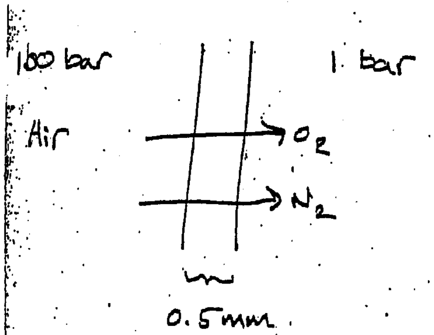
Part A

Solutions 11/29/00



a)

$$O_2 \text{ flux} = \frac{\mathscr{D}}{l}\Delta c$$

$$= \frac{(0.21 \cdot 10^{-9})}{(0.5 \cdot 10^{-3})}\left[(0.21)(100)(3.12 \cdot 10^{-3}) - \underset{\text{approximately}}{0}\right]$$

$$= 2.75 \cdot 10^{-8} \text{ kmol/m}^2 \cdot \text{s}$$

b)

$$N_2 \text{ flux} = \frac{\mathscr{D}}{l}\Delta c$$

$$= \left(\frac{0.15 \cdot 10^{-9}}{0.5 \cdot 10^{-3}}\right)\left((0.79)(100)(1.56 \cdot 10^{-3}) - 0\right)$$

$$= 3.7 \cdot 10^{-8} \text{ kmol/m}^2 \cdot \text{s}$$

c) $$X_{O_2} = \frac{2.75}{2.75 + 3.70} = 0.43$$

This enriches the O_2 concentration...
in principle, it works... but the rate is <u>very</u> small.

Part B

A) $x_{naph} = \frac{0.70\text{mmHg}}{760\text{mmHg}} = 9.2 \cdot 10^{-4} << 1$

Convection can be ignored: the naphthalene is dilute.

B) r is the distance coordinate from the center of the sphere, and R is the sphere's radius.

$$\nabla^2 c = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right) = 0$$

$$r^2\frac{\partial c}{\partial r} = k_1 \text{ or } \frac{\partial c}{\partial r} = \frac{k_1}{r^2}$$

$$c = \frac{-k_1}{r} + k_2$$

Since when $r \to \infty$, $c \to 0$; $k_2 = 0$.

At $r = R$, $c = c_0$.

$$\frac{c}{c_0} = \frac{R}{r} \text{ (concentration profile for } r > R\text{, outside the sphere)}$$

$$\text{Dissolution rate} = 4\pi R^2\left(-D\frac{\partial c}{\partial r}\Big|_R\right) = 4\pi R^2(-D)\frac{-c_0}{R}$$

$$4\pi R D c_0 = -\rho_{M,S}\frac{d}{dt}\left(\frac{4}{3}\pi R^3\right) = -\rho_{M,S}\frac{4}{3}(3\pi R^2)\frac{dR}{dt}$$

$$Dc_0 = -\rho_{M,S} R\frac{dR}{dt} \text{ and } Dc_0\int_0^t dt = -\rho_{M,S}\int_{R_0}^0 R\,dR$$

$$t = \frac{1}{2}\frac{\rho_{M,S}}{c_0}\frac{R^2}{D}$$

$$\rho_{MS} = \frac{1150}{128}\frac{\text{kg}-\text{mol}}{\text{m}^3}$$

$$c_0 = \left(\frac{1\text{g}-\text{mol}}{22.4\text{L}}\right)\left(\frac{1\text{kg}-\text{mol}}{1000\text{g}-\text{mol}}\right)\left(\frac{1000\text{L}}{\text{m}^3}\right)\left(\frac{273}{273+23}\right)\left(\frac{0.7}{760}\right)$$

$t = 3.7 \cdot 10^5$ seconds or 103 hours

C) For quasi-steady behavior to be justified, $\frac{\partial c}{\partial t} << D\frac{\partial^2 c}{\partial r^2}$

$$\frac{\partial c}{\partial r}\frac{\partial R}{\partial t} << D\frac{\partial^2 c}{\partial r^2}$$

$$\frac{c_0}{R}\frac{D}{R}\frac{c_0}{\rho_{M,S}} << D\frac{c_0}{R^2}$$

$$\frac{c_0}{\rho_{M,S}} << 1$$

$$\frac{(4.12 \cdot 10^{-2})\frac{0.7}{760}(128)}{1150} = 4 \cdot 10^{-6} << 1\text{, therefore quasi-steady is valid.}$$

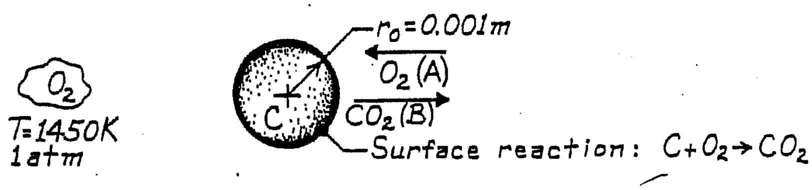
Part C

PROBLEM 14.19

KNOWN: Radius of coal particles burning in oxygen atmosphere of prescribed pressure and temperature.

FIND: (a) Radial distributions of O_2 and CO_2, (b) O_2 molar consumption rate.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Uniform total molar concentration, (3) No homogeneous chemical reactions, (4) Coal is pure carbon, (5) Surface reaction rate is infinite (hence concentration of O_2 at surface, C_A, is zero), (6) Constant D_{AB}, (7) Perfect gas behavior.

PROPERTIES: *Table A.8*, $CO_2 \rightarrow O_2$: D_{AB} (273 K) $= 0.14\times10^{-4}$ m^2/s; D_{AB} (1450 K) $= D_{AB}$ (273 K)$(1450/273)^{3/2} = 1.71\times10^{-4}$ m^2/s.

ANALYSIS: (a) For the assumed conditions, Equation 14.53 reduces to

$$\frac{d}{dr}\left(r^2\frac{dC_A}{dr}\right) = 0$$

$$r^2(dC_A/dr) = C_1 \quad \text{or} \quad C_A = -(C_1/r) + C_2.$$

From the boundary conditions:

$$C_A(\infty) = C \rightarrow \quad C_2 = C$$

$$C_A(r_o) = 0 \rightarrow \quad 0 = -C_1/r_o + C \qquad C_1 = Cr_o.$$

Hence, recognizing that $C = C_A + C_B$,

$$C_A = C - C(r_o/r) = C(1 - r_o/r) \qquad C_B = C - C_A = C(r_o/r).$$ ◁

(b) The conditions correspond to equimolar, counter diffusion ($N''_A = -N''_B$), with

$$N_{A,r} = N''_{A,r}4\pi r^2 = -CD_{AB}4\pi r^2\frac{dx_A}{dr} = -D_{AB}4\pi r^2\frac{dC_A}{dr} = -4\pi D_{AB}r^2\left(+\frac{Cr_o}{r^2}\right) = -4\pi D_{AB}Cr_o.$$

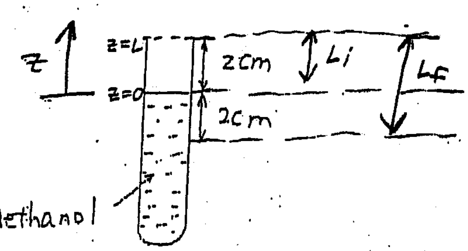
With

$$C = \frac{p}{\Re T} = \frac{1 \text{ atm}}{8.205\times10^{-2}\ \text{m}^3\cdot\text{atm/kmol}\cdot\text{K}\times1450\ \text{K}} = 8.405\times10^{-3}\ \text{kmol/m}^3$$

find

$$N_{A,r} = -1.71\times10^{-4}\ \text{m}^2/\text{s}\times4\pi\times8.405\times10^{-3}\ \text{kmol/m}^3(10^{-3}\ \text{m})$$

$$N_{A,r} = 1.81\times10^{-8}\ \text{kmol/s}.$$ ◁



Part D

Interfacial balance

MB $N''_M A = \frac{\rho_M}{MW_M} A \frac{dL}{dt} \Rightarrow N''_M = \frac{\rho_M}{MW} \frac{dL}{dt}$ (1)

Need N''_M:

(2) $\underbrace{\cancel{\frac{\partial C_M}{\partial t}}}_{\text{steady state}} + \underline{V}^* \nabla C_M = D \nabla^2 C_M + \underbrace{\cancel{\dot{N}_M} - x_M (\cancel{\dot{N}_M + \dot{N}_{AIR}})}_{\text{No rxn} \ldots}$

$\Rightarrow \underbrace{-D \nabla^2 C_M + \underline{V}^* \nabla C_M}_{} = 0$

$\Rightarrow \nabla N''_M = 0$

$\nabla N''_M = \frac{dN''_M}{dz} = 0$ (3)

14.30 $N''_M = -D \nabla C_M + V^* C_M$ (4)

14.27 $CV^* = x_M (N''_M + N''_{AIR})$

Substitute into (4)

Air insoluble in Methanol

$N''_M = -D \nabla C_M + x_M (N''_M + \cancel{N''_{AIR}})$

Solve for N''_M

(5) $\Rightarrow N''_M = -\frac{D_M}{1-x_M} \nabla C_M = \frac{-D_M C}{1-x_M} \nabla x_M = -\frac{D_M C}{1-x_M} \frac{dx_M}{dz}$

Substitute into (3)

$\frac{dN''_M}{dz} = \frac{d}{dz}\left(\frac{-D_M C}{1-x_M} \frac{dx_M}{dz}\right) = 0$

$\Rightarrow \frac{d}{dz}\left(\frac{1}{1-x_M} \frac{dx_M}{dz}\right) = 0$

Integrating $\frac{1}{1-x_M}\frac{dx_M}{dz} = C_1 \leftarrow \frac{dx_M}{dz} = C_1(1-x_M)$ (6)

Integrating again $-\ln(1-x_M) = C_1 z + C_2$

BC ① $x_M|_{z=0} = x_{M0}$ $\quad -\ln(1-x_{M0}) = C_2$

② $x_M|_{z=L} = x_{ML}$ $\quad -\ln(1-x_{ML}) = C_1 L + C_2$

$$\Rightarrow C_1 = \frac{1}{L}\ln\left(\frac{1-x_{M0}}{1-x_{ML}}\right)$$

Equation (5)

from (6)

$$N''_M = -\frac{D_M C}{1-x_M}\frac{dx_M}{dz} = \frac{-D_M C}{\cancel{1-x_M}} C_1 (\cancel{1-x_M})$$

$$N''_M = \frac{-D_M C}{L}\ln\left(\frac{1-x_{M0}}{1-x_{ML}}\right) \qquad (14.77)$$

Substitute into (1)

$$-\frac{D_M C}{L}\ln\left(\frac{1-x_{M0}}{1-x_{ML}}\right) = \frac{\rho_M}{MW_M}\frac{dL}{dt}$$

$$\int_0^t dt = \frac{\rho_M}{MW_M}\frac{1}{D_M C}\frac{1}{\ln\left(\frac{1-x_{ML}}{1-x_{M0}}\right)}\int_{L_i}^{L_f} L\,dL$$

$$C = \frac{P}{RT}$$

$$t = \frac{1}{2}\frac{\rho_M}{MW_M}\frac{RT}{D_M P}\frac{1}{\ln\left(\frac{1-x_{ML}}{1-x_{M0}}\right)}\left(L_f^2 - L_i^2\right)$$

$$t = \frac{1}{2}\frac{0.8\ \frac{g}{cm^3}}{32\ \frac{g}{mol}}\frac{\left(82.05\ \frac{cm^3\cdot atm}{mol\cdot K}\right)(323K)}{\left(0.2\ \frac{cm^2}{s}\right)(1\ atm)}\frac{1}{\ln\left(\frac{1-0}{1-\frac{400}{760}}\right)}\left[(4cm)^2-(2cm)^2\right]$$

$$\boxed{t = 26{,}601\,s = 7.4\,hr}$$

Is quasi-steady valid?

$$\cancel{\phi}\left[\frac{\partial x_M}{\partial t}+v^*\nabla x_M\right]=\nabla(\cancel{\phi}D x_M)$$

$$\frac{\partial x_M}{\partial t}+v^*\frac{\partial x_M}{\partial z}=D\frac{\partial^2 x_M}{\partial z^2}$$

$$\frac{\partial x_M}{\partial t}+\frac{v^*}{L}\frac{\partial x_M}{\partial (z/L)}=\frac{D}{L^2}\frac{\partial^2 x_M}{\partial (\frac{z}{L})^2}$$

$$\frac{L^2}{D}\frac{\partial(x_M/\Delta x_M)}{\partial t}+\frac{v^* L}{D}\frac{\partial(x_M/\Delta x_M)}{\partial(\frac{z}{L})}=\frac{\partial^2(x_M/\Delta x_M)}{\partial(\frac{z}{L})^2}$$

OOM $\quad \frac{L^2}{D}\frac{\partial(x_M/\Delta x_M)}{\partial t}<<1$

Diffussion and Convection same order of magnitude

$$\frac{L^2}{D}\frac{\partial(x_M/\Delta x_M)}{\partial L}\frac{\partial L}{\partial t}<<1$$

$$\left(\frac{\cancel{L^2}}{\cancel{D}}\right)\left(\frac{1}{\cancel{L}}\right)\left(\frac{\cancel{D}C}{\frac{\rho}{MW}\cancel{L}}\right)\ln(1-x_{MO})<<1$$

$$\frac{C_{MO}}{\frac{\rho}{MW}}<<1$$

$$\frac{P_{MO}/RT}{\rho/MW}=\frac{\frac{400/760\ atm}{(82.05)(323K)}}{\frac{0.8\ \frac{g}{cm^3}}{32\ \frac{g}{mol}}}=8\times10^{-4}<<1$$

$\therefore$ quasi-steady is valid

Also $t_{system}=\frac{\rho}{MW}\frac{L^2}{DC}\frac{1}{\ln(1-x)}$

$t_{diff}=\frac{L^2}{D}$

$$\frac{t_{Diff}}{t_{system}}\sim\frac{C_{MO}}{\frac{\rho}{MW}}<<1$$

same result!