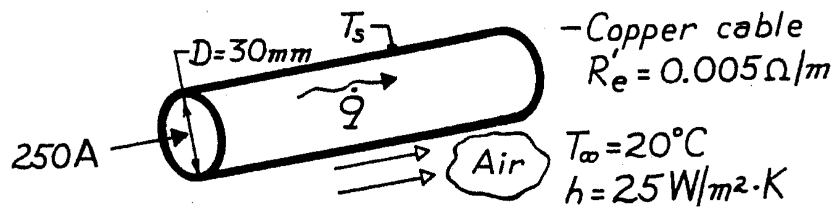
PROBLEM 3.79

KNOWN: Current flow through a copper cable exposed to air.

FIND: Surface and centerline temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Uniform generation, (5) Negligible radiation.

PROPERTIES: *Table A-1*, Copper ($T \approx 300K$): $k = 401$ W/m·K.

ANALYSIS: The surface temperature may be obtained from the energy balance result, Eq. 3.55,

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h},$$

where

$$\dot{q} = \frac{I^2 R_e}{(\pi D^2/4)L} = \frac{I^2 R_e'}{\pi D^2/4} = \frac{4(250A)^2(0.005\Omega/m)}{\pi\,(0.03m)^2} = 4.42\times10^5\ W/m^3 .$$

Hence,

$$T_s = 20°C + \frac{4.42\times10^5 W/m^3 \times 0.015m}{2\times 25\ W/m^2\cdot K} = 152.6°C . \quad \triangleleft$$

From Eq. 3.53 the centerline temperature is

$$T_o = \frac{\dot{q}\, r_o^2}{4k} + T_s = \frac{4.42\times10^5\ W/m^3\ (0.015m)^2}{4\times 401\ W/m\cdot K} + 152.6°C$$

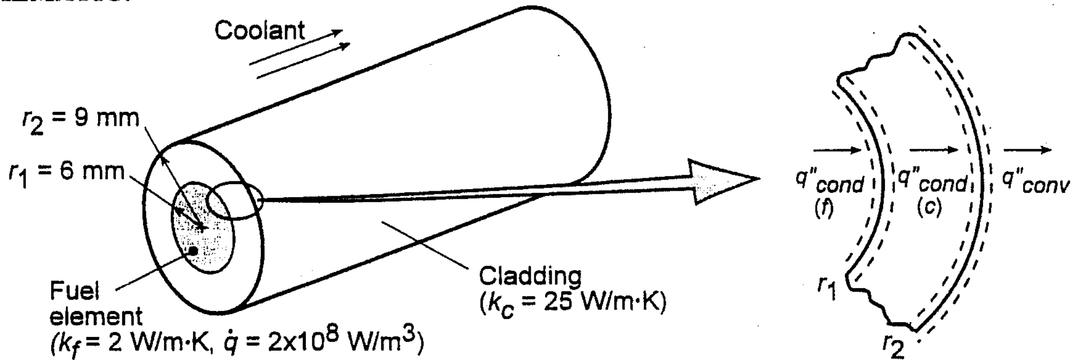
$$T_o = 0.06 + 152.6 = 152.7°C . \quad \triangleleft$$

COMMENTS: The temperature of the rod is determined primarily by surface convection effects and is nearly uniform due to the large value of k and small value of $\dot{q}$. The temperature may be reduced by increasing h.

KNOWN: Radii and thermal conductivities of reactor fuel element and cladding. Fuel heat generation rate. Temperature and convection coefficient of coolant.

FIND: (a) Expressions for temperature distributions in fuel and cladding. (b) Maximum fuel element temperature for prescribed conditions, (c) Effect of h on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible contact resistance, (4) Constant properties.

ANALYSIS: (a) From Eqs. 3.49 and 3.23, the heat equations for the fuel and cladding are

$$\frac{1}{r}\frac{d}{dr}\left(rk_f\frac{dT_f}{dr}\right)=-\dot{q} \qquad (0\le r\le r_1) \qquad\qquad \frac{1}{r}\frac{d}{dr}\left(k_c r\frac{dT_c}{dr}\right)=0 \qquad (r_1\le r\le r_2)$$

Hence, integrating both equations twice,

$$\frac{dT_f}{dr}=-\frac{\dot{q}r}{2k_f}+\frac{C_1}{k_f r} \qquad T_f=-\frac{\dot{q}r^2}{4k_f}+\frac{C_1}{k_f}\ln r+C_2 \qquad (1,2)$$

$$\frac{dT_c}{dr}=\frac{C_3}{k_c r} \qquad T_c=\frac{C_3}{k_c}\ln r+C_4 \qquad (3,4)$$

The corresponding boundary conditions are:

$$dT_f/dr)_{r=0}=0 \qquad T_f(r_1)=T_c(r_1) \qquad (5,6)$$

$$-k_f\frac{dT_c}{dr}\bigg)_{r=r_1}=-k_c\frac{dT_c}{dr}\bigg)_{r=r_1} \qquad -k_c\frac{dT_c}{dr}\bigg)_{r=r_2}=h\left[T_c(r_2)-T_\infty\right] \qquad (7,8)$$

Note that Eqs. (7) and (8) are obtained from surface energy balances at r_1 and r_2, respectively. Applying Eq. (5) to Eq. (1), it follows that $C_1 = 0$. Hence,

$$T_f=-\frac{\dot{q}r^2}{4k_f}+C_2 \qquad (9)$$

From Eq. (6), it follows that

$$-\frac{\dot{q}r_1^2}{4k_f}+C_2=\frac{C_3\ln r_1}{k_c}+C_4 \qquad (10)$$

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Also, from Eq. (7),

$$\frac{\dot{q}r_1}{2} = -\frac{C_3}{r_1} \qquad \text{or} \qquad C_3 = -\frac{\dot{q}r_1^2}{2} \qquad (11)$$

Finally, from Eq. (8), $-\frac{C_3}{r_2} = h\left[\frac{C_3}{k_c}\ln r_2 + C_4 - T_\infty\right]$ or, substituting for C_3 and solving for C_4

$$C_4 = \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty \qquad (12)$$

Substituting Eqs. (11) and (12) into (10), it follows that

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} - \frac{\dot{q}r_1^2 \ln r_1}{2k_c} + \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty$$

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h}T_\infty \qquad (13)$$

Substituting Eq. (13) into (9),

$$T_f = \frac{\dot{q}}{4k_f}\left(r_1^2 - r^2\right) + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \qquad (14) <$$

Substituting Eqs. (11) and (12) into (4),

$$T_c = \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty . \qquad (15) <$$

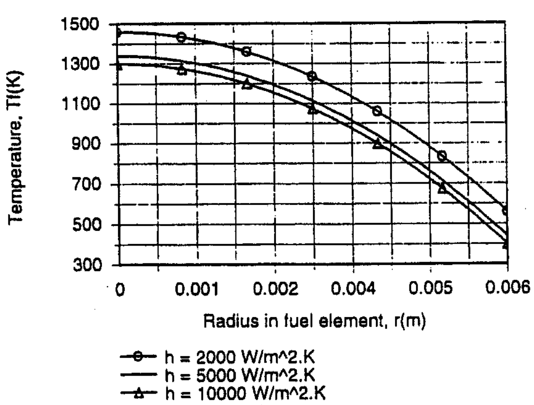
(b) Applying Eq. (14) at r = 0, the maximum fuel temperature for h = 2000 W/m^2·K is

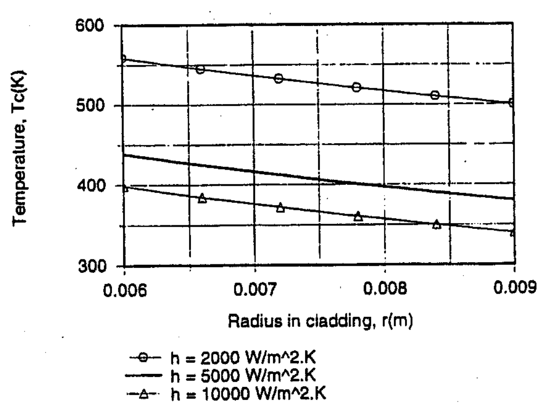
$$T_f(0) = \frac{2\times10^8\,\text{W/m}^3 \times (0.006\,\text{m})^2}{4\times 2\,\text{W/m}\cdot\text{K}} + \frac{2\times10^8\,\text{W/m}^3 \times (0.006\,\text{m})^2}{2\times 25\,\text{W/m}\cdot\text{K}}\ln\frac{0.009\,\text{m}}{0.006\,\text{m}}$$

$$+\frac{2\times10^8\,\text{W/m}^3 (0.006\,\text{m})^2}{2\times(0.09\,\text{m})2000\,\text{W/m}^2\cdot\text{K}} + 300\,\text{K}$$

$$T_f(0) = (900 + 58.4 + 200 + 300)\text{K} = 1458\,\text{K}. \qquad <$$

(c) Temperature distributions for the prescribed values of h are as follows:





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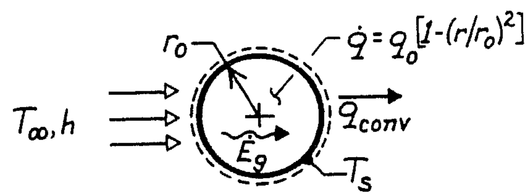
PROBLEM 3.89

KNOWN: Radial distribution of heat dissipation of a spherical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left[1-\left(\frac{r}{r_o}\right)^2\right].$$

Hence

$$r^2\frac{dT}{dr} = -\frac{\dot{q}_o}{k}\left(\frac{r^3}{3} - \frac{r^5}{5r_o^2}\right) + C_1$$

$$T = -\frac{\dot{q}_o}{k}\left(\frac{r^2}{6} - \frac{r^4}{20r_o^2}\right) - \frac{C_1}{r} + C_2.$$

From the boundary conditions,

$$dT/dr\big|_{r=0} = 0 \quad \text{and} \quad -kdT/dr\big|_{r=r_o} = h[T(r_o) - T_\infty]$$

if follows that $C_1 = 0$ and

$$\dot{q}_o\left(\frac{r_o}{3} - \frac{r_o}{5}\right) = h\left[-\frac{\dot{q}_o}{k}\left(\frac{r_o^2}{6} - \frac{r_o^2}{20}\right) + C_2 - T_\infty\right]$$

$$C_2 = \frac{2r_o\dot{q}_o}{15h} + \frac{7\dot{q}_o r_o^2}{60k} + T_\infty.$$

Hence

$$T(r) = T_\infty + \frac{2r_o\dot{q}_o}{15h} + \frac{\dot{q}r_o^2}{k}\left[\frac{7}{60} - \frac{1}{6}\left(\frac{r}{r_o}\right)^2 + \frac{1}{20}\left(\frac{r}{r_o}\right)^4\right]. \quad \triangleleft$$

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty + (2r_o\dot{q}_o/15h).$$

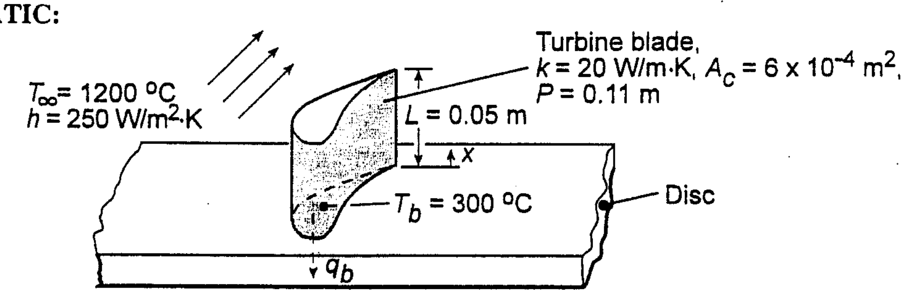
The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at $r = 0$.

PROBLEM 3.104

KNOWN: Dimensions and thermal conductivity of a gas turbine blade. Temperature and convection coefficient of gas stream. Temperature of blade base and maximum allowable blade temperature.

FIND: (a) Whether blade operating conditions are acceptable, (b) Heat transfer to blade coolant.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in blade, (2) Constant k, (3) Adiabatic blade tip, (4) Negligible radiation.

ANALYSIS: Conditions in the blade are determined by Case B of Table 3.4.

(a) With the maximum temperature existing at x = L, Eq. 3.75 yields

$$\frac{T(L)-T_\infty}{T_b-T_\infty}=\frac{1}{\cosh mL}$$

$$m=\left(hP/kA_c\right)^{1/2}=\left(250\,W/m^2\cdot K\times 0.11m/20\,W/m\cdot K\times 6\times 10^{-4}m^2\right)^{1/2}$$

$$m = 47.87\ m^{-1} \quad \text{and} \quad mL = 47.87\ m^{-1}\times 0.05\ m = 2.39$$

From Table B.1, cosh mL = 5.51. Hence,

$$T(L) = 1200^\circ C + (300-1200)^\circ C/5.51 = 1037^\circ C$$ <

and the operating conditions are acceptable.

(b) With $M=\left(hPkA_c\right)^{1/2}\Theta_b=\left(250\,W/m^2\cdot K\times 0.11m\times 20\,W/m\cdot K\times 6\times 10^{-4}m^2\right)^{1/2}\left(-900^\circ C\right)=-517W$, Eq. 3.76 and Table B.1 yield

$$q_f = M\tanh mL = -517W(0.983) = -508W$$

Hence, $q_b = -q_f = 508W$ <

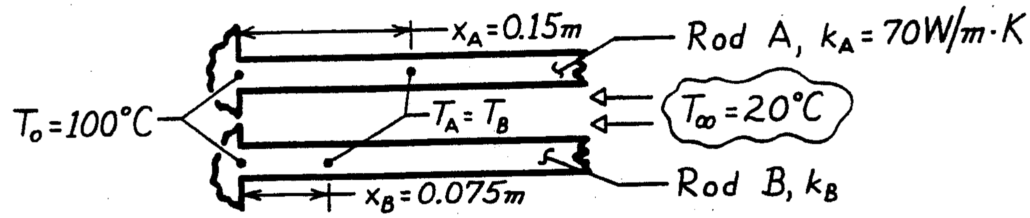
COMMENTS: Radiation losses from the blade surface and convection from the tip will contribute to reducing the blade temperatures.

PROBLEM 3.111

KNOWN: Positions of equal temperature on two long rods of the same diameter, but different thermal conductivity, which are exposed to the same base temperature and ambient air conditions.

FIND: Thermal conductivity of rod B, k_B.

SCHEMATIC:



ASSUMPTIONS (1) Steady-state conditions, (2) Rods are infinitely long fins of uniform cross-sectional area, (3) Uniform heat transfer coefficient, (4) Constant properties.

ANALYSIS: The temperature distribution for the infinite fin has the form

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_o - T_\infty} = e^{-mx} \qquad m = \left[\frac{hP}{kA_c}\right]^{1/2}. \qquad (1,2)$$

For the two positions prescribed, x_A and x_B, it was observed that

$$T_A(x_A) = T_B(x_B) \qquad \text{or} \qquad \theta_A(x_A) = \theta_B(x_B). \qquad (3)$$

Since θ_b is identical for both rods, Eq. (1) with the equality of Eq. (3) requires that

$$m_A x_A = m_B x_B$$

Substituting for m from Eq. (2) gives the form

$$\left[\frac{hP}{k_A A_c}\right]^{1/2} x_A = \left[\frac{hP}{k_B A_c}\right]^{1/2} x_B.$$

Recognizing that h, P and A_c are identical for each rod and rearranging gives

$$k_B = \left[\frac{x_B}{x_A}\right]^2 k_A$$

$$k_B = \left[\frac{0.075m}{0.15m}\right]^2 \times 70\ W/m\cdot K = 17.5\ W/m\cdot K. \qquad \triangleleft$$

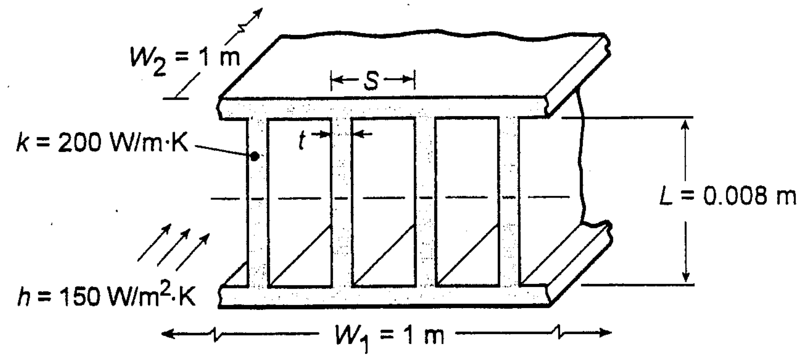
COMMENTS: This approach was used in earlier times as a method for obtaining thermal conductivity measurements. It has the attractive feature of not requiring power or temperature measurements, assuming of course, a reference material of known thermal conductivity is available.

PROBLEM 3.115

KNOWN: Conditions associated with an array of straight rectangular fins.

FIND: Thermal resistance of the array.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Uniform convection coefficient, (3) Symmetry about midplane.

ANALYSIS: (a) Considering a one-half section of the array, the corresponding resistance is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $A_t = NA_f + A_b$. With S = 4 mm and t = 1 mm, it follows that $N = W_1/S = 250$, $A_f = 2(L/2)W_2 = 0.008\ m^2$, $A_b = W_2(W_1 - Nt) = 0.75\ m^2$, and $A_t = 2.75\ m^2$.

The overall surface efficiency is

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$$

where the fin efficiency is

$$\eta_f = \frac{\tanh m(L/2)}{m(L/2)} \quad \text{and} \quad m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left[\frac{h(2t + 2W_2)}{ktW_2}\right]^{1/2} \approx \left(\frac{2h}{kt}\right)^{1/2} = 38.7\text{m}^{-1}$$

With $m(L/2) = 0.155$, it follows that $\eta_f = 0.992$ and $\eta_o = 0.994$. Hence

$$R_{t,o} = \left(0.994 \times 150\text{W/m}^2\cdot\text{K} \times 2.75\text{m}^2\right)^{-1} = 2.44 \times 10^{-3}\text{K/W}$$ <

(b) The requirements that $t \geq 0.5$ m and $(S - t) > 2$ mm are based on manufacturing and flow passage restriction constraints. Repeating the foregoing calculations for representative values of t and (S - t), we obtain

S (mm)	N	t (mm)	$R_{t,o}$ (K/W)
2.5	400	0.5	0.00169
3	333	0.5	0.00193
3	333	1	0.00202
4	250	0.5	0.00234
4	250	2	0.00268
5	200	0.5	0.00264
5	200	3	0.00334

COMMENTS: Clearly, the thermal performance of the fin array improves ($R_{t,o}$ decreases) with increasing N. Because $\eta_f \approx 1$ for the entire range of conditions, there is a slight degradation in performance ($R_{t,o}$ increases) with increasing t and fixed N. The reduced performance is associated with the reduction in surface area of the exposed base.