2K\sqrt{2.5\times10^{-7}\frac{m^2}{s}\cdot100s}}{1.2\ W/mK} = 0.4167$$

③ continued

from I+D 5.60

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}(\eta) - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right] \cdot \left[\operatorname{erfc}\left(\eta + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$\frac{hx}{k} = \frac{(100\ W/m^2K)(0.0175m)}{1.2\ W/mK} = 1.4583$$

$$\frac{T(\frac{H}{2},100s)-25°C}{275°C-25°C} = (\cancel{1}\ 1)_{0} - [\exp(1.4583 + 0.1736)] \cdot \underbrace{[\operatorname{erfc}(1.75 + 0.4167)]}_{\sim 0}$$

$$\cong 0$$

$$\underline{T(\tfrac{H}{2},100s) = T_{mid} \cong 25°C}$$

c) with mid plane (center) $T = 60°C$,

$\frac{10}{10}$ $Fo > 0.2$, check after

Since $Bi = 2.1875$,

use plot of dimensionless center T in Appendix D

(+4) of I+D for plane wall (Figure D1) or use similar one from handout or use 1st term approximation

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{60°C - 275°C}{25°C - 275°C} = 0.86$$

(+4) for $Bi = 2.1875$, find that $Fo \cong 0.3$ from handout

$Bi^{-1} = 0.457$

(3) continued

$$Fo = \frac{\alpha t}{(H/2)^2} = 0.3$$

(+2) $$t = \frac{0.3\,(0.0175\,m)^2}{(2.5\times10^{-7}\,m^2/s)} = \boxed{367.5\,s \text{ or } 6\text{ min}}$$

10/10 d) volumetric energy balance around steak from microwave contribution.

(+5) $$\dot{q}V = \rho C_p V_{steak}\frac{dT}{dt} \quad [\text{energy in} = \text{storage}]$$

$\dot{q}V$ ⇓ 1500W

$$1500\,W = (1200\,kg/m^3)(4000\,J/kg\,K)(0.035m\cdot0.1m\cdot0.2m)\frac{dT}{dt}$$

(+5) integrate from $t=0$ to $100s$

$\boxed{\Delta T = 44.6°C}$ from microwave

e) this is not a good idea. At steady-state with only convection, $T_{steak} \Rightarrow T_{oven}$ (275°C)!

10/10

not necessary: Steady-state occurs near $Fo \sim 1.5$

Given convection alone, using plot of θ_o^* as in part b) with $Fo = 1.5$ and $Bi = 2.19$

$\rightarrow \theta_o^* = 0.2$ or $T_o = 225°C$!

The surface of the steak would be even higher.

With microwave as in part d), temperatures would be 45°C higher. The steak would be extremely dry if not burnt to a crisp.