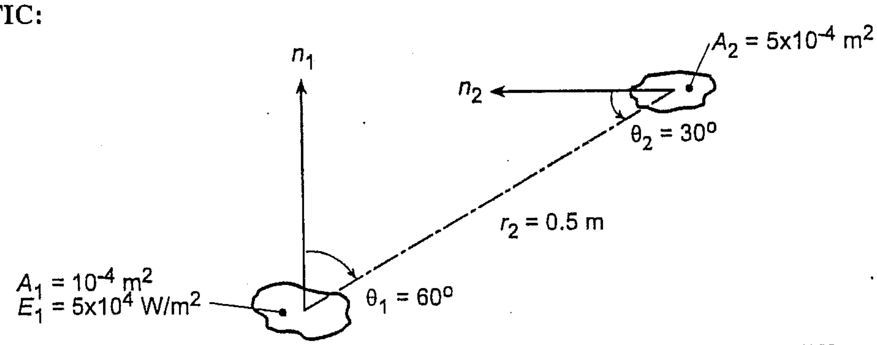
PROBLEM 12.2

KNOWN: A diffuse surface of area $A_1 = 10^{-4} m^2$ emits diffusely with total emissive power $E = 5 \times 10^4$ W/m^2.

FIND: (a) Rate this emission is intercepted by small surface of area $A_2 = 5 \times 10^{-4}$ m^2 at a prescribed location and orientation, (b) Irradiation G_2 on A_2, and (c) Compute and plot G_2 as a function of the separation distance r_2 for the range $0.25 \le r_2 \le 1.0$ m for zenith angles $\theta_2 = 0$, 30 and 60°.

SCHEMATIC:



ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) A_1 may be approximated as a differential surface area and that $A_2/r_2^2 << 1$.

ANALYSIS: (a) The rate at which emission from A_1 is intercepted by A_2 follows from Eq. 12.5 written on a total rather than spectral basis.

$$q_{1\to2} = I_{e,1}(\theta,\phi)A_1\cos\theta_1 d\omega_{2-1}. \quad (1)$$

Since the surface A_1 is diffuse, it follows from Eq. 12.13 that

$$I_{e,1}(\theta,\phi) = I_{e,1} = E_1/\pi. \quad (2)$$

The solid angle subtended by A_2 with respect to A_1 is

$$d\omega_{2-1} \approx A_2 \cdot \cos\theta_2/r_2^2. \quad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1) with numerical values gives

$$q_{1\to2} = \frac{E_1}{\pi}\cdot A_1\cos\theta_1\cdot\frac{A_2\cos\theta_2}{r_2^2} = \frac{5\times10^4\,W/m^2}{\pi\,sr}\times\left(10^{-4}m^2\times\cos60°\right)\times\left[\frac{5\times10^{-4}m^2\times\cos30°}{(0.5m)^2}\right]sr \quad (4)$$

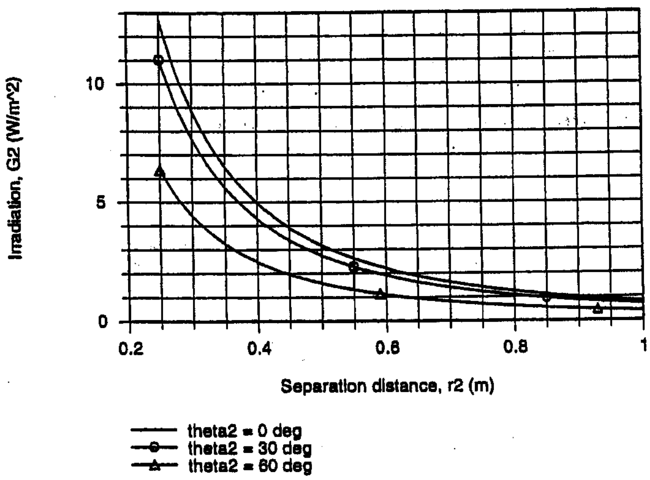
$$q_{1\to2} = 15{,}915\,W/m^2sr\times\left(5\times10^{-5}m^2\right)\times1.732\times10^{-3}sr = 1.378\times10^{-3}\,W. \quad <$$

(b) From section 12, 2.3, the irradiation is the rate at which radiation is incident upon the surface per unit surface area,

$$G_2 = \frac{q_{1\to2}}{A_2} = \frac{1.378\times10^{-3}W}{5\times10^{-4}m^2} = 2.76\,W/m^2 \quad (5) <$$

(c) Using the IHT workspace with the foregoing equations, the G_2 was computed as a function of the separation distance for selected zenith angles. The results are plotted below.

Continued...



For all zenith angles, G_2 decreases with increasing separation distance r_2. From Eq. (3), note that $d\omega_{2\text{-}1}$ and, hence G_2, vary inversely as the square of the separation distance. For any fixed separation distance, G_2 is a maximum when $\theta_2 = 0°$ and decreases with increasing θ_2, proportional to $\cos\theta_2$.

COMMENTS: (1) For a diffuse surface, the intensity, I_e, is independent of direction and related to the emissive power as $I_e = E/\pi$. Note that π has the units of $[sr]$ in this relation.

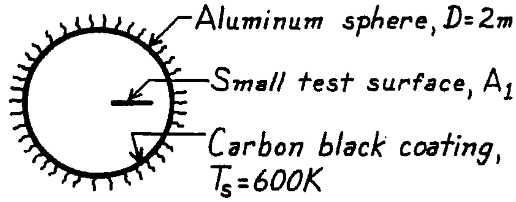
(2) Note that Eq. 12.5 is an important relation for determining the radiant power leaving a surface in a prescribed manner. It has been used here on a total rather than spectral basis.

PROBLEM 12.15

KNOWN: Evacuated, aluminum sphere (D = 2m) serving as a radiation test chamber.

FIND: Irradiation on a small test object when the inner surface is lined with carbon black and at 600K. What effect will surface coating have?

SCHEMATIC:



ASSUMPTIONS: (1) Sphere walls are isothermal, (2) Test surface area is small compared to the enclosure surface.

ANALYSIS: It follows from the discussion of Section 13.3 that this isothermal sphere is an enclosure behaving as a blackbody. For such a condition, see Fig. 12.12(c), the irradiation on a small surface within the enclosure is equal to the blackbody emissive power at the temperature of the enclosure. That is,

$$G_1 = E_b(T_s) = \sigma T_s^4$$

$$G_1 = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(600\mathrm{K})^4 = 7348\,\mathrm{W/m^2}. \quad \triangleleft$$

The irradiation is independent of the nature of the enclosure surface coating properties.

COMMENTS: (1) The irradiation depends only upon the enclosure surface temperature and is independent of the enclosure surface properties.

(2) Note that the test surface area must be small compared to the enclosure surface area. This allows for inter-reflections to occur such that the radiation field, within the enclosure will be uniform (diffuse) or isotropic.

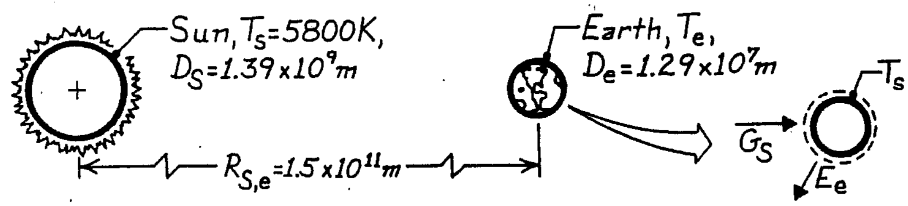
(3) The irradiation level would be the same if the enclosure were not evacuated since, in general, air would be a non-participating medium.

PROBLEM 12.17

KNOWN: Sun has equivalent blackbody temperature of 5800 K. Diameters of sun and earth as well as separation distance are prescribed.

FIND: Temperature of the earth assuming the earth is black.

SCHEMATIC:



ASSUMPTIONS: (1) Sun and earth emit as blackbodies, (2) No attenuation of solar irradiation enroute to earth, and (3) Earth atmosphere has no effect on earth energy balance.

ANALYSIS: Performing an energy balance on the earth,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_{e,p} \cdot G_S = A_{e,s} \cdot E_b(T_e)$$

$$(\pi D_e^2/4) G_S = \pi D_e^2 \sigma T_e^4 \qquad \pi r_e^2 G_S = 4\pi r_e^2 \sigma T_e^4$$

$$T_e = (G_S/4\sigma)^{1/4}$$

where $A_{e,p}$ and $A_{e,s}$ are the projected area and total surface area of the earth, respectively. To determine the irradiation G_S at the earth's surface, perform an energy balance on the control volume bounded by the spherical surface shown in the sketch.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\pi D_S^2 \cdot \sigma T_S^4 = 4\pi [R_{S,e} - D_e/2]^2 G_S \qquad G_S = \frac{q_{S \to E}}{A_{S \to E}}$$

$$\pi (1.39 \times 10^9 \text{ m})^2 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (5800 \text{ K})^4$$

$$= 4\pi [1.5 \times 10^{11} - 1.29 \times 10^7 /2]^2 \text{m}^2 \times G_S$$

$$G_S = 1377.5 \text{ W/m}^2.$$

Substituting numerical values, find

$$T_e = (1377.5 \text{ W/m}^2 / 4 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)^{1/4} = 279 \text{ K}. \qquad \triangleleft$$

COMMENTS: (1) The average earth's temperature is greater than 279 K since the effect of the atmosphere is to reduce the heat loss by radiation.

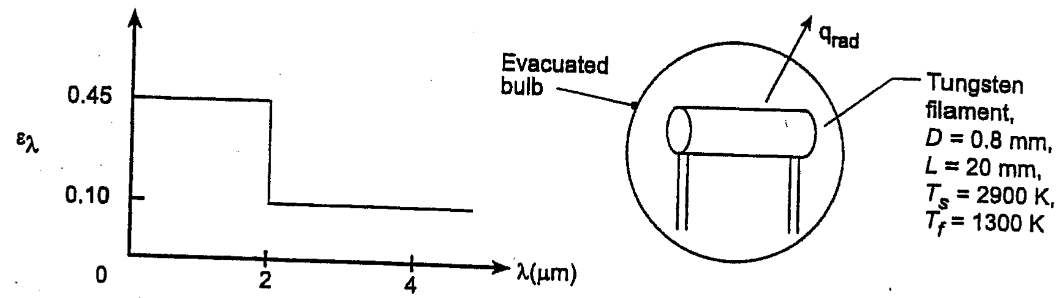
(2) Note carefully the different areas used in the earth energy balance. Emission occurs from the total spherical area, while solar irradiation is absorbed by the projected spherical area.

PROBLEM 12.25

KNOWN: Spectral emissivity, dimensions and initial temperature of a tungsten filament.

FIND: (a) Total hemispherical emissivity, ε, when filament temperature is $T_s = 2900$ K; (b) Initial rate of cooling, dT_s/dt, assuming the surroundings are at $T_{sur} = 300$ K when the current is switched off; (c) Compute and plot ε as a function of T_s for the range $1300 \le T_s \le 2900$ K; and (d) Time required for the filament to cool from 2900 to 1300 K.

SCHEMATIC:



ASSUMPTIONS: (1) Filament temperature is uniform at any time (lumped capacitance), (2) Negligible heat loss by conduction through the support posts, (3) Surroundings large compared to the filament, (4) Spectral emissivity, density and specific heat content over the temperature range, (5) Negligible convection.

PROPERTIES: *Table A-1*, Tungsten (2900 K); $\rho = 19{,}300\ \mathrm{kg/m^3}$, $c_p \approx 185\ \mathrm{J/kg\cdot K}$.

ANALYSIS: (a) The total emissivity at Ts = 2900 K follows from Eq. 12.38 using Table 12.1 for the band emission factors,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_s)d\lambda = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2(1 - F_{0\to 2\mu m}) \qquad (1)$$

$$\varepsilon = 0.45 \times 0.72 + 0.1\,(1 - 0.72) = 0.352 \qquad <$$

where $F_{(0\to 2\mu m)} = 0.72$ at $\lambda T = 2\mu m \times 2900\ K = 5800\ \mu m\cdot K$.

(b) Perform an energy balance on the filament at the instant of time at which the current is switched off,

$$\dot{E}_{in} - \dot{E}_{out} = Mc_p \frac{dT_s}{dt}$$

$$-q_{rad} = -\varepsilon A_s \sigma\left(T_s^4 - T_{sur}^4\right) = Mc_p\, dT_s/dt$$

and find the change in temperature with time where $A_s = \pi DL$, $M = \rho\forall$, and $\forall = (\pi D^2/4)L$,

$$\frac{dT_s}{dt} = -\frac{\varepsilon\pi DL\sigma(T_s^4 - T_{sur}^4)}{\rho(\pi D^2/4)Lc_p} = -\frac{4\varepsilon\sigma}{\rho c_p D}(T_s^4 - T_{sur}^4)$$

$$\frac{dT_s}{dt} = -\frac{4\times 0.352\times 5.67\times 10^{-8}\ \mathrm{W/m^2\cdot K^4}(2900^4 - 300^4)\mathrm{K^4}}{19{,}300\ \mathrm{kg/m^2}\times 185\ \mathrm{J/kg\cdot K}\times 0.0008\mathrm{m}} = -1977\ \mathrm{K/s} \qquad <$$

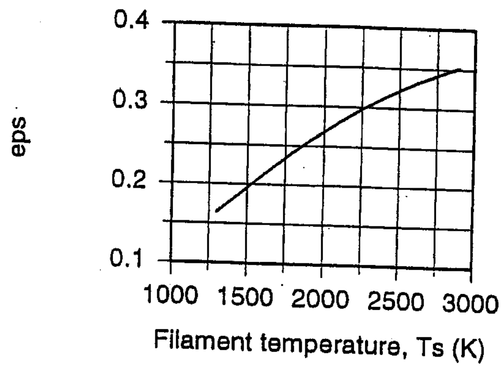
(c) Using the *IHT Tool, Radiation, Band Emission Factor*, and Eq. (1), a model was developed to calculate and plot ε as a function of T_s. See plot below.

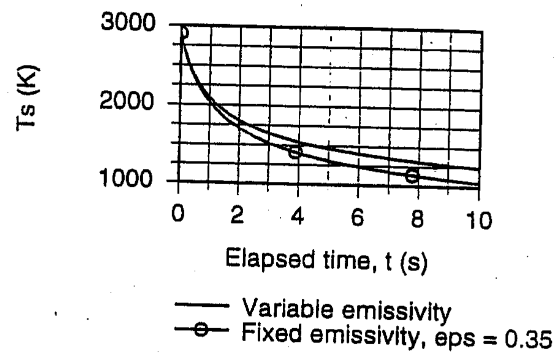
Continued...

(d) Using the IHT *Lumped Capacitance Model* along with the IHT workspace for part (c) to determine ε as a function of T_s, a model was developed to predict T_s as a function of cooling time. The results are shown below for the variable emissivity case (ε vs. T_s as per the plot below left) and the case where the emissivity is fixed at ε(2900 K) = 0.352. For the variable and fixed emissivity cases, the times to reach T_s = 1300 K are

$t_{var} = 8.3$ s $\qquad t_{fix} = 5.1$ s

<





COMMENTS: (1) From the ε vs. T_s plot, note that ε increases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution, ε_λ vs. λ?

(2) How do you explain the result that $t_{var} > t_{fix}$?

(3) The IHT workspace used to generate the T_s vs. t plot is copied below.

```
// Lumped Capacitance Model - Radiation Only
/* Conservation of energy requirement on the control volume, CV. */
Edotin - Edotout = Edotst
Edotin = 0
Edotout = As * ( + q"rad )
Edotst = rho * vol * cp * Der(Ts,t)
// Radiation exchange between CS and large surroundings
q"rad = eps * sigma * ( Ts^4 - Tsur^4)
sigma = 5.67e-8  // Stefan-Boltzmann constant, W/m^2·K^4
/* The independent variables for this system and their assigned numerical values are */
As =   pi * D * L            // surface area, m^2
vol =   pi * D^2 / 4 * L     // vol, m^3
D = 0.0008                   // filament diameter, m
L = 0.020                    // filament length, m
rho =  19300                 // density, kg/m^3
cp =    185                  // specific heat, J/kg·K
// Radiation exchange with large surroundings, CS
//eps =                      // emissivity; from band emission tool below
Tsur = 300                   / surroundings temperature, K
// Radiation Tool - Band Emission Factor
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Ts = F_lambda_T(lambda1,Ts)          // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
// Emissivity Calculation:
eps = eps1 * FL1Ts  +  eps2 * (1 - FL1Ts)        // Eq (1)
// Other Assigned Variables
eps1 = 0.45                  // Spectral emissivity; lambda1 <= 2 mum
eps2 = 0.1                   // Spectral emissivity; lambda1 >= 2 mum
lambda1 = 2                  // Wavelength, mum
```