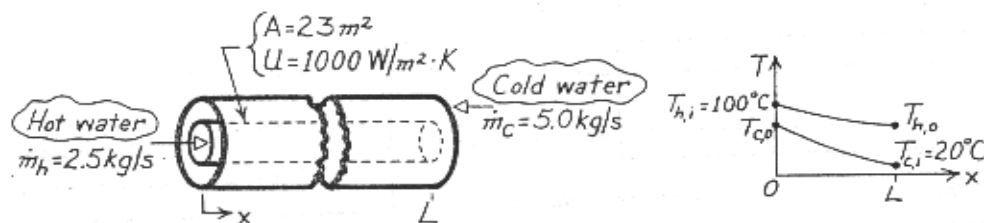


PROBLEM 11.53

KNOWN: Concentric tube, counter-flow heat exchanger.

FIND: Total heat transfer rate and outlet temperatures of both fluids.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: Table A-6, Water ($\bar{T}_h = 68^\circ\text{C} = 340\text{K}$): $c_h = 4188 \text{ J/kg} \cdot \text{K}$; Table A-6, Water ($\bar{T}_c = 37^\circ\text{C} = 310\text{K}$): $c_c = 4178 \text{ J/kg} \cdot \text{K}$.

ANALYSIS: Using the ϵ -NTU method, begin by evaluating the capacity rates.

$$\left. \begin{aligned} c_h &= \dot{m}_h C_h = 2.5 \text{ kg/s} \times 4188 \text{ J/kg} \cdot \text{K} = 10,470 \text{ W/K} \\ c_c &= \dot{m}_c C_c = 5.0 \text{ kg/s} \times 4178 \text{ J/kg} \cdot \text{K} = 20,890 \text{ W/K} \end{aligned} \right\} \begin{aligned} C_{\min} &= C_h \\ C_{\min}/C_{\max} &= 0.50 \end{aligned}$$

From the definition, Eq. 11.25,

$$\text{NTU} = UA/C_{\min} = 1000 \text{ W/m}^2 \cdot \text{K} \times 23 \text{ m}^2 / (10,470 \text{ W/K}) = 2.20.$$

Using values of NTU and $C_{\min}/C_{\max}$, find from Fig. 11.15, that

$$\epsilon = 0.80.$$

From the definition of ϵ , Eq. 11.20, it follows that

$$q = \epsilon q_{\max} = \epsilon C_{\min} (T_{h,i} - T_{c,i}) = 0.80 \times 10,470 \text{ W/K} (100 - 20) \text{ K} = 670 \text{ kW}.$$

Performing energy balances on both fluids, find

$$T_{c,o} = T_{c,i} + q/C_c = 20^\circ\text{C} + 670 \text{ kW} / 20,890 \text{ W/K} = 52.1^\circ\text{C}$$

$$T_{h,o} = T_{h,i} - q/C_h = 100^\circ\text{C} - 670 \text{ kW} / 10,470 \text{ W/K} = 36.0^\circ\text{C}.$$

COMMENTS: (1) Note that $\bar{T}_c = (20 + 52.1)^\circ\text{C} / 2 = 310\text{K}$ and $\bar{T}_h = (100 + 36)^\circ\text{C} / 2 = 341\text{K}$ and that these values agree well with those used to evaluate the properties.

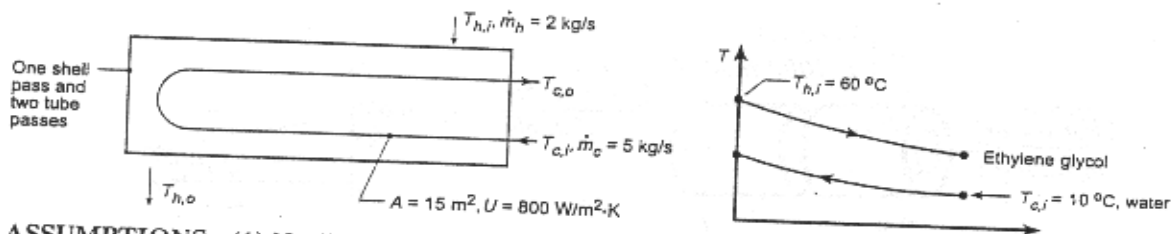
(2) Eq. 11.30 could be used to evaluate ϵ ; the result gives $\epsilon = 0.800$.

PROBLEM 11.56

KNOWN: Inlet temperatures and flow rates of water (c) and ethylene glycol (h) in a shell-and-tube heat exchanger (one shell pass and two tube passes) of prescribed area and overall heat transfer coefficient.

FIND: (a) Heat transfer rate and fluid outlet temperatures and (b) Compute and plot the effectiveness, ϵ , and fluid outlet temperatures, $T_{c,o}$ and $T_{h,o}$ as a function of the flow rate of ethylene glycol, $\dot{m}_h$, for the range $0.5 \leq \dot{m}_h \leq 5 \text{ kg/s}$.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, and (4) Overall coefficient remains unchanged.

PROPERTIES: Table A-5, Ethylene glycol ($\bar{T}_m = 40^\circ\text{C}$): $c_p = 2474 \text{ J/kg}\cdot\text{K}$; Table A-6, Water ($\bar{T}_m = 15^\circ\text{C}$): $c_p = 4186 \text{ J/kg}\cdot\text{K}$.

ANALYSIS: (a) Using the ϵ -NTU method we first obtain

$$C_h = (\dot{m}_h c_{p,h}) = (2 \text{ kg/s} \times 2474 \text{ J/kg}\cdot\text{K}) = 4948 \text{ W/K}$$

$$C_c = (\dot{m}_c c_{p,c}) = (5 \text{ kg/s} \times 4186 \text{ J/kg}\cdot\text{K}) = 20,930 \text{ W/K}$$

Hence with $C_{\min} = C_h = 4948 \text{ W/K}$ and $C_r = C_{\min}/C_{\max} = 0.236$,

$$NTU = \frac{UA}{C_{\min}} = \frac{(800 \text{ W/m}^2\cdot\text{K})15 \text{ m}^2}{4948 \text{ W/K}} = 2.43$$

From Fig. 11.16, $\epsilon = 0.81$ and from Eq. 11.23

$$q = \epsilon C_{\min} (T_{h,i} - T_{c,i}) = 0.81(4948 \text{ W/K})(60 - 10)^\circ\text{C} = 2 \times 10^5 \text{ W}$$

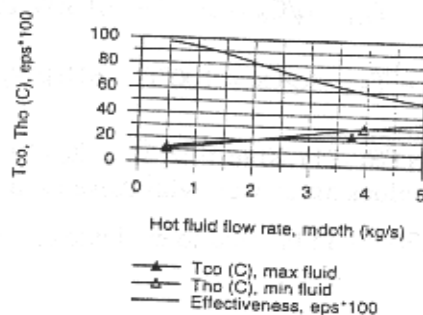
From Eqs. 11.6 and 11.7, energy balances on the fluids,

$$T_{h,o} = T_{h,i} - \frac{q}{C_h} = 60^\circ\text{C} - \frac{2 \times 10^5 \text{ W}}{4948 \text{ W/K}} = 19.6^\circ\text{C}$$

$$T_{c,o} = T_{c,i} + \frac{q}{C_c} = 10^\circ\text{C} + \frac{2 \times 10^5 \text{ W}}{20,930 \text{ W/K}} = 19.6^\circ\text{C}$$

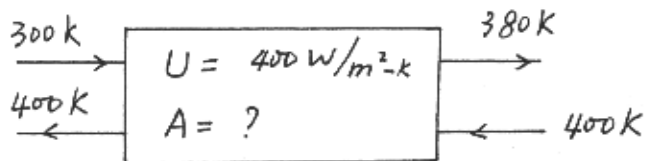
(b) Using the *IHT Heat Exchanger Tool, Shell and Tube*, and the *Properties Tool* for Water and Ethylene Glycol, $T_{c,o}$, $T_{h,o}$, and ϵ as a function of $\dot{m}_h$ were computed and plotted.

At very low $C_{\min}$ (low $\dot{m}_h$) note that $\epsilon \rightarrow 1$ while $T_{h,o} \rightarrow T_{c,i}$. As $\dot{m}_h$ increases, both fluid outlet temperatures increase and the effectiveness decreases.



Part B of Pset 10

A.



1/4

LMTD Method

$F = 1$ for condenser.

$$\Delta T_{lm} = \frac{(400 - 300) - (400 - 380)}{\ln \frac{400 - 300}{400 - 380}}$$

$$= 49.7 \text{ K}$$

$$q_T = \dot{m}_c C_{p,c} (T_{c,o} - T_{c,i})$$

$$= (5)(2000)(380 - 300)$$

$$= 8 \times 10^5 \text{ J/s}$$

$$A = \frac{q_T}{U \Delta T_{lm}}$$

$$= \frac{8 \times 10^5}{(400)(49.7)}$$

$$= \boxed{40.2 \text{ m}^2}$$

ϵ -NTU Method

for condenser,

$$C_{min} = C_c \quad C_{min}/C_{max} = 0$$

$$\epsilon = \frac{q}{q_{max}} = \frac{C_c (T_{c,o} - T_{c,i})}{C_h (T_h - T_{c,i})} = \frac{380 - 300}{400 - 300} = 0.8$$

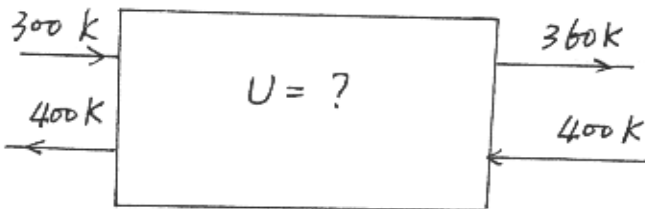
from Equ. 11-36 b

$$NTU = -\ln(1 - \epsilon) = -\ln(1 - 0.8) = 1.61$$

Therefore

$$\begin{aligned}
 A &= \frac{NTU \cdot C_{\min}}{U} \\
 &= \frac{(1.61)(2000 \cdot 5)}{400} \\
 &= \boxed{40.2 \text{ m}^2}
 \end{aligned}$$

B.

Determine U_{new} LMTD Method $F = 1$ for condenser

$$\begin{aligned}
 \Delta T_{\text{Lm}} &= \frac{(400 - 300) - (400 - 360)}{\ln \frac{400 - 300}{400 - 360}} \\
 &= 65.5 \text{ K}
 \end{aligned}$$

$$\begin{aligned}
 q_T &= \dot{m}_c C_{p,c} (T_{c,o} - T_{c,i}) \\
 &= (5)(2000)(360 - 300) \\
 &= 6 \times 10^5 \text{ J/s}
 \end{aligned}$$

$$\begin{aligned}
 U &= \frac{q_T}{A \Delta T_{\text{Lm}}} \\
 &= \frac{6 \times 10^5}{(40.2)(65.5)} = \boxed{227.9 \text{ W/m}^2 \cdot \text{K}}
 \end{aligned}$$

 ϵ -NTU Method

for condenser

3/4

$$C_{\min} = C_c \quad C_{\min} / C_{\max} = 0$$

$$\varepsilon = \frac{q}{q_{\max}} = \frac{C_c (T_{c,o} - T_{c,i})}{C_c (T_h - T_{c,i})} = \frac{360 - 300}{400 - 300} = 0.6$$

from Equ. 11-36 b

$$NTU = -\ln(1 - \varepsilon) = -\ln(1 - 0.6) = 0.92$$

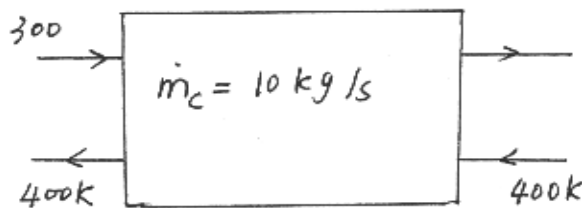
$$\begin{aligned} U &= \frac{NTU \cdot C_{\min}}{A} \\ &= \frac{0.92 \cdot (5 \cdot 2000)}{40.2} \\ &= \boxed{227.9 \text{ W/m}^2 \cdot \text{K}} \end{aligned}$$

Based on

$$\frac{1}{U_{\text{new}}} = \frac{1}{U_{\text{old}}} + R_f$$

$$R_f = \frac{1}{U_{\text{new}}} - \frac{1}{U_{\text{old}}} = \frac{1}{227.9} - \frac{1}{400} = \boxed{1.89 \times 10^{-3} \frac{\text{m}^2 \cdot \text{K}}{\text{W}}}$$

C.



Determine U_{new}

$$\begin{aligned} U_{\text{new}} &= U_{\text{old}} \left(\frac{\dot{m}_{c,\text{new}}}{\dot{m}_{c,\text{old}}} \right)^{0.8} \\ &= 400 \left(\frac{10}{5} \right)^{0.8} \\ &= 696.4 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \end{aligned}$$

for condenser

$$C_{\min} = C_c \quad C_{\min} / C_{\max} = 0$$

$$\begin{aligned}
 NTU &= \frac{UA}{C_{\min}} \\
 &= \frac{(696.4)(40.2)}{(2000 \cdot 10)} \\
 &= 1.40
 \end{aligned}$$

4/4

from Equ. 11.36 a

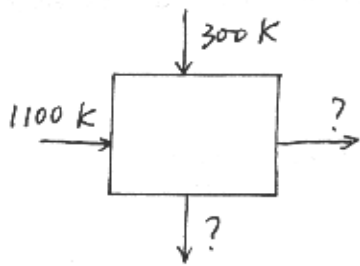
$$\begin{aligned}
 \epsilon &= 1 - \exp(-NTU) \\
 &= 0.753
 \end{aligned}$$

Therefore

$$q = \epsilon q_{\max} \Rightarrow \dot{m}_c C_{p,c} (T_{c,o} - T_{c,i}) = \epsilon \dot{m}_c C_{p,c} (T_h - T_{c,i})$$

$$\begin{aligned}
 T_{c,o} &= \epsilon (T_h - T_{c,i}) + T_{c,i} \\
 &= 0.753 (400 - 300) + 300 \\
 &= \boxed{375.3 \text{ K}}
 \end{aligned}$$

A.



$$C_h = C_c = 15 \cdot 1075 = 16125 \text{ J/K}\cdot\text{s}$$

$$C_{\min} = 1.6125 \times 10^4 \text{ J/K}\cdot\text{s}$$

$$C_{\min} / C_{\max} = 1$$

$$NTU = \frac{UA}{C_{\min}} = \frac{(100)(500)}{1.6125 \times 10^4} = 3.1$$

From Equ. 11.33

$$\varepsilon = 1 - \exp \left[(NTU)^{0.22} \{ \exp[-NTU^{0.78}] - 1 \} \right] \quad (Cr=1)$$

$$= 1 - \exp \left[3.1^{0.22} \{ \exp[-3.1^{0.78}] - 1 \} \right]$$

$$= 0.69$$

$$\varepsilon = \frac{q}{q_{\max}} = \frac{Q_c (T_{c,o} - T_{c,i})}{C_{p,h} (T_{h,i} - T_{c,i})}$$

$$T_{c,o} = \varepsilon (T_{h,i} - T_{c,i}) + T_{c,i}$$

$$= 0.69 (1100 - 300) + 300$$

$$= \boxed{851.3 \text{ K}}$$

B.

$$C_h = C_c$$

Therefore, both air and gas can attain the maximum temperature difference.

$$T_{c,o} = T_{h,i} = \boxed{1100 \text{ K}}$$

C. In this case, $T_{c,o} = 1000 \text{ K} > T_{c,o} = 851.3 \text{ (K)}$ from part A, therefore, a higher flowrate of combustion gas is needed.

So, C_h will be higher.

$$C_{\min} = C_c = 15 \cdot 1075 = 1.6125 \times 10^4 \frac{\text{W}}{\text{K}}$$

$$NTU = \frac{UA}{C_{\min}} = \frac{(100)(500)}{1.6125 \times 10^4} = 3.1$$

$$\begin{aligned} \epsilon &= \frac{q}{q_{\max}} = \frac{C_{\min} (T_{c,o} - T_{c,i})}{C_{\min} (T_{h,i} - T_{c,i})} \\ &= \frac{(1000 - 300)}{(1100 - 300)} \\ &= 0.875 \end{aligned}$$

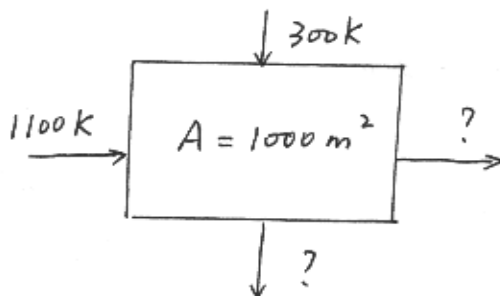
Using Figure 11.18

$$\frac{C_{\min}}{C_{\max}} = 0.36$$

$$\frac{\dot{m}_c C_{p,c}}{\dot{m}_h C_{p,h}} = 0.36$$

$$\dot{m}_h = \frac{\dot{m}_c}{0.36} = \frac{15}{0.36} = \boxed{41.7 \text{ kg/s}}$$

D. (1)



$$C_{\min} = C_c = C_h = 1.6125 \times 10^4 \frac{\text{W}}{\text{K}}$$

$$C_{\min} / C_{\max} = 1$$

$$NTU = \frac{UA}{C_{\min}} = \frac{(100)(1000)}{1.6125 \times 10^4} = 6.20$$

From Equ. 11.33

3/4

$$C_{\min}/C_{\max} = 1$$

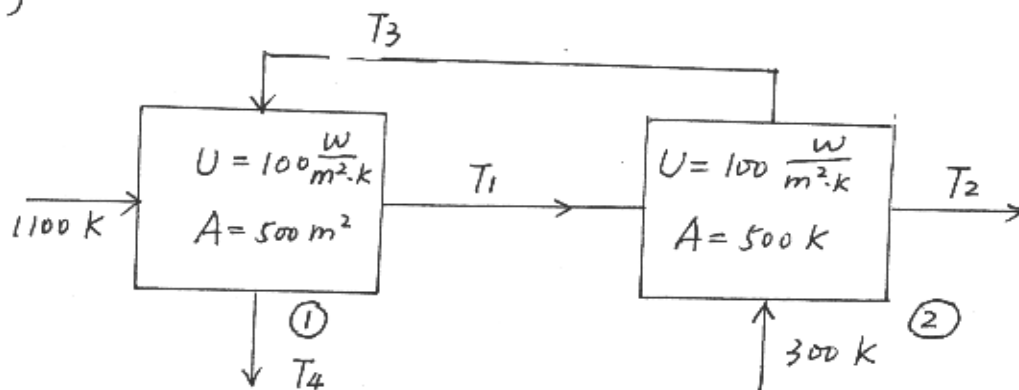
$$\begin{aligned}\epsilon &= 1 - \exp[(NTU)^{0.22} \{ \exp[-NTU^{0.78}] - 1 \}] \\ &= 1 - \exp[6.2^{0.22} \{ \exp[-6.2^{0.78}] - 1 \}]\end{aligned}$$

$$= 0.77$$

$$\epsilon = \frac{q}{q_{\max}} = \frac{q_c (T_{c,i} - T_{c,i})}{q_c (T_{h,i} - T_{c,i})}$$

$$\begin{aligned}T_{c,i} &= T_{c,i} + \epsilon (T_{h,i} - T_{c,i}) \\ &= 300 + 0.77 (1100 - 300) \\ &= \boxed{916.1 \text{ K}}\end{aligned}$$

(ii)



Based on the calculation procedure of Part A

$$\epsilon_1 = 0.69$$

$$\epsilon_2 = 0.69$$

Therefore

$$\epsilon_1 = \frac{T_4 - T_3}{1100 - T_3} = 0.69 \quad \text{for heat exchanger ①} \quad (1)$$

$$\epsilon_2 = \frac{T_3 - 300}{T_1 - 300} = 0.69 \quad \text{for heat exchanger ②} \quad (2)$$

Energy balance for the whole system ①

$$\dot{m}_c \cancel{C_{p,c}} (T_{c,o} - T_{c,i}) = \dot{m}_h \cancel{C_{p,h}} (T_{h,i} - T_{h,o})$$

$$T_4 - T_3 = 1100 - T_1 \quad (3)$$

Combine Equ. (1), (2), and (3), and get

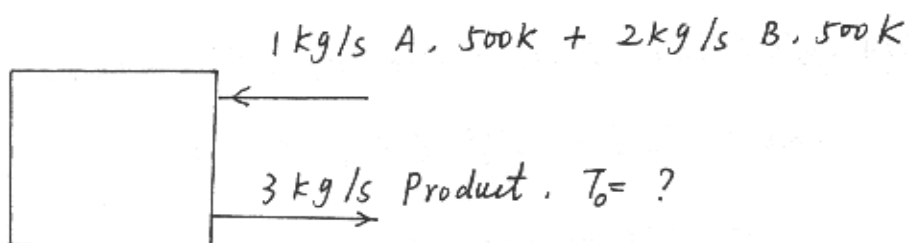
$$T_1 = 773 \text{ K}$$

$$T_3 = 626 \text{ K}$$

$$\boxed{T_4 = 953 \text{ K}}$$

Therefore, scheme (ii) gives the higher air exit temperature, 953 K than that of scheme (i), 916.1 K.

A.

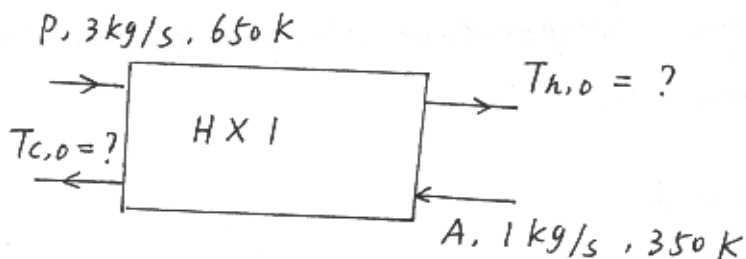


Assume the reactor to be well insulated,

$$\dot{m} C_p (T_o - T_i) = \Delta H_{rxn}$$

$$\begin{aligned} T_o &= \frac{\Delta H_{rxn}}{\dot{m} C_p} + T_i \\ &= \frac{(900,000)(1 \text{ kg})}{(1+2) \cdot 2000} + T_i \\ &= 150 + 500 \\ &= \boxed{650 \text{ K}} \end{aligned}$$

B.



Assuming the reactor effluent temperature to be 650 K,

$$C_c = (1)(2000) = 2000 \frac{\text{W}}{\text{K}}$$

$$C_h = (3)(2000) = 6000 \frac{\text{W}}{\text{K}}$$

$$C_{min} = C_c$$

Therefore, the maximum temperature difference can be attained by the cold flow, A.

$$T_{c,o} = T_{h,i} = 650 \text{ K}$$

Energy Balance for both streams

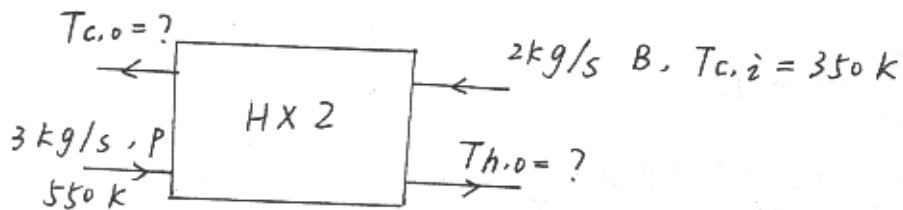
2/5

$$\dot{m}_A c_{p,A} (T_{c,o} - T_{c,i}) = \dot{m}_P c_{p,P} (T_{h,i} - T_{h,o})$$

$$T_{h,o} = T_{h,i} - \frac{\dot{m}_A}{\dot{m}_P} (T_{c,o} - T_{c,i})$$

$$= 650 - \frac{1}{3} (650 - 350)$$

$$= 550 \text{ K}$$



$$C_c = (2000)(2) = 4000 \frac{\text{W}}{\text{K}}$$

$$C_h = (2000)(3) = 6000 \frac{\text{W}}{\text{K}}$$

$$C_{\min} = C_c$$

Therefore, the maximum temperature difference can be attained by the cold flow, B.

$$T_{c,o} = T_{h,i} = 550 \text{ K}$$

The temperature of A+B stream

$$T_{\text{feed}} = (T_A \dot{m}_A + T_B \dot{m}_B) / (\dot{m}_A + \dot{m}_B)$$

$$= \frac{650 \cdot 1 + 550 \cdot 2}{3}$$

$$= \boxed{583.3 \text{ K}} > 500 \text{ K}$$

So it will be above 500 K.

C. the same procedure as A and B

$$\begin{aligned}
 T_o &= \frac{\Delta H_{rxn}}{\dot{m} C_p} + T_i \\
 &= \frac{240,000}{(3)(2000)} + 500 \\
 &= 540 \text{ K}
 \end{aligned}$$

$$T_A = 540 \text{ K}$$

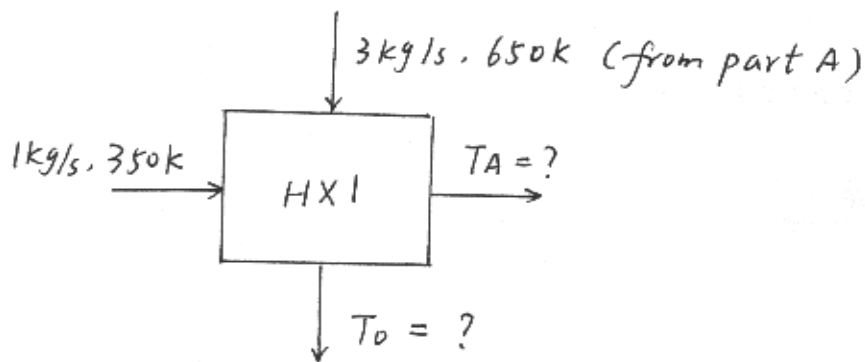
$$\begin{aligned}
 T_p &= 540 - \frac{1}{3} (540 - 350) \\
 &= 476.7 \text{ K}
 \end{aligned}$$

$$T_B = 476.7 \text{ K}$$

$$\begin{aligned}
 T_{feed} &= \frac{1}{3} \cdot 540 + \frac{2}{3} \cdot 476.7 \\
 &= \boxed{497.8 \text{ K}}
 \end{aligned}$$

The feed temperature doesn't reach 500 K.

D.



$$C_c = (1)(2000) = 2000 \frac{\text{W}}{\text{K}}$$

$$C_h = (3)(2000) = 6000 \frac{\text{W}}{\text{K}}$$

$$C_{min} = 2000 \frac{\text{W}}{\text{K}}$$

$$C_{min} / C_{max} = \frac{1}{3}$$

$$NTU = \frac{UA}{C_{min}}$$

$$= \frac{(600)(10)}{2000}$$

$$= 3$$

for equ. 11.33

$$\varepsilon = 1 - \exp \left[\left(\frac{1}{C_r} \right) (NTU)^{0.22} \{ \exp [- C_r (NTU)^{0.78}] - 1 \} \right]$$

$$= 0.875$$

$$\varepsilon = \frac{q}{q_{max}} = \frac{\dot{m}_A C_{p,A} (T_A - 350)}{\dot{m}_A C_{p,A} (650 - 350)}$$

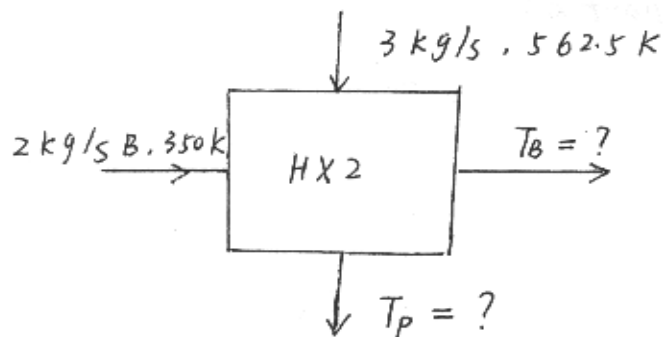
$$T_A = 612.5 \text{ K}$$

Energy Balance for A and P

$$\dot{m}_A C_{p,A} (T_A - 350) = \dot{m}_P C_{p,P} (650 - T_0)$$

$$T_0 = 650 - \frac{612.5 - 350}{3}$$

$$= 562.5 \text{ K}$$



$$C_{min} = (2)(2000) = 4000 \frac{W}{K}$$

$$\frac{C_{min}}{C_{max}} = \frac{2}{3}$$

$$NTU = \frac{UA}{C_{min}} = \frac{(600)(10)}{4000} = \frac{3}{2}$$

From Equ. 11.33

$$\varepsilon = 1 - \exp \left[\left(\frac{1}{C_r} \right) (NTU)^{0.22} \left\{ \exp \left[-C_r (NTU)^{0.78} \right] - 1 \right\} \right]$$

$$= 0.626$$

$$\varepsilon = \frac{q}{q_{\max}} = \frac{T_B - 350}{562.5 - 350}$$

$$\boxed{T_B = 483.0 \text{ K}}$$

The feed temperature is

$$T_{\text{feed}} = \frac{1}{3} (612.5) + \frac{2}{3} (483.0)$$

$$= \boxed{526.2 \text{ K}}$$

It is above 500 K!