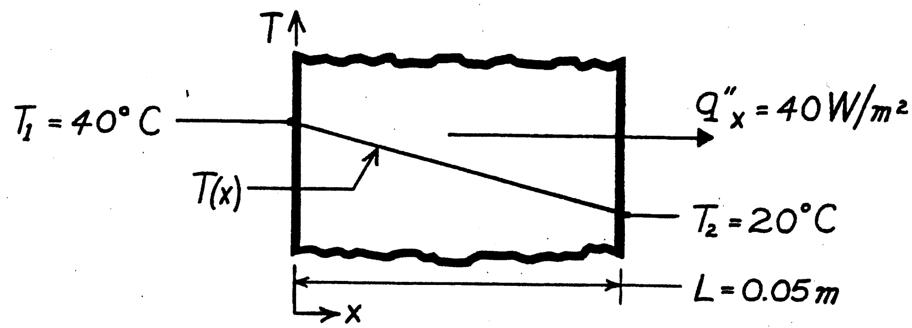
PROBLEM 1.3

KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q_x'' \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40-20)\,^\circ C}$$

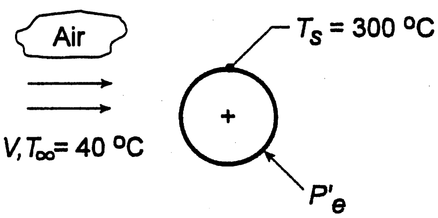
$$k = 0.10 \text{ W/m·K} .$$ ◁

COMMENTS: Note that the °C or K temperature units may be used interchangeably when evaluating a temperature difference.

KNOWN: Power required to maintain the surface temperature of a long, 25-mm diameter cylinder with an imbedded electrical heater for different air velocities.

FIND: (a) Determine the convection coefficient for each of the air velocity conditions and display the results graphically, and (b) Assuming that the convection coefficient depends upon air velocity as $h = CV^n$, determine the parameters C and n.

SCHEMATIC:



V(m/s)	1	2	4	8	12
P'_e (W/m)	450	658	983	1507	1963
h (W/m^2·K)	22.0	32.2	48.1	73.8	96.1

ASSUMPTIONS: (1) Temperature is uniform over the cylinder surface, (2) Negligible radiation exchange between the cylinder surface and the surroundings, (3) Steady-state conditions.

ANALYSIS: (a) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newtons law of cooling on a per unit length basis,

$$P'_e = h(\pi D)(T_s - T_\infty)$$

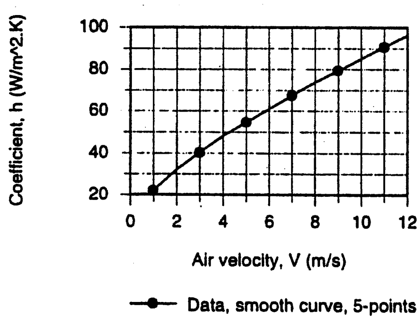
where P'_e is the electrical power dissipated per unit length of the cylinder. For the V = 1 m/s condition, using the data from the table above, find

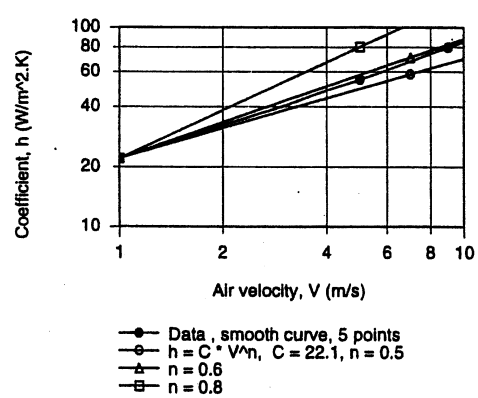
$$h = 450\,W/m/\pi \times 0.025\,m(300-40)^\circ C = 22.0\,W/m^2{\cdot}K$$ <

Repeating the calculations, find the convection coefficients for the remaining conditions which are tabulated above and plotted below. Note that h is not linear with respect to the air velocity.

(b) To determine the (C,n) parameters, we plotted h vs. V on log-log coordinates. Choosing C = 22.12 W/m^2·K(s/m)^n, assuring a match at V = 1, we can readily find the exponent n from the slope of the h vs. V curve. From the trials with n = 0.8, 0.6 and 0.5, we recognize that n = 0.6 is a reasonable choice.

Hence, C = 22.12 and n = 0.6. <



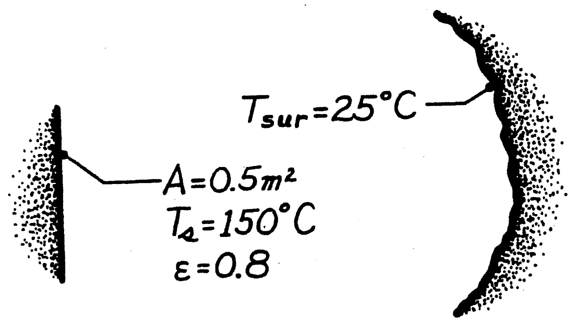


PROBLEM 1.19

KNOWN: Area, emissivity and temperature of a surface placed in a large, evacuated chamber of prescribed temperature.

FIND: (a) Rate of surface radiation emission, (b) Net rate of radiation exchange between surface and chamber walls.

SCHEMATIC:



ASSUMPTIONS: (1) Area of the enclosed surface is much less than that of chamber walls.

ANALYSIS: (a) From Eq. 1.5, the rate at which radiation is emitted by the surface is

$$q_{emit} = q''_{emit} \cdot A = \varepsilon A \sigma T_s^4$$

$$q_{emit} = 0.8(0.5\ \text{m}^2)\ 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ [(150 + 273)\text{K}]^4$$

$$q_{emit} = 726\ \text{W}.$$ ◁

(b) From Eq. 1.7, the *net* rate at which radiation is transferred *from* the surface to the chamber walls is

$$q = \varepsilon A \sigma (T_s^4 - T_{sur}^4)$$

$$q = 0.8(0.5\ \text{m}^2)\ 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ [(423\text{K})^4 - (298\text{K})^4]$$

$$q = 547\ \text{W}.$$ ◁

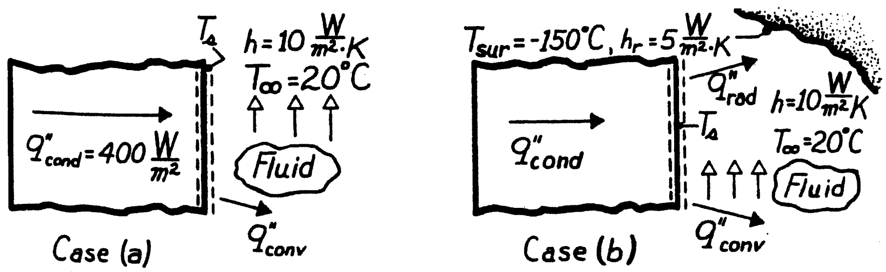
COMMENTS: The foregoing result gives the net heat loss from the surface which occurs at the instant the surface is placed in the chamber. The surface would, of course, cool due to this heat loss and its temperature, as well as the heat loss, would decrease with increasing time. Steady-state conditions would eventually be achieved when the temperature of the surface reached that of the surroundings.

PROBLEM 1.42

KNOWN: Heat flux through a plane wall.

FIND: Surface temperature with these conditions at the surface: (a) Convection process with prescribed h, T_∞ and (b) Convection and radiation processes with prescribed h, T_∞ and h_r, T_{sur}, respectively.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with no internal generation, (3) Heat flux through wall is same for both cases.

ANALYSIS: (a) Using Newton's law of cooling, the surface temperature T_s can be expressed in terms of the convective heat flux,

$$q''_{conv} = h(T_s - T_\infty) \qquad \text{or} \qquad T_s = T_\infty + q''_{conv}/h .$$

Performing a surface energy balance, at an instant of time, on a per unit area basis, find

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad \text{or} \qquad q''_{cond} - q''_{conv} = 0 .$$

Since, q''_{cond} is known, it follows that

$$T_s = T_\infty + \frac{q''_{cond}}{h} = 20°C + \frac{400\ W/m^2}{10\ W/m^2\cdot K} = 20°C + 40°C = 60°C . \qquad \triangleleft$$

(b) Considering now radiative heat transfer between the surface at T_s and the surroundings at T_{sur}, the surface energy balance is

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad \text{or} \qquad q''_{cond} - q''_{conv} - q''_{rad} = 0 .$$

Using the linearized form of the radiation rate equation, Eq. 1.8, the energy balance and surface temperature are

$$q''_{cond} - h(T_s - T_\infty) - h_r(T_s - T_{sur}) = 0$$

$$T_s = + \frac{h\,T_\infty + h_r\,T_{sur}}{h + h_r} + \frac{q''_{cond}}{h + h_r}$$

$$T_s = \frac{10\ W/m^2\cdot K \times 20°C + 5\ W/m^2\cdot K(-150°C)}{(10+5)\ W/m^2\cdot K} + \frac{400\ W/m^2}{(10+5)\ W/m^2\cdot K} = -10°C . \qquad \triangleleft$$

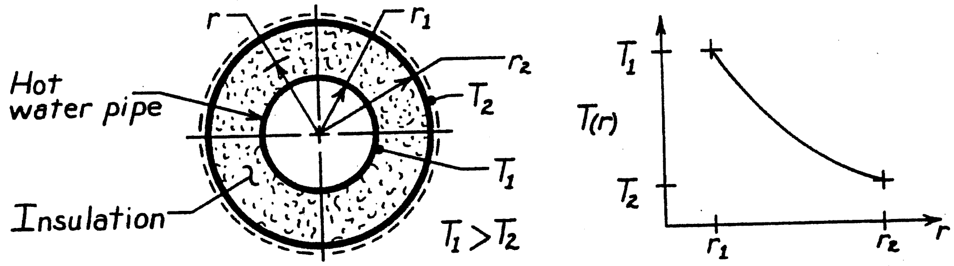
COMMENTS: Note that q''_{conv} for Case (b) is into the surface. The effect of the radiation exchange, part (b), is, as expected, to reduce the surface temperature.

PROBLEM 2.2

KNOWN: Hot water pipe covered with thick layer of insulation.

FIND: Sketch temperature distribution and give brief explanation to justify shape.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

$$q_r = -kA_r \frac{dT}{dr} = -k\,(2\pi r\ell)\,\frac{dT}{dr}$$

where $A_r = 2\pi r\ell$ and ℓ is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires $\dot{E}_{in} = \dot{E}_{out}$ since $\dot{E}_g = \dot{E}_{st} = 0$ and hence

$$q_r = \text{Constant.}$$

That is, q_r is independent of radius (r). Since the thermal conductivity is also constant, it follows that

$$r\left(\frac{dT}{dr}\right) = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius, r, remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the above, right sketch.

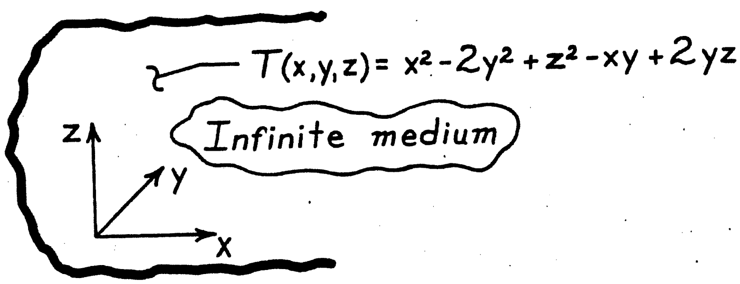
COMMENTS: (1) Note that while q_r is a constant and independent of r, q_r'' is not a constant. How does $q_r''(r)$ vary with r? (2) Recognize that the radial temperature gradient, dT/dr, decreases with increasing radius.

PROBLEM 2.20

KNOWN: Temperature distribution, T(x,y,z), within an infinite, homogeneous body at a given instant of time.

FIND: Regions where the temperature changes with time.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties of infinite medium and (2) No internal heat generation.

ANALYSIS: The temperature distribution throughout the medium, at any instant of time, must satisfy the heat equation. For the three-dimensional cartesian coordinate system, with constant properties and no internal heat generation, the heat equation, Eq. 2.15, has the form

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}. \qquad (1)$$

When T(x,y,z) satisfies this relation, then conservation of energy at every point in the medium is satisfied. Substituting T(x,y,z) into the Eq. (1), first find the gradients, $\partial T/\partial x$, $\partial T/\partial y$, etc.,

$$\frac{\partial}{\partial x}(2x-y) + \frac{\partial}{\partial y}(-4y-x+2z) + \frac{\partial}{\partial z}(2z+2y) = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Performing the differentiation, find

$$2-4+2 = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

That is,

$$\frac{\partial T}{\partial t} = 0$$

which implies that, for the given instant of time, the temperature will everywhere not change.

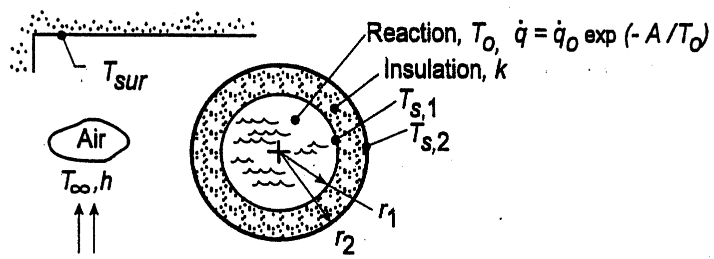
COMMENTS: Since we do not know the initial and boundary conditions, we cannot determine the temperature distribution, T(x,y,z), at any future time. We only can determine that, for this special instant of time, the temperature will not change.

PROBLEM 2.39

KNOWN: Spherical container with an exothermic reaction enclosed by an insulating material whose outer surface experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that the prescribed temperature distribution for the insulation satisfies the appropriate form of the heat diffusion equation; sketch the temperature distribution and label key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the insulation layer, q_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the container and obtain an alternative expression for q_r in terms of $\dot{q}$ and r_1; (c) Apply a surface energy balance around the outer surface of the insulation to obtain an expression to evaluate $T_{s,2}$; (d) Determine $T_{s,2}$ for the specified geometry and operating conditions; (e) Compute and plot the variation of $T_{s,2}$ as a function of the outer radius for the range $201 \le r_2 \le 210$ mm; explore approaches for reducing $T_{s,2} \le 45°C$ to eliminate potential risk for burn injuries to personnel.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial spherical conduction, (2) Isothermal reaction in container so that $T_o = T_{s,1}$, (2) Negligible thermal contact resistance between the container and insulation, (3) Constant properties in the insulation, (4) Surroundings large compared to the insulated vessel, and (5) Steady-state conditions.

ANALYSIS: The appropriate form of the heat diffusion equation (HDE) for the insulation follows from Eq. 2.23,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad (1) <$$

The temperature distribution is given as

$$T(r) = T_{s,1} - \left(T_{s,1} - T_{s,2}\right)\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right] \qquad (2)$$

Substitute T(r) into the HDE to see if it is satisfied:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\left[0-\left(T_{s,1}-T_{s,2}\right)\frac{0+(r_1/r^2)}{1-(r_1/r_2)}\right]\right) ?=?0$$

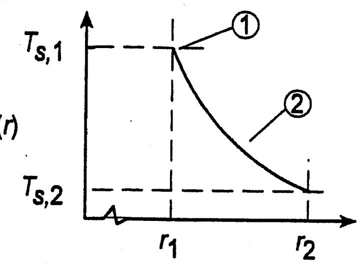
$$\frac{1}{r^2}\frac{d}{dr}\left(+\left(T_{s,1}-T_{s,2}\right)\frac{r_1}{1-(r_1/r_2)}\right) ?=?0 \qquad <$$

and since the expression in parenthesis is independent of r, T(r) does indeed satisfy the HDE. The temperature distribution in the insulation and its key features are as follows:

Continued...

(1) $T_{s,1} > T_{s,2}$

(2) Decreasing gradient with increasing radius, r, since the heat rate is constant through the insulation.



(b) Using Fourier's law for the radial-spherical coordinate, the heat rate through the insulation is

$$q_r = -kA_r \frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr}$$ <

and substituting for the temperature distribution, Eq. (2),

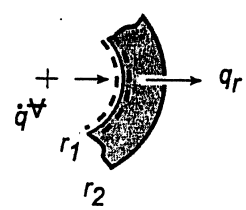
$$q_r = -k\pi r^2 \left[0 - \left(T_{s,1} - T_{s,2}\right)\frac{0 + \left(r_1/r^2\right)}{1 - \left(r_1/r_2\right)}\right]$$

$$q_r = \frac{4\pi k\left(T_{s,1} - T_{s,2}\right)}{\left(1/r_1\right) - \left(1/r_2\right)} \qquad (3) <$$

Applying an energy balance to a control surface about the container at $r = r_1$,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{q}\forall - q_r = 0$$



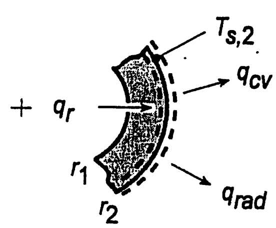
where $\dot{q}\forall$ represents the generated heat in the container,

$$q_r = (4/3)\pi r_1^3 \dot{q} \qquad (4) <$$

(c) Applying an energy balance to a control surface placed around the outer surface of the insulation,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_r - q_{cv} - q_{rad} = 0$$



$$q_r - hA_s\left(T_{s,2} - T_\infty\right) - \varepsilon A_s \sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \qquad (5) <$$

Continued...

where

$$A_s = 4\pi r_2^2 \tag{6}$$

These relations can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider the reactor system operating under the following conditions:

$r_1 = 200$ mm	$h = 5\ W/m^2 \cdot K$	$\varepsilon = 0.9$
$r_2 = 208$ mm	$T_\infty = 25°C$	$T_{sur} = 35°C$
$k = 0.05\ W/m \cdot K$		

The heat generated by the exothermic reaction provides for a volumetric heat generation rate,

$$\dot{q} = \dot{q}_o \exp(-A/T_o) \qquad q_o = 5000\,W/m^3 \qquad A = 75\,K \tag{7}$$

where the temperature of the reaction is that of the inner surface of the insulation, $T_o = T_{s,1}$. The following system of equations will determine the operating conditions for the reactor.

Conduction rate equation, insulation, Eq. (3),

$$q_r = \frac{4\pi \times 0.05\,W/m \cdot K(T_{s,1} - T_{s,2})}{(1/0.200\,m - 1/0.208\,m)} \tag{8}$$

Heat generated in the reactor, Eqs. (4) and (7),

$$q_r = 4/3\pi(0.200\,m)^3 \dot{q} \tag{9}$$

$$\dot{q} = 5000\,W/m^3 \exp(-75\,K/T_{s,1}) \tag{10}$$

Surface energy balance, insulation, Eqs. (5) and (6),

$$q_r - 5\,W/m^2 \cdot K\,A_s(T_{s,2} - 298\,K) - 0.9A_s 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\left(T_{s,2}^4 - (308\,K)^4\right) = 0 \tag{11}$$

$$A_s = 4\pi(0.208\,m)^2 \tag{12}$$

Solving these equations simultaneously, find that

$$T_{s,1} = 94.3°C \qquad T_{s,2} = 52.5°C$$ <

That is, the reactor will be operating at $T_o = T_{s,1} = 94.3°C$, very close to the desired 95°C operating condition.

(e) From the above analysis, we found the outer surface temperature $T_{s,2} = 52.5°C$ represents a potential burn risk to plant personnel. Using the above system of equations, Eqs. (8)-(12), we have explored the effects of changes in the convection coefficient, h, and the insulation thermal conductivity, k, as a function of insulation thickness, $t = r_2 - r_1$.

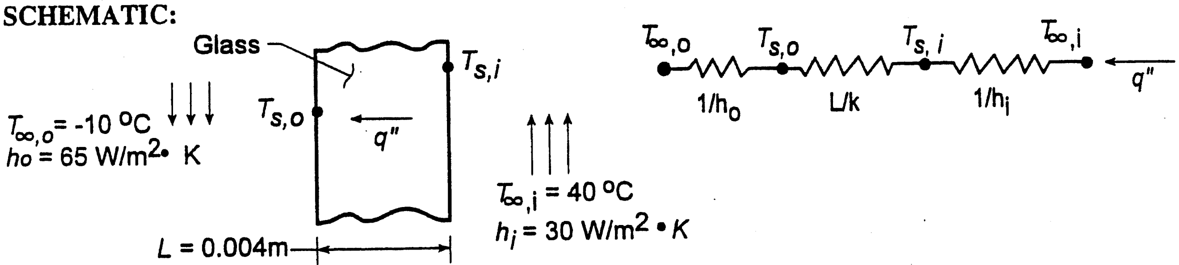
Continued...

PROBLEM 3.2

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ C - (-10^\circ C)}{\frac{1}{65\,W/m^2 \cdot K} + \frac{0.004\,m}{1.4\,W/m \cdot K} + \frac{1}{30\,W/m^2 \cdot K}}$$

$$q'' = \frac{50^\circ C}{(0.0154 + 0.0029 + 0.0333)m^2 \cdot K/W} = 968\,W/m^2.$$

Hence, with $q'' = h_i(T_{\infty,i} - T_{\infty,o})$, the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ C - \frac{968\,W/m^2}{30\,W/m^2 \cdot K} = 7.7^\circ C$$ <

Similarly for the outer surface temperature with $q'' = h_o(T_{s,o} - T_{\infty,o})$ find

$$T_{s,o} = T_{\infty,o} - \frac{q''}{h_o} = -10^\circ C - \frac{968\,W/m^2}{65\,W/m^2 \cdot K} = 4.9^\circ C$$ <

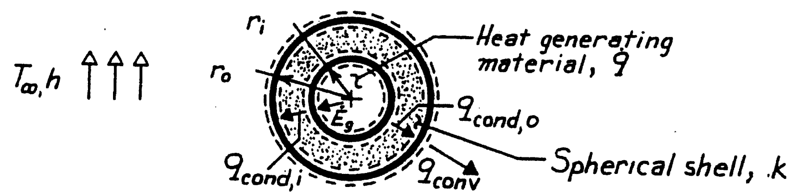
~~(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of h_o = 2, 65, and 100 W/m²·K. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with h_o = 2 W/m²·K, $T_{s,i}$ - $T_{s,o}$, is too small to show on the plot.~~

PROBLEM 3.61

KNOWN: Volumetric heat generation occurring within the cavity of a spherical shell of prescribed dimensions. Convection conditions at outer surface.

FIND: Expression for steady-state temperature distribution in shell.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Steady-state conditions, (3) Constant properties, (4) Uniform generation within the shell cavity, (5) Negligible radiation.

ANALYSIS: For the prescribed conditions, the appropriate form of the heat equation is

$$\frac{d}{dr}\left[r^2 \frac{dT}{dr}\right] = 0$$

Integrate twice to obtain,

$$r^2 \frac{dT}{dr} = C_1 \qquad \text{and} \qquad T = -\frac{C_1}{r} + C_2 . \qquad (1,2)$$

The boundary conditions may be obtained from energy balances at the inner and outer surfaces. At the inner surface (r_i),

$$\dot{E}_g = \dot{q}(4/3\pi r_i^3) = q_{cond,i} = -k(4\pi r_i^2)\, dT/dr)_{r_i} \qquad dT/dr)_{r_i} = -\dot{q}r_i/3k. \qquad (3)$$

At the outer surface (r_o),

$$q_{cond,o} = -k4\pi r_o^2\, dT/dr)_{r_o} = q_{conv} = h4\pi r_o^2\, [T(r_o)-T_\infty]$$

$$dT/dr)_{r_o} = -(h/k)\, [T(r_o)-T_\infty]. \qquad (4)$$

From Eqs. (1) and (3), $C_1 = -\dot{q}r_i^3/3k$. From Eqs. (1), (2) and (4)

$$-\frac{\dot{q}r_i^3}{3kr_o^2} = -\left[\frac{h}{k}\right]\left[\frac{\dot{q}r_i^3}{3r_o k} + C_2 - T_\infty\right]$$

$$C_2 = \frac{\dot{q}r_i^3}{3hr_o^2} - \frac{\dot{q}r_i^3}{3r_o k} + T_\infty.$$

Hence, the temperature distribution is

$$T = \frac{\dot{q}r_i^3}{3k}\left[\frac{1}{r} - \frac{1}{r_o}\right] + \frac{\dot{q}r_i^3}{3hr_o^2} + T_\infty. \qquad \triangleleft$$

COMMENTS: Note that $\dot{E}_g = q_{cond,i} = q_{cond,o} = q_{conv}$.