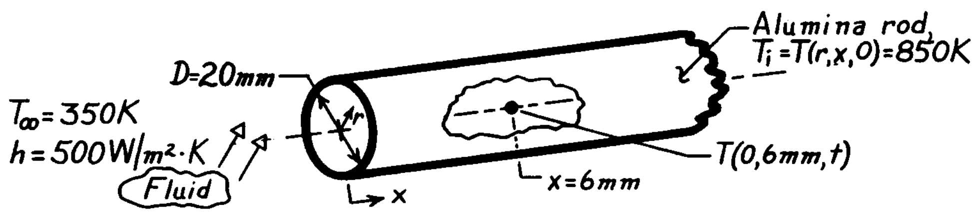
PROBLEM 5.77

KNOWN: A long alumina rod, initially at a uniform temperature of 850K, is suddenly exposed to a cooler fluid.

FIND: Temperature of the rod after 30s, at an exposed end, T(0,0,t), and at an axial distance 6mm from the end, T(0, 6mm, t).

SCHEMATIC:



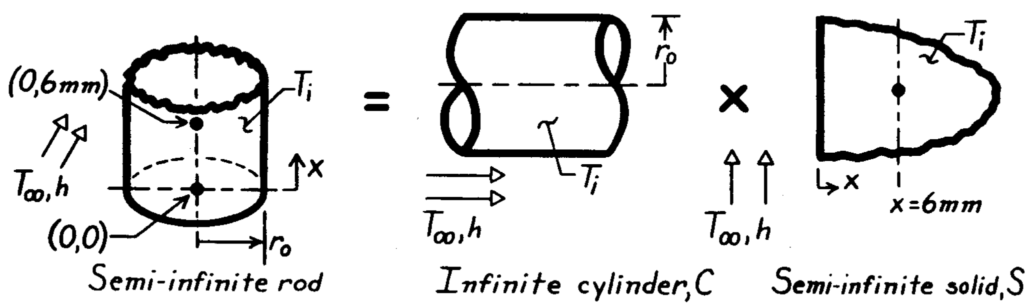
ASSUMPTIONS: (1) Two-dimensional conduction in (r,x) directions, (2) Constant properties, (3) Convection coefficient is same on end and cylindrical surfaces.

PROPERTIES: *Table A-2,* Alumina, polycrystalline aluminum oxide (assume $\bar{T} \approx (850+600)K/2 = 725K$): $\rho = 3970\ kg/m^3$, $c = 1154\ J/kg \cdot K$, $k = 12.4\ W/m \cdot K$.

ANALYSIS: First, check if system behaves as a lumped capacitance. Find

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{500\ W/m \cdot K\ (0.010m/2)}{12.4\ W/m \cdot K} = 0.202\ .$$

Since Bi > 0.1, rod does not behave as spacewise isothermal object. Hence, treat rod as a semi-infinite cylinder, the multi-dimensional system Case (f), Fig. 5.11.



The product solution can be written as

$$\theta^*(r,x,t) = \frac{\theta(r,x,t)}{\theta_i} = \frac{\theta(r,t)}{\theta_i} \times \frac{\theta(x,t)}{\theta_i} = C(r^*,t^*) \times S(x^*,t^*)$$

Infinite cylinder, $C(r^*,t^*)$. Using the Heisler charts with $r^* = r = 0$ and

$$Bi^{-1} = \left[\frac{h\,r_o}{k}\right]^{-1} = \left[\frac{500\ W/m^2 \cdot K \times 0.01m}{12.4\ W/m \cdot K}\right]^{-1} = 2.48\ .$$

Evaluate $\alpha = k/\rho c = 2.71 \times 10^{-6} m^2/s$, find $Fo = \alpha t/r_o^2 = 2.71 \times 10^{-6} m^2/s \times 30s/(0.01m)^2 = 0.812$. From the Heisler chart, Fig. C.4, with $Bi^{-1} = 2.48$ and $Fo = 0.812$, read $C(0,t^*) = \theta(0,t)/\theta_i = 0.61$.

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Semi-infinite medium, $S(x^*,t^*)$. Recognize this as Case (3), Fig. 5.7. From Eq. 5.60, note that the LHS needs to be transformed as follows,

$$\frac{T-T_i}{T_\infty-T_i} = 1 - \frac{T-T_\infty}{T_i-T_\infty} \qquad S(x,t) = \frac{T-T_\infty}{T_i-T_\infty}\ .$$

Thus,

$$S(x,t) = 1 - \left\{\operatorname{erfc}\left[\frac{x}{2(\alpha t)^{1/2}}\right] - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}} + \frac{h(\alpha t)^{1/2}}{k}\right)\right]\right\}.$$

Evaluating this expression at the surface (x=0) and 6mm from the exposed end, find

$$S(0,30s) = 1 - \left\{\operatorname{erfc}(0) - \left[\exp\left(0 + \frac{(500\ W/m^2\cdot K)^2\ 2.71\times10^{-6}\ m^2/s \times 30s}{(12.4\ W/m\cdot K)^2}\right)\right]\right.$$

$$\left.\left[\operatorname{erfc}\left(0 + \frac{500\ W/m^2\cdot K\ (2.71\times10^{-6}\ m^2/s \times 30s)^{1/2}}{12.4\ W/m\cdot K}\right)\right]\right\}$$

$$S(0,30s) = 1 - \left\{1 - [\exp(0.1322)]\ [\operatorname{erfc}(0.3636)]\right\} = 0.693\ .$$

Note that Table B.2 was used to evaluate the complementary error function, erfc(w).

$$S(6mm,30s) = 1 - \left\{\operatorname{erfc}\left[\frac{0.006m}{2(2.71\times10^{-6}\ m^2/s\times30s)^{1/2}}\right] - \right.$$

$$\left.\left[\exp\left(\frac{500W/m^2\cdot K\times0.006m}{12.4\ W/m\cdot K} + 0.1322\right)\right]\ [\operatorname{erfc}(0.3327 + 0.3636)]\right\} = 0.835.$$

The product solution can now be evaluated for each location. At (0,0),

$$\theta^*(0,0,t) = \frac{T(0,0,30s)-T_\infty}{T_i-T_\infty} = C(0,t^*) \times S(0,t^*) = 0.61 \times 0.693 = 0.423\ .$$

Hence,

$$T(0,0,30s) = T_\infty + 0.423(T_i-T_\infty) = 350K + 0.423(850-350)K = 561K\ . \qquad \triangleleft$$

At (0, 6mm),

$$\theta^*(0,6mm,t) = C(0,t^*) \times S(6mm,t^*) = 0.61 \times 0.835 = 0.509$$

$$T(0,6mm,30s) = 604K. \qquad \triangleleft$$

COMMENTS: Note that the temperature at which the properties were evaluated was a good estimate.