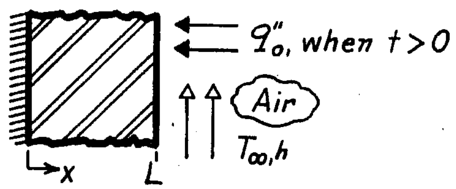
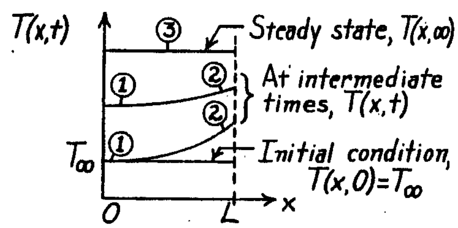
PROBLEM 5.2

KNOWN: Plane wall whose inner surface is insulated and outer surface is exposed to an airstream at T_∞. Initially, the wall is at a uniform temperature equal to that of the airstream. Suddenly, a radiant source is switched on applying a uniform flux, q''_o, to the outer surface.

FIND: (a) Sketch temperature distribution on T-x coordinates for initial, steady-state, and two intermediate times, (b) Sketch heat flux at the outer surface, $q''_x(L,t)$, as a function of time.

SCHEMATIC:

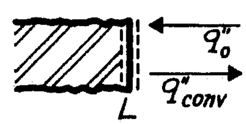




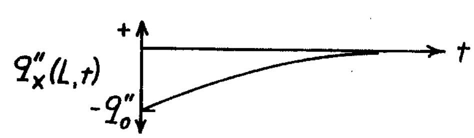
ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal generation, $\dot{E}_g=0$, (4) Surface at x=0 is perfectly insulated, (5) All incident radiant power is absorbed, negligible radiation exchange with surroundings.

ANALYSIS: (a) The temperature distributions are shown on the T-x coordinates and labeled accordingly. Note these special features: ① Gradient at x=0 is always zero, ② gradient is more steep at early times and ③ for steady-state conditions, the radiant flux is equal to the convective heat flux; this follows from an energy balance on the CS at x=L,

$$q''_o = q''_{conv} = h[T(L,\infty)-T_\infty] .$$



(b) The heat flux at the outer surface, $q''_x(L,t)$, as a function of time appears as shown below.



COMMENTS: The sketch must reflect the initial and boundary conditions:

$T(x,0) = T_\infty$ uniform initial temperature.

$-k\frac{\partial T}{\partial x}\Big|_{x=0} = 0$ insulated at x=0.

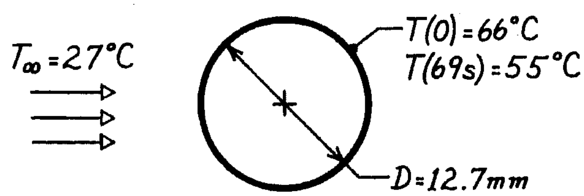
$-k\frac{\partial T}{\partial x}\Big|_{x=L} = h[T(L,t)-T_\infty] - q''_o$ surface energy balance at x=L.

PROBLEM 5.6

KNOWN: The temperature-time history of a pure copper sphere in an air stream.

FIND: The heat transfer coefficient between the sphere and the air stream.

SCHEMATIC:



ASSUMPTIONS: (1) Temperature of sphere is spatially uniform, (2) Negligible radiation exchange, (3) Constant properties.

PROPERTIES: *Table A-1*, Pure copper (333K): $\rho = 8933\ \text{kg/m}^3$, $c_p = 389\ \text{J/kg·K}$, $k = 398\ \text{W/m·K}$.

ANALYSIS: The time-temperature history is given by Eq. 5.6 with Eq. 5.7.

$$\frac{\theta(t)}{\theta_i} = \exp\left(-\frac{t}{R_t C_t}\right) \qquad \text{where} \qquad R_t = \frac{1}{hA_s} \qquad A_s = \pi D^2$$

$$C_t = \rho V c_p \qquad V = \frac{\pi D^3}{6}$$

$$\theta = T - T_\infty .$$

Recognize that when $t = 69\text{s}$,

$$\frac{\theta(t)}{\theta_i} = \frac{(55-27)\,^\circ\text{C}}{(66-27)\,^\circ\text{C}} = 0.718 = \exp\left(-\frac{t}{\tau_t}\right) = \exp\left(-\frac{69\text{s}}{\tau_t}\right)$$

and noting that $\tau_t = R_t C_t$ find

$$\tau_t = 208\text{s} .$$

Hence,

$$h = \frac{\rho V c_p}{A_s \tau_t} = \frac{8933\ \text{kg/m}^3\ (\pi 0.0127^3\ \text{m}^3/6)\ 389\text{J/kg·K}}{\pi 0.0127^2 \text{m}^2 \times 208\text{s}}$$

$$h = 35.3\ \text{W/m}^2\text{·K}. \qquad \triangleleft$$

COMMENTS: Note that with $L_c = D_o/6$,

$$Bi = \frac{hL_c}{k} = 35.3\ \text{W/m}^2\text{·K} \times \frac{0.0127}{6}\ \text{m}/398\ \text{W/m·K} = 1.88\times10^{-4} .$$

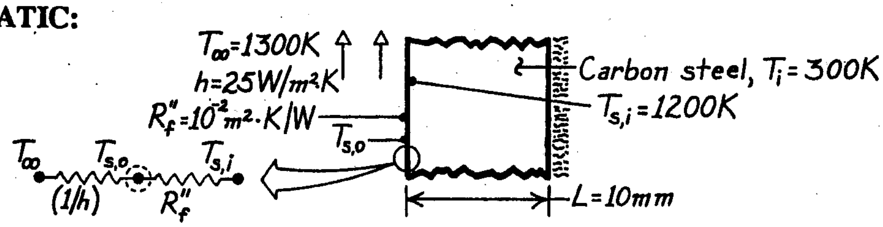
Hence $Bi < 0.1$ and the spatially isothermal assumption is reasonable.

PROBLEM 5.14

KNOWN: Thickness and properties of furnace wall. Thermal resistance of film on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of film surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible film thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel (given): $\rho = 7850\ \text{kg/m}^3$, $c = 430$ J/kg·K, $k = 60$ W/m·K.

ANALYSIS: The overall coefficient for heat transfer from the surface of the steel to the gas is

$$U = (R''_{tot})^{-1} = \left(\frac{1}{h} + R''_f\right)^{-1} = \left(\frac{1}{25\ \text{W/m}^2\cdot\text{K}} + 10^{-2}\text{m}^2\cdot\text{K/W}\right)^{-1} = 20\ \text{W/m}^2\cdot\text{K}.$$

Hence,

$$Bi = \frac{UL}{k} = \frac{20\ \text{W/m}^2\cdot\text{K}\times 0.01\ \text{m}}{60\ \text{W/m}\cdot\text{K}} = 0.0033$$

and the lumped capacitance method can be used.

(a) It follows that

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-t/\tau_t) = \exp(-t/RC) = \exp(-Ut/\rho Lc)$$

$$t = -\frac{\rho Lc}{U}\ln\frac{T - T_\infty}{T_i - T_\infty} = -\frac{7850\ \text{kg/m}^3(0.01\ \text{m})430\ \text{J/kg}\cdot\text{K}}{20\ \text{W/m}^2\cdot\text{K}}\ln\frac{1200 - 1300}{300 - 1300}$$

$t = 3886\text{s} = 1.08\text{h}.$ ◁

(b) Performing an energy balance at the outer surface (s,o),

$$h(T_\infty - T_{s,o}) = (T_{s,o} - T_{s,i})/R''_f$$

$$T_{s,o} = \frac{hT_\infty + T_{s,i}/R''_f}{h + (1/R''_f)} = \frac{25\ \text{W/m}^2\cdot\text{K}\times 1300\ \text{K} + 1200\ \text{K}/10^{-2}\text{m}^2\cdot\text{K/W}}{(25 + 100)\text{W/m}^2\cdot\text{K}}$$

$T_{s,o} = 1220$ K. ◁

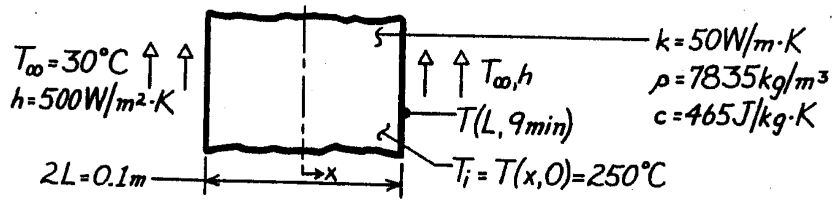
COMMENTS: The film increases τ_t by increasing R_t but not C_t.

PROBLEM 5.31

KNOWN: Plane wall, initially at a uniform temperature, is suddenly immersed in an oil bath and subjected to a convection cooling process.

FIND: Surface temperature of the wall nine minutes after immersion, T(L,9 min).

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in wall, (2) Constant properties.

ANALYSIS: The Biot number for the plane wall is

$$Bi = \frac{hL_c}{k} = \frac{500\ W/m^2\cdot K \times 0.05m}{50\ W/m\cdot K} = 0.50$$

Since Bi > 0.1, lumped capacitance analysis is not appropriate. Using Figure C.1 with

$$Fo = \frac{\alpha t}{L^2} = \frac{(k/\rho c)t}{L^2} = \frac{(50\ W/m\cdot K/7835\ kg/m^3 \times 465\ J/kg\cdot K) \times (9 \times 60)s}{(0.05m)^2} = 2.96$$

and $Bi^{-1} = 1/0.50 = 2$, find

$$\frac{\theta_o}{\theta_i} = \frac{T(0,t) - T_\infty}{T_i - T_\infty} \approx 0.3\ . \qquad (1)$$

From Figure C.2 with $Bi^{-1} = 1/0.50 = 2$ and for $x/L = 1$, find

$$\frac{\theta(1,t)}{\theta_o} \approx 0.8\ . \qquad (2)$$

By combining Eqs. (1) and (2),

$$\theta(1,t) = 0.8\theta_o = 0.8(0.3\theta_i) = 0.24\theta_i\ .$$

Recalling that $\theta = T(L,t) - T_\infty$ and $\theta_i = T_i - T_\infty$, it follows that

$$T(L,t) = T_\infty + 0.24(T_i - T_\infty) = 30°C + 0.24(250-30)°C = 83°C. \qquad \triangleleft$$

COMMENTS: (1) Note that Figure C.2 provides a relationship between the temperature at any x/L and the centerline temperature as a function of only the Biot number. Figure C.1 applies to the centerline temperature which is a function of the Biot number and the Fourier number. The centerline temperature at t=9min follows from Eq. (1) with

$$T(0,t) - T_\infty = 0.3(T_i - T_\infty) = 0.3(250-30)°C = 66°C \qquad T(0,t) = 96°C.$$

(2) Since $Fo \geq 0.2$, the approximate analytical solution for θ^* is valid. From Table 5.1 with $Bi = 0.50$, find $\zeta_1 = 0.6533$ rad and $C_1 = 1.0701$. Substituting numerical values into Eqs. 5.40 and 5.41,

$$\theta_o^* = 0.303 \qquad \text{and} \qquad \theta^*(1,Fo) = 0.240.$$

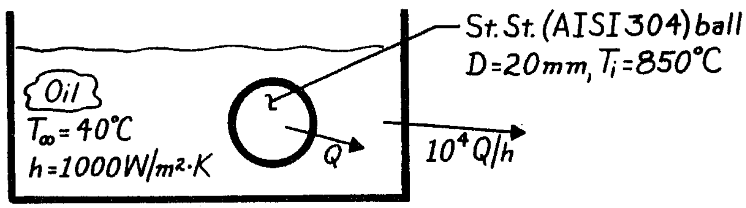
From this value, find T(L,9 min) = 83°C which is identical to graphical result.

PROBLEM 5.51

KNOWN: Diameter and initial temperature of ball bearings to be quenched in an oil bath.

FIND: (a) Time required for surface to cool to 100°C and the corresponding center temperature, (b) Oil bath cooling requirements.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction in ball bearings, (2) Constant properties.

PROPERTIES: *Table A-1,* St. St., AISI 304, ($T \approx 500°C$): $k = 22.2$ W/m·K, $c_p = 579$ J/kg·K, $\rho = 7900$ kg/m³, $\alpha = 4.85\times10^{-6}$ m²/s.

ANALYSIS: (a) To determine whether use of the lumped capacitance method is suitable, first compute

$$Bi = \frac{h(r_o/3)}{k} = \frac{1000\ W/m^2\cdot K(0.010m/3)}{22.2\ W/m\cdot K} = 0.15\ .$$

We conclude that, although the lumped capacitance method could be used as a first approximation, the Heisler charts should be used in the interest of improving accuracy. Hence, with

$$Bi^{-1} = \frac{k}{hr_o} = \frac{22.2\ W/m\cdot K}{1000\ W/m^2\cdot K(0.01m)} = 2.22 \qquad \text{and} \qquad \frac{r}{r_o} = 1\ ,$$

Fig. C.8 gives

$$\frac{\theta(r_o,t)}{\theta_o(t)} \approx 0.80\ .$$

Hence, with

$$\frac{\theta(r_o,t)}{\theta_i} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = \frac{100-40}{850-40} = 0.074\ ,$$

Continued .....

it follows that

$$\frac{\theta_o}{\theta_i} = \frac{\theta(r_o,t)/\theta_i}{\theta(r_o,t)/\theta_o} = \frac{0.074}{0.80} = 0.093 .$$

From Fig. C.7, with $\theta_o/\theta_i = 0.093$ and $Bi^{-1} = k/hr_o = 2.22$, find

$$t^* = Fo \approx 2.0$$

$$t = \frac{r_o^2 Fo}{\alpha} = \frac{(0.01m)^2(2.0)}{4.85\times10^{-6}m^2/s} = 41s.$$ ◁

Also,

$$\theta_o = T_o - T_\infty = 0.093(T_i - T_\infty) = 0.093(850-40) = 75°C$$

$$T_o = 115°C$$ ◁

(b) With $Bi^2Fo = (1/2.2)^2 \times 2.0 = 0.41$, where $Bi \equiv (hr_o/k) = 0.45$, it follows from Fig. C.9 that for a single ball

$$\frac{Q}{Q_o} \approx 0.93 .$$

Hence, from Eq. 5.44,

$$Q = 0.93\, \rho c_p V(T_i - T_\infty)$$

$$Q = 0.93 \times 7900\ kg/m^3 \times 579\ J/kg\cdot K \times \frac{\pi}{6}(0.02m)^3 \times 810°C$$

$$Q = 1.44\times10^4 J$$

is the amount of energy transferred from a single ball during the cooling process. Hence, the oil bath cooling rate must be

$$q = 10^4 Q/3600s$$

$$q = 4\times10^4 W = 40\ kW.$$ ◁

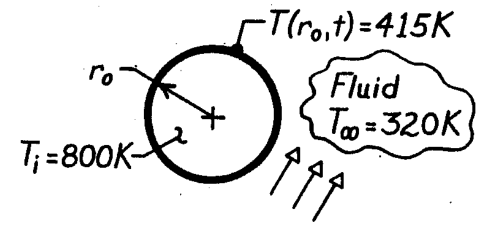
COMMENTS: If the lumped capacitance method is used, the cooling time, obtained from Eq. 5.5, would be $t = 39.7s$, where the ball is assumed to be uniformly cooled to 100°C. This result, and the fact that $T_o - T(r_o) = 15°C$ at the conclusion, suggests that use of the lumped capacitance method would have been reasonable. Note that, when using the Heisler charts, accuracy to better than 5% is seldom possible.

PROBLEM 5.54

KNOWN: Two spheres, A and B, initially at uniform temperatures of 800K and simultaneously quenched in large, constant temperature baths each maintained at 320K; properties of the spheres and convection coefficients are prescribed.

FIND: (a) Show in a qualitative manner, on T-t coordinates, temperatures at the center and the outer surface for each sphere; explain features of the curves; (b) Time required for the outer surface of each sphere to reach 415K, (c) Energy gained by each bath during process of the sphere's cooling to a surface temperature of 415K.

SCHEMATIC:



	Sphere A	Sphere B
r_o (mm)	150	15
ρ (kg/m^3)	1600	400
c (J/kg·K)	400	1600
k (W/m·K)	170	1.7
h (W/m^2·K)	5	50

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Uniform properties, (3) Constant convection coefficient.

ANALYSIS: (a) From knowledge of the Biot number and the thermal time constant, it is possible to qualitatively represent the temperature distributions. From Eq. 5.10, with $L_c = r_o/3$, find

$$Bi \equiv \frac{h(r_o/3)}{k} \qquad Bi_A = \frac{5\ W/m^2\cdot K(0.150m/3)}{170\ W/m\cdot K} = 1.47\times10^{-3} \qquad (1)$$

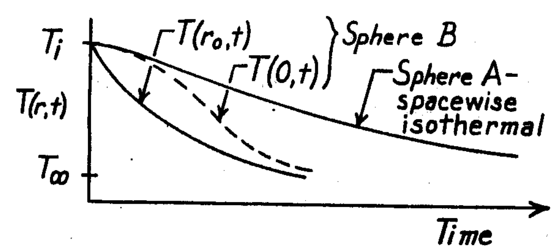
$$Bi_B = \frac{50\ W/m^2\cdot K(0.015m/3)}{1.7\ W/m\cdot K} = 0.147 \qquad (2)$$

The thermal time constant for a lumped capacitance system from Eq. 5.7 is

$$\tau = \left[\frac{1}{hA_s}\right](\rho Vc) \qquad \tau_A = \frac{1600\ kg/m^3 \times (0.150m)400\ J/kg\cdot K}{3\times5\ W/m^2\cdot K} = 6400s \qquad (3)$$

$$\tau = \frac{\rho\, r_o\, c}{3h} \qquad \tau_B = \frac{400\ kg/m^3 \times (0.015m)1600\ J/kg\cdot K}{3\times50\ W/m^2\cdot K} = 64s \qquad (4)$$

When Bi $\ll$ 0.1, the sphere will cool in a spacewise isothermal manner (Sphere A). For sphere B, Bi > 0.1, hence gradients will be important. Note that the thermal time constant of A is much larger than for B; hence, A will cool much slower. See sketch for these features.



(b) Recognizing that $Bi_A < 0.1$, Sphere A can be treated as spacewise isothermal and analyzed using the lumped capacitance method. From Eq. 5.6 and 5.7, with T = 415 K

$$\frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i-T_\infty} = \exp(-t/\tau) \qquad (5)$$

Continued .....

$$t_A = -\tau_A\left[\ln\frac{T-T_\infty}{T_i-T_\infty}\right] = -6400s\left[\ln\frac{415-320}{800-320}\right] = 10{,}367s = 2.88h. \quad \triangleleft$$

Note that since the sphere is nearly isothermal, the surface and inner temperatures are approximately the same.

Since $Bi_B > 0.1$, *Sphere B* must be treated by the Heisler chart method of solution beginning with Figure C.8 using

$$Bi_B \equiv \frac{hr_o}{k} = \frac{50\ W/m^2\cdot K \times (0.015m)}{1.7\ W/m\cdot K} = 0.44 \qquad \text{or} \qquad Bi_B^{-1} = 2.27,$$

find that for $r/r_o = 1$,

$$\frac{\theta(1,t)}{\theta_o} = \frac{T(r_o,t)-T_\infty}{\theta_o} = \frac{(415-320)}{\theta_o} \approx 0.8. \qquad (6)$$

Using Eq. (6) and Figure C.7, find the Fourier number,

$$\frac{\theta_o}{\theta_i} = \frac{(T(r_o,t)-T_\infty)/0.8}{T_i-T_\infty} = \frac{(415-320)K/0.8}{(800-320)K} = 0.25 \qquad Fo = \frac{\alpha t}{r_o^2} = 1.3.$$

$$t_B = \frac{Fo\, r_o^2}{\alpha} = \frac{1.3\ (0.015m)^2}{2.656\times10^{-6} m^2/s} = 110s = 1.8\ min \quad \triangleleft$$

where $\alpha = k/\rho c = 1.7\ W/m\cdot K/400\ kg/m^3 \times 1600\ J/kg\cdot K = 2.656\times10^{-6} m^2/s$.

(c) To determine the energy change by the spheres during the cooling process, apply the conservation of energy requirement on a time interval basis.

Sphere A:

$$E_{in} - E_{out} = \Delta E \qquad -Q_A = \Delta E = E(t) - E(0).$$

$$Q_A = \rho cV[T(t)-T_i] = 1600kg/m^3 \times 400J/kg\cdot K \times (4/3)\pi(0.150m)^3[415-800]K$$

$$Q_A = 3.483\times10^6 J. \quad \triangleleft$$

Note that this simple expression is a consequence of the spacewise isothermal behavior.

Sphere B:

$$E_{in} - E_{out} = \Delta E \qquad -Q_B = E(t) - E(0).$$

For the nonisothermal sphere, the Groeber chart, Figure C.9, can be used to evaluate Q_B. With $Bi = 0.44$ and $Bi^2Fo = (0.44)^2\times1.3 = 2.52$, find $Q/Q_o = 0.74$. The energy transfer from the sphere during the cooling process, using Eq. 5.44, is

$$Q_B = 0.74\ Q_o = 0.74[\rho cV(T_i-T_\infty)]$$

$$Q_B = 0.75 \times 400kg/m^3 \times 1600J/kg\cdot K(4/3)\pi(0.015m)^3(800-320)K = 3257\ J. \quad \triangleleft$$

COMMENTS: (1) In summary:

Sphere	$Bi = hr_o/k$	τ(s)	t(s)	Q(J)
A	4.41×10^{-3}	6400	10,370	3.48×10^6
B	0.44	64	110	3257