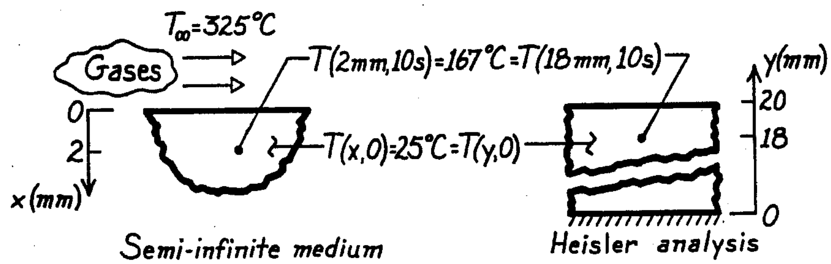
PROBLEM 5.60

KNOWN: Temperature response of an in-depth thermocouple in a block, initially at a uniform temperature, embedded in the channel suddenly exposed to hot gases.

FIND: The surface convection heat flux assuming the block is a semi-infinite medium. Compare result with that obtained from a Heisler analysis.

SCHEMATIC:



ASSUMPTIONS: (1) Block behaves as semi-infinite medium, (2) Constant properties.

PROPERTIES: Block (given): $k = 15$ W/m·K, $\alpha = 2.0\times10^{-5}$ m^2/s.

ANALYSIS: Treat the block as a semi-infinite medium, initially at a uniform temperature $T(x,0) = T_i$, and suddenly exposed to a surface convective flux (T_∞, h) (Fig. 5.7, case 3). Evaluate,

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \frac{(167-25)°C}{(325-25)°C} = 0.4733$$

$$\frac{x}{2(\alpha t)^{1/2}} = \frac{0.002 \text{ m}}{2(2\times10^{-5} \text{ m}^2/\text{s}\times10\text{s})^{1/2}} = 0.0707$$

and from Fig. 5.8, find

$$\frac{h(\alpha t)^{1/2}}{k} \approx 0.8$$

$$h = \frac{0.8\text{ k}}{(\alpha t)^{1/2}} = \frac{0.8\times15 \text{ W/m·K}}{(2\times10^{-5} \text{ m}^2/\text{s}\times10\text{s})^{1/2}} = 849 \text{ W/m}^2\text{·K}.$$

From Figure 5.8, read that at x = 0,

$$\frac{T(0,t)-T_i}{T_\infty - T_i} \approx 0.52 \qquad \text{or} \qquad T(0,10\text{s}) = 181°C.$$

Hence, the surface convective heat flux at t = 10 s is

$$q''_x(0,10\text{s}) = h[T_\infty - T(0,t)] \tag{1}$$

$$q''_x(0,10\text{s}) = 849 \text{ W/m}^2\text{·K}[325-181]°C = 122.3\text{kW/m}^2. \quad \triangleleft$$

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Treating the block as a plane wall, consider the coordinate system shown above using the approximate solution, Eq. 5.40,

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo) \cos \zeta_1 y^* \tag{2}$$

where

$$\theta^* = \frac{T(y^*, 10s) - T_\infty}{T_i - T_\infty} = \frac{(167 - 325)°C}{(25 - 325)°C} = 0.527$$

$$Fo = \frac{\alpha t}{L^2} = \frac{2\times10^{-5}\ m^2/s \times 10s}{(0.020\ m)^2} = 0.50$$

then

$$0.527 = C_1 \exp(-\zeta_1^2 \times 0.50) \cos(0.9\zeta_1) \tag{3}$$

where $y^* = 18/20 = 0.9$ and $(C_1, \zeta_1) = f(Bi)$. Since $Bi = hL/k$ and h is presently unknown, an iterative solution is necessary. Let $h = 850\ W/m^2 \cdot K$ for which

$$Bi = 850\ W/m^2 \cdot K \times 0.020\ m / 15\ W/m \cdot K = 1.133.$$

Find values for C_1 and ζ_1 from Table 5.1 to see if Eq. (3) is satisfied. Here are several tabulated results where RHS denotes the RHS of Eq. (3):

Bi	ζ_1 (rad)	C_1	RHS
1.0	0.8603	1.1191	0.553
0.9	0.8274	1.1107	0.580
2.0	1.0769	1.1795	0.374

By linear interpolation, find $Bi \approx 1.145$ or $h = 859\ W/m^2 \cdot K$. The surface temperature, from Eq. 5.40 with $y^* = 1$ is

$$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo) \cos(1 \times \zeta_1)$$

$$\theta_o^* = \frac{T(0, 10s) - 325}{25 - 325} = 1.1279 \exp(-0.8917^2 \times 0.5) \cos(0.8917) = 0.4760$$

$$T(20mm, 10s) = 182°C$$

where $C_1 = 1.1279$ and $\zeta_1 = 0.8917$ determined from interpolation in Table 5.1 with $Bi = 1.145$. Hence the convection heat flux is

$$q_x''(20mm, 10s) = h[T_\infty - T(20mm, 10s)]$$

$$q_x'' = 859\ W/m^2 (325 - 182)°C = 122.8\ kW/m^2.$$ ◁

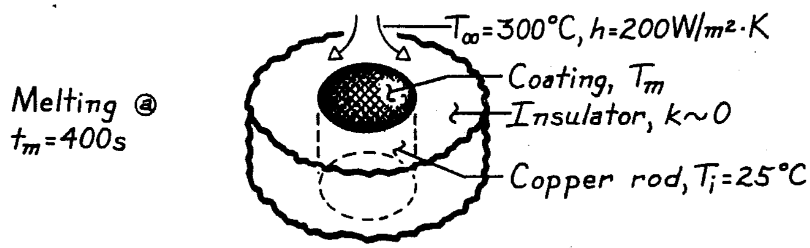
COMMENTS: For these conditions, the block behavior does approximate that of a semi-infinite medium. This method is used to determine surface convective heat flux in many applications. The purpose of imbedding the thermocouple is to provide protection from the gas stream.

PROBLEM 5.63

KNOWN: Procedure for measuring convection heat transfer coefficient, which involves melting of a surface coating.

FIND: Melting point of coating for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in solid rod (negligible losses to insulation), (2) Rod approximation as semi-infinite medium, (3) Negligible surface radiation, (4) Constant properties, (5) Negligible thermal resistance of coating.

PROPERTIES: Copper rod (Given): k = 400 W/m·K, $\alpha = 10^{-4}$ m²/s.

ANALYSIS: Problem corresponds to transient conduction in a semi-infinite solid. Thermal response is given by

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}}\right) - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}}+\frac{h(\alpha t)^{1/2}}{k}\right)\right].$$

For x = 0, erfc(0) = 1 and $T(x,t) = T(0,t) = T_s$. Hence

$$\frac{T_s - T_i}{T_\infty - T_i} = 1 - \exp\left(\frac{h^2\alpha t}{k^2}\right)\operatorname{erfc}\left(\frac{h(\alpha t)^{1/2}}{k}\right)$$

with

$$\frac{h(\alpha t_m)^{1/2}}{k} = \frac{200\ \text{W/m}^2\cdot\text{K}(10^{-4}\text{m}^2/\text{s}\times 400\ \text{s})^{1/2}}{400\ \text{W/m}\cdot\text{K}} = 0.1$$

$$T_s = T_m = T_i + (T_\infty - T_i)[1-\exp(0.01)\operatorname{erfc}(0.1)]$$

$$T_s = 25°\text{C} + 275°\text{C}[1 - 1.01\times 0.888] = 53.5°\text{C}. \qquad \triangleleft$$

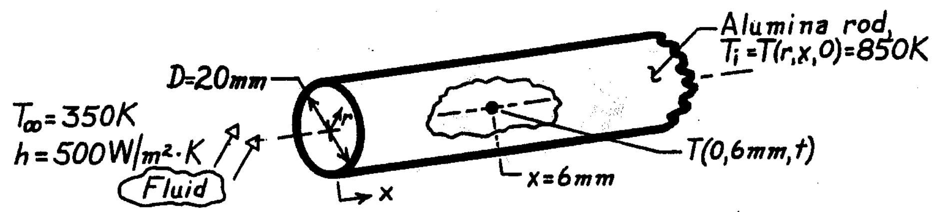
COMMENTS: Use of the procedure to evaluate h from measurement of t_m necessitates iterative calculations.

PROBLEM 5.77

KNOWN: A long alumina rod, initially at a uniform temperature of 850K, is suddenly exposed to a cooler fluid.

FIND: Temperature of the rod after 30s, at an exposed end, T(0,0,t), and at an axial distance 6mm from the end, T(0, 6mm, t).

SCHEMATIC:



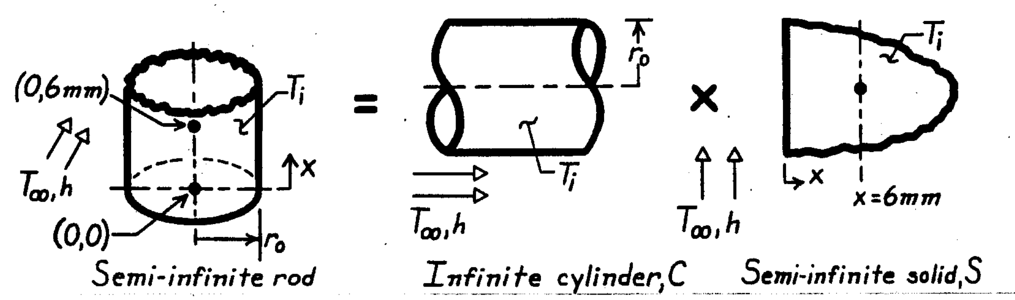
ASSUMPTIONS: (1) Two-dimensional conduction in (r,x) directions, (2) Constant properties, (3) Convection coefficient is same on end and cylindrical surfaces.

PROPERTIES: *Table A-2,* Alumina, polycrystalline aluminum oxide (assume $\bar{T} \approx (850+600)K/2 = 725K$): $\rho = 3970\ kg/m^3$, $c = 1154\ J/kg \cdot K$, $k = 12.4\ W/m \cdot K$.

ANALYSIS: First, check if system behaves as a lumped capacitance. Find

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{500\ W/m \cdot K\ (0.010m/2)}{12.4\ W/m \cdot K} = 0.202 .$$

Since Bi > 0.1, rod does not behave as spacewise isothermal object. Hence, treat rod as a semi-infinite cylinder, the multi-dimensional system Case (f), Fig. 5.11.



The product solution can be written as

$$\theta^*(r,x,t) = \frac{\theta(r,x,t)}{\theta_i} = \frac{\theta(r,t)}{\theta_i} \times \frac{\theta(x,t)}{\theta_i} = C(r^*,t^*) \times S(x^*,t^*)$$

Infinite cylinder, $C(r^*,t^*)$. Using the Heisler charts with $r^* = r = 0$ and

$$Bi^{-1} = \left[\frac{h\, r_o}{k}\right]^{-1} = \left[\frac{500\ W/m^2 \cdot K \times 0.01m}{12.4\ W/m \cdot K}\right]^{-1} = 2.48 .$$

Evaluate $\alpha = k/\rho c = 2.71 \times 10^{-6} m^2/s$, find $Fo = \alpha t/r_o^2 = 2.71 \times 10^{-6} m^2/s \times 30s/(0.01m)^2 = 0.812$. From the Heisler chart, Fig. C.4, with $Bi^{-1} = 2.48$ and $Fo = 0.812$, read $C(0,t^*) = \theta(0,t)/\theta_i = 0.61$.

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Semi-infinite medium, $S(x^*, t^*)$. Recognize this as Case (3), Fig. 5.7. From Eq. 5.60, note that the LHS needs to be transformed as follows,

$$\frac{T-T_i}{T_\infty-T_i} = 1 - \frac{T-T_\infty}{T_i-T_\infty} \qquad S(x,t) = \frac{T-T_\infty}{T_i-T_\infty}.$$

Thus,

$$S(x,t) = 1 - \left\{ \text{erfc}\left[\frac{x}{2(\alpha t)^{1/2}}\right] - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\right]\left[\text{erfc}\left(\frac{x}{2(\alpha t)^{1/2}} + \frac{h(\alpha t)^{1/2}}{k}\right)\right]\right\}.$$

Evaluating this expression at the surface (x=0) and 6mm from the exposed end, find

$$S(0,30s) = 1 - \left\{ \text{erfc}(0) - \left[\exp\left(0 + \frac{(500\ W/m^2\cdot K)^2\ 2.71\times10^{-6}\ m^2/s \times 30s}{(12.4\ W/m\cdot K)^2}\right)\right]\right.$$

$$\left.\left[\text{erfc}\left(0 + \frac{500\ W/m^2\cdot K\ (2.71\times10^{-6}\ m^2/s \times 30s)^{1/2}}{12.4\ W/m\cdot K}\right)\right]\right\}$$

$$S(0,30s) = 1 - \left\{1 - [\exp(0.1322)]\ [\text{erfc}(0.3636)]\right\} = 0.693.$$

Note that Table B.2 was used to evaluate the complementary error function, erfc(w).

$$S(6mm, 30s) = 1 - \left\{ \text{erfc}\left[\frac{0.006m}{2(2.71\times10^{-6}\ m^2/s\times30s)^{1/2}}\right] - \right.$$

$$\left.\left[\exp\left(\frac{500W/m^2\cdot K\times0.006m}{12.4\ W/m\cdot K} + 0.1322\right)\right]\ [\text{erfc}(0.3327 + 0.3636)]\right\} = 0.835.$$

The product solution can now be evaluated for each location. At (0,0),

$$\theta^*(0,0,t) = \frac{T(0,0,30s)-T_\infty}{T_i-T_\infty} = C(0,t^*) \times S(0,t^*) = 0.61 \times 0.693 = 0.423.$$

Hence,

$$T(0,0,30s) = T_\infty + 0.423(T_i-T_\infty) = 350K + 0.423(850-350)K = 561K.$$ ◁

At (0,6mm),

$$\theta^*(0,6mm,t) = C(0,t^*) \times S(6mm,t^*) = 0.61 \times 0.835 = 0.509$$

$$T(0,6mm,30s) = 604K.$$ ◁

COMMENTS: Note that the temperature at which the properties were evaluated was a good estimate.