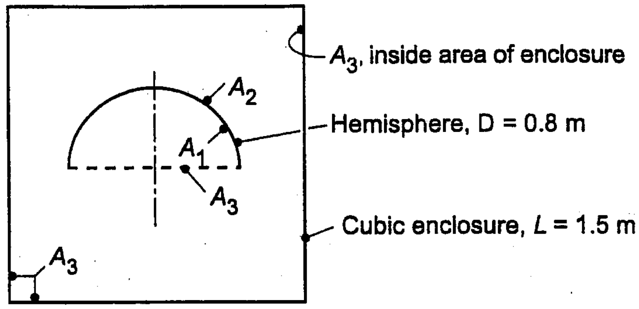
KNOWN: Thin hemispherical shell inside of a 1.5 m-cubical enclosure.

FIND: View factors F_{11}, F_{22} and F_{33}.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces, (2) Wall of hemisphere is very thin.

ANALYSIS: Define the surface $A_{3'}$ as the plane circular area at the base of the hemisphere. The areas are,

$$A_1 = A_2 = \pi D^2/2 = \pi(0.8\,\text{m})^2/2 = 1.005\,\text{m}^2$$

$$A_{3'} = \pi D^2/4 = 0.503\,\text{m}^2$$

$$A_3 = 6L^2 = 6(1.5\text{m})^2 = 13.500\,\text{m}^2$$

All the radiation leaving A_1 passes through $A_{3'}$ and intercepts A_3. Hence,

$$F_{13} = F_{13'} \qquad (1)$$

and using reciprocity,

$$F_{13'} = A_{3'}F_{3'1}/A_1 = 0.503\,\text{m}^2 \times 1.0/1.005 = 0.500 \qquad (2)$$

where, by inspection, $F_{3'1} = 1$. From the summation rule on A_1,

$$F_{11} + F_{12} + F_{13} = 0$$

$$F_{11} = 1 - F_{12} - F_{13} = 1 - 0 - 0.500 = 0.500 \qquad (3) <$$

since $F_{12} = 0$. By inspection,

$$F_{22} = 0 \qquad <$$

Using reciprocity, where $F_{23} = 1$, and then the summation rule on A_3

$$F_{32} = A_2F_{23}/A_3 = 0.074 \qquad (4)$$

$$F_{31} = A_1F_{13}/A_3 = 0.037 \qquad (5)$$

$$F_{33} = 1 - F_{31} - F_{32} = 1 - 0.037 - 0.074 = 0.889 \qquad <$$

Continued...

COMMENTS: An alternative solution begins with the summation rules for A_1, A_2, A_3 identifying those view factors determined by inspection. For the first relation below, the enclosure is within the hemisphere.

$$F_{11} + F_{12} + F_{13'} = 1$$

$$F_{21} + F_{22} + F_{23'} = 1$$

$$F_{31} + F_{32} + F_{33} = 1$$

By inspection, it follows that

$$F_{13'} = F_{13} \qquad\qquad F_{22} = 0 \qquad <$$

Using the reciprocity rule for three different combinations; *Areas A_1 and $A_{3'}$*

$$A_1 F_{13'} = A_{3'} F_{3'1}$$

where $F_{3'1} = 1$, find $F_{13'} = 0.500$ and hence

$$F_{11} = 1 - F_{13'} = 0.500 \qquad <$$

Areas A_1 and A_3:

$$A_3 F_{31} = A_1 F_{13}$$

where $F_{13} = F_{13'} = 0.500$, find $F_{31} = 0.037$. *Areas A_2 and A_3*:

$$A_2 F_{23} = A_3 F_{32}$$

where $F_{23} = 1$, find $F_{32} = 0.074$

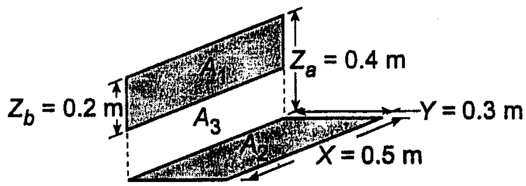
From the summation rule for A_3,

$$F_{33} = 1 - F_{31} - F_{32} = 1 - 0.037 - 0.074 = 0.889 \qquad <$$

KNOWN: Two perpendicular rectangles not having a common edge.

FIND: (a) Shape factor, F_{12}, and (b) Compute and plot F_{12} as a function of Z_b for $0.05 \le Z_b \le 0.4$ m; compare results with the view factor obtained from the two-dimensional relation for perpendicular plates with a common edge, Table 13.1

SCHEMATIC:



ASSUMPTIONS: (1) All surfaces are diffuse, (2) Plane formed by $A_1 + A_3$ is perpendicular to plane of A_2.

ANALYSIS: (a) Introducing the hypothetical surface A_3, we can write

$$F_{2(3,1)} = F_{23} + F_{21} \quad (1)$$

Using Fig. 13.6, applicable to perpendicular rectangles with a common edge, find

$$F_{23} = 0.19: \quad \text{with } Y = 0.3, \quad X = 0.5, \quad Z = Z_a - Z_b = 0.2, \text{ and } \frac{Y}{X} = \frac{0.3}{0.5} = 0.6, \quad \frac{Z}{X} = \frac{0.2}{0.5} = 0.4$$

$$F_{2(3,1)} = 0.25: \quad \text{with } Y = 0.3, X = 0.5, Z_a = 0.4, \text{ and } \frac{Y}{X} = \frac{0.3}{0.5} = 0.6, \quad \frac{Z}{X} = \frac{0.4}{0.5} = 0.8$$

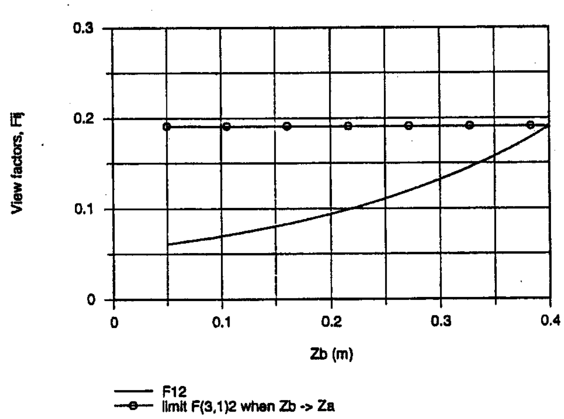
Hence from Eq. (1)

$$F_{21} = F_{2(3,1)} - F_{23} = 0.25 - 0.19 = 0.06$$

By reciprocity,

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{0.5 \times 0.3\,\text{m}^2}{0.5 \times 0.2\,\text{m}^2} \times 0.06 = 0.09 \quad (2) <$$

(b) Using the *IHT Tool - View Factors* for *Perpendicular Rectangles with a Common Edge* and Eqs (1,2) above, F_{12} was computed as a function of Z_b. Also shown on the plot below is the view factor $F_{(3,1)2}$ for the limiting case $Z_b \to Z_a$.

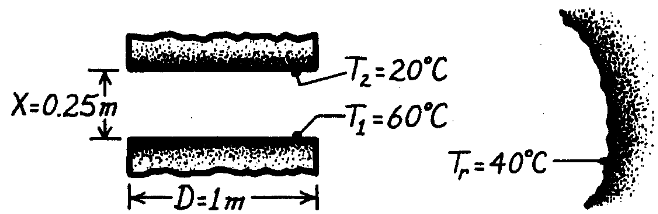


PROBLEM 13.17

KNOWN: Two black, parallel disks of 1m diameter separated by 0.25m at 60°C and 20°C are situated in a large room at 40°C.

FIND: (a) Net radiation exchange between the disks, (b) Net radiation exchange between the disks and the room.

SCHEMATIC:



ASSUMPTIONS: (1) Exterior of disks are well insulated, (b) All surfaces are black.

ANALYSIS: (a) The net heat exchange between disks 1 and 2 follows from Eq. 13.13:

$$q_{12} = A_1 F_{12} \sigma [T_1^4 - T_2^4] \quad \text{where} \quad A_1 = \pi D^2/4$$

and F_{12} follows from Fig. 13.5 with $\frac{r_i}{L} = \frac{1/2}{0.25} = 2$, $\frac{L}{r_i} = \frac{0.25}{1/2} = 0.5$ giving $F_{12} = 0.62$

$$q_{12} = (\pi\, 1^2\, m^2/4) 0.62 \times 5.667 \times 10^{-8} W/m^2 K^4 \, [(273+60)^4 + (273+20)^4] \, K^4$$

$$q_{12} = 136 W. \qquad \triangleleft$$

(b) The net exchange between the disks and the room (r) is determined from two exchange expressions:

Disk 1 to Room: $\quad q_{1r} = A_1 F_{1r} \sigma [T_1^4 - T_r^4]$
where $F_{1r} = 1 - F_{12} = 1 - 0.62 = 0.38$. Hence,

$$q_{1r} = (\pi\, 1^2\, m^2/4) 0.38 \times 5.667 \times 10^{-8} W/m^2 \cdot K^4 \, [(273+60)^4 - (273+40)^4] \, K^4$$

$$q_{1r} = 45.6 \, W. \qquad \triangleleft$$

Disk 2 to Room: $\quad q_{2r} = A_2 F_{2r} \sigma [T_2^4 - T_r^4]$
where $F_{2r} = 1 - F_{1r} = 0.38$. Hence,

$$q_{1r} = (\pi\, 1^2\, m^2/4) 0.38 \times 5.667 \times 10^{-8} W/m^2 \cdot K^4 \, [(273+20)^4 - (273+40)^4] \, K^4$$

$$q_{1r} = -37.7 \, W. \qquad \triangleleft$$

The *net* heat exchange between the disks and the room is

$$q_{(1,2)r} = q_{1r} + q_{2r} = 45.6 - 37.7 = 7.9 \, W. \qquad \triangleleft$$

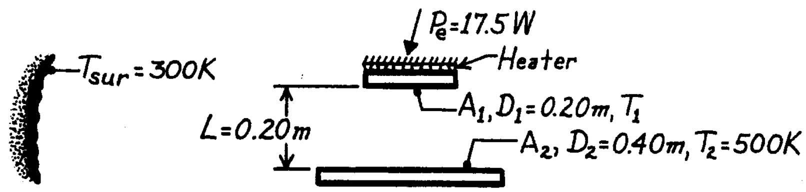
COMMENTS: How much power must be supplied or removed from each of the disks in order that they will be maintained at their respective temperatures?

PROBLEM 13.21

KNOWN: Coaxial, parallel black plates with surroundings. Lower plate (A_2) maintained at prescribed temperature T_2 while electrical power supplied to upper plate (A_1).

FIND: Temperature of the upper plate T_1.

SCHEMATIC:



ASSUMPTIONS: (1) Plates are black surfaces of uniform temperature, and (2) Backside of heater on A_1 insulated.

ANALYSIS: The net radiation heat rate leaving A_i is

$$P_e = \sum_{j=1}^{N} q_{ij} = A_1 F_{12} \sigma(T_1^4 - T_2^4) + A_1 F_{13}\sigma(T_1^4 - T_3^4)$$

$$P_e = A_1 \sigma[F_{12}(T_1^4 - T_2^4) + F_{13}(T_1^4 - T_{sur}^4) \qquad (1)$$

From Figure 13.5 for coaxial disks (see Table 13.2),

$$R_1 = r_1/L = 0.10\text{ m}/0.20\text{ m} = 0.5 \qquad R_2 = r_2/L = 0.20\text{ m}/0.20\text{ m} = 1.0$$

$$S = 1 + \frac{1+R_2^2}{R_1{}^2} = 1 + \frac{1+1^2}{(0.5)^2} = 9.0$$

$$F_{12} = \frac{1}{2}\{S - [S^2 - 4(r_2/r_1)^2]^{1/2}\} = \frac{1}{2}\{9 - [9^2 - 4(0.2/0.1)^2]^{1/2}\} = 0.469.$$

From the summation rule for the enclosure A_1, A_2 and A_3 where the last area represents the surroundings with $T_3 = T_{sur}$,

$$F_{12} + F_{13} = 1 \qquad F_{13} = 1 - F_{12} = 1 - 0.469 = 0.531.$$

Substituting numerical values into Eq. (1), with $A_1 = \pi D_1^2/4 = 3.142\times10^{-2}\text{ m}^2$,

$$17.5\text{ W} = 3.142\times10^{-2}\text{ m}^2 \times 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4[0.469(T_1^4 - 500^4)\text{K}^4$$

$$+ 0.531(T_1^4 - 300^4)\text{K}^4]$$

$$9.823\times10^9 = 0.469(T_1^4 - 500^4) + 0.531(T_1^4 - 300^4)$$

find by trail-and-error that

$$T_1 = 456\text{ K}. \qquad \triangleleft$$

COMMENTS: Note that if the upper plate were adiabatic, $T_1 = 427$ K.

Part C

(a) for surface 1

$F_{1-1} = 0$ (plane surface)

$F_{1-3} = 0$

$F_{1-4} = 0$

$\Rightarrow F_{1-2} = 1$

for surface 2

$$F_{2-1} = \frac{F_{1-2} A_1}{A_2} = \frac{1 \cdot 2}{\frac{\pi}{2} \cdot 15} = 0.085$$

$$F_{2-4} = \frac{13}{2} \cdot F_{2-1} = 0.552$$

$$F_{2-2} = 0.363$$

for surface 3

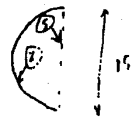
$F_{3-3} = 0$ (convex surface)

$F_{3-1} = 0$

$F_{3-2} = 0$

$F_{3-4} = 1$

Find F_{22} $F_{22} + F_{25} = 1$ by ...



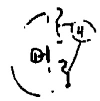
$F_{52} = 1$ by inspection

$A_5 F_{52} = A_2 F_{25}$ by reciprocity

$$F_{25} = \frac{A_5}{A_2} F_{52} \qquad A_5 = 15 \cdot L \qquad A_2 = \frac{15\pi}{2} \cdot L \quad \left(\tfrac{1}{2} \cdot 2\pi r \cdot L\right.$$

$$F_{25} = \frac{15 \cdot L}{\frac{15\pi}{2} \cdot L} (1) = \frac{2}{\pi} = 0.637$$

$$F_{22} = 1 - F_{25} = 1 - \frac{2}{\pi} = 0.363$$



$F_{25} = F_{21} + F_{24}$ (eq 13.5) $\Rightarrow F_{24} = F_{25} - F_{21}$

$A_2 F_{21} = A_1 F_{12}$ $F_{12} = 1$ by inspection

$$F_{21} = \frac{A_1}{A_2} = \frac{2 \cdot L}{\frac{15\pi}{2} \cdot L} = \frac{4}{15\pi}$$

$$F_{24} = \frac{2}{\pi} - \frac{4}{15\pi} = \frac{26}{15\pi} = 0.552$$

(b) r — reflector   steady state

s — surrounding $E_{in} = E_{out}$

h — heater

$$A_2 F_{2-1} \sigma (T_h^4 - T_r^4) = A_2 F_{2-4} \sigma (T_r^4 - T_s^4) + A_3 F_{3-4} \sigma (T_r^4 - T_s^4)$$

$$\Rightarrow T_r^4 = \frac{F_{2-1} T_h^4 + F_{2-4} T_s^4 + F_{3-4} T_s^4}{F_{2-1} + F_{2-4} + F_{3-4}}$$

$$= \frac{0.085 \cdot (1100+273)^4 + 0.552 (20+273)^4 + 1 (20+273)^4}{0.085 + 0.552 + 1}$$

$$T_r = 661.5 \text{ K} = 388.5 ^\circ\text{C}$$

(c) At steady state

$E_{in} = E_{out}$

$$A_2 F_{2-1} \sigma (T_h^4 - T_r^4) = A_2 F_{2-4} \sigma (T_r^4 - T_s^4) + A_3 T_{3-4} \sigma (T_r^4 - T_s^4) + A_2 h (T_r - T_s) + A_3 h (T_r - T_s)$$

$$F_{2-1} \sigma (T_h^4 - T_r^4) = F_{2-4} \sigma (T_r^4 - T_s^4) + F_{3-4} \sigma (T_r^4 - T_s^4) + 2h (T_r - T_s)$$

Using trial and error method

$T_r = 566.6\ K$

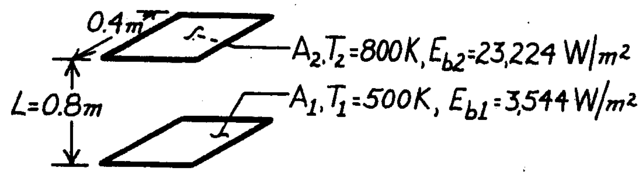
$= 293.6°C$

PROBLEM 13.68

KNOWN: Two aligned, parallel square plates with prescribed temperatures.

FIND: Net radiative transfer from surface 1 for these plate conditions: (a) black, surroundings at 0K, (b) black with connecting, re-radiating walls, (c) diffuse-gray with radiation-free surroundings at 0K, (d) diffuse-gray with re-radiating walls.

SCHEMATIC:



ASSUMPTIONS: (1) Plates are black or diffuse-gray, (2) Surroundings are at 0K.

ANALYSIS: (a) The view factor for the aligned, parallel plates follows from Figure 13.4, $X/L = 0.4\ m/0.8\ m = 0.5$, $Y/L = 0.4\ m/0.8\ m = 0.5$, $F_{12} = F_{21} \approx 0.075$. When the plates are *black with surroundings at 0K*, from Eq. 13.13,

$$q_1 = q_{12} + q_{1(sur)} = A_1 F_{12}(E_{b1} - E_{b2}) + A_1 F_{1(sur)}(E_{b1} - E_{b(sur)})$$

$$q_1 = (0.4 \times 0.4) m^2 [0.075(3,544 - 23,224) + (1 - 0.075)(3,544 - 0)] W/m^2 = 288\ W. \quad \triangleleft$$

(b) When the plates are *black with connecting re-radiating walls*, from Eq. 13.30 with $F_{1R} = R_{2R} = 1 - F_{12} = 0.925$,

$$q_1 = \frac{A_1[E_{b1} - E_{b2}]}{[F_{12} + (1/F_{1R} + 1/F_{2R})^{-1}]^{-1}} = \frac{(0.4\ m)^2[3,544 - 23,224]\ W/m^2}{[0.075 + (1/0.925 + 1/0.925)^{-1}]^{-1}} = -1,692\ W. \quad \triangleleft$$

(c) When the plates are *diffuse-gray ($\varepsilon_1 = 0.6$ and $\varepsilon_2 = 0.8$)* with the *surroundings at 0K*, using Eq. 13.20 or Eq. 13.19, with $E_{b3} = J_3 = 0$,

$$q_1 = A_1 F_{12}(J_1 - J_2) + A_1 F_{13}(J_1 - J_3) = (E_{b1} - J_1)/[(1 - \varepsilon_1)/\varepsilon_1 A_1].$$

The radiosities must be determined from energy balances, Eq. 13.21, on each of the surfaces,

$$\frac{E_{b1} - J_1}{(1 - \varepsilon_1)/\varepsilon_1} = F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3) \qquad \frac{E_{b2} - J_2}{(1 - \varepsilon_2)/\varepsilon_2} = F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)$$

$$\frac{3,544 - J_1}{(1 - 0.6)/0.6} = 0.075(J_1 - J_2) + 0.925 J_1 \qquad \frac{23,224 - J_2}{(1 - 0.8)/0.8} = 0.075(J_2 - J_1) + 0.925 J_2.$$

Find $J_1 = 2,682\ W/m^2$ and $J_2 = 18,542\ W/m^2$. Combining these results,

$$q_1 = (0.4\ m)^2(0.075)(2682 - 18,542)\ W/m^2 + (0.4\ m)^2(0.925)(2682 - 0)\ W/m^2 = 207\ W. \quad \triangleleft$$

(d) When the plates are *diffuse-gray with connecting re-radiating walls*, use Eq. 13.30,

$$q_1 = \frac{A_1[E_{b1} - E_{b2}]}{(1 - \varepsilon_1/\varepsilon_1) + [F_{12} + (1/F_{1R} + 1/F_{2R})^{-1}]^{-1} + (1 - \varepsilon_2/\varepsilon_2)}$$

$$q_1 = \frac{(0.4\ m)^2[3,544 - 23,244]\ W/m^2}{(1 - 0.6/0.6) + [0.075 + (1/0.925 + 1/0.925)^{-1}]^{-1} + (1 - 0.8/0.8)} = -1,133\ W. \quad \triangleleft$$