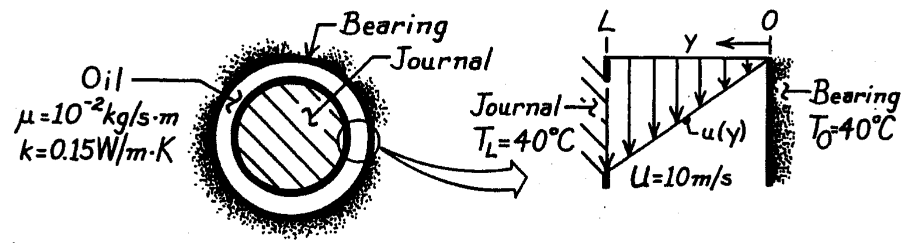
PROBLEM 6.16

KNOWN: Oil properties, journal and bearing temperatures, and journal speed for a lightly loaded journal bearing.

FIND: Maximum oil temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Clearance is much less than journal radius and flow is Couette.

ANALYSIS: The temperature distribution corresponds to the result obtained in the text Example on Couette flow,

$$T(y) = T_0 + \frac{\mu}{2k} U^2 \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right].$$

The position of maximum temperature is obtained from

$$\frac{dT}{dy} = 0 = \frac{\mu}{2k} U^2 \left[\frac{1}{L} - \frac{2y}{L^2} \right]$$

or, $y = L/2.$

The temperature is a maximum at this point since $d^2T/dy^2 < 0$. Hence,

$$T_{max} = T(L/2) = T_0 + \frac{\mu}{2k} U^2 \left[\frac{1}{2} - \frac{1}{4} \right] = T_0 + \frac{\mu U^2}{8k}.$$

$$T_{max} = 40°C + \frac{10^{-2} kg/s{\cdot}m (10 m/s)^2}{8 \times 0.15\ W/m{\cdot}K}$$

$$T_{max} = 40.83°C.$$ ◁

COMMENTS: Note that T_{max} *increases* with *increasing* μ and U, *decreases* with *increasing* k, and is *independent* of L.

PROBLEM 6.25

KNOWN: The convection conservation equations.

FIND: (a) Identify conservation equations and describe terms, (b) Identify approximations and special conditions used to reduce these equations to the boundary layer equations of Section 6.5, (c) Conditions for which momentum and energy boundary layer equations have the same form and the analogy applies.

ANALYSIS: (a) The *conservation of mass requirement* has the form

$$\underset{①}{\frac{\partial(\rho u)}{\partial x}} + \underset{②}{\frac{\partial(\rho v)}{\partial y}} = 0$$

The terms, as identified, have the following significance:

1. Net change of mass flow in the x-direction, ◁
2. Net change of mass flow in the y-direction.

The expression for *conservation of momentum* in the *x-direction* has the form

$$\rho\left[\underset{①}{u\frac{\partial u}{\partial x}} + \underset{②}{v\frac{\partial u}{\partial y}}\right] = \frac{\partial}{\partial x}(\underset{③}{\sigma_{xx}} - \underset{④}{p}) + \underset{⑤}{\frac{\partial \tau_{yx}}{\partial y}} + \underset{⑥}{X}$$

The terms, as identified, have the following significance:

1. Net rate in x-momentum of fluid leaving control volume in x-direction, ◁
2. Net rate in x-momentum of fluid leaving control volume in y-direction,
3. Change of normal viscous stresses in x-direction,
4. Change of static pressure in x-direction,
5. Change of shear stresses in x-direction,
6. Body force in the x-direction.

The expression for *conservation of energy* has the form

$$\underset{1a}{\rho u\frac{\partial i}{\partial x}} + \underset{1b}{\rho v\frac{\partial i}{\partial y}} = \underset{2a}{\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right]} + \underset{2b}{\frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right]}$$

$$+ \underset{③}{\left[u\frac{\partial p}{dx} + v\frac{\partial p}{\partial y}\right]} + \underset{④}{\mu\Phi} + \underset{⑤}{\dot{q}}$$

Continued .....

The terms, as identified, have the following significance:

1. Change of enthalpy (thermal + flow work) advected in x and y directions, ◁
2. Change of conduction rate in x and y directions,
3. Work done by static pressure forces,
4. Work done by viscous dissipation,
5. Rate of energy generation.

(b) The above conservation equations reduce to the boundary layer form when these assumptions are made

constant properties, ◁
incompressible fluid,
negligible body forces,
no energy generation,

and special conditions relating to flow near a surface. The latter are referred to as *boundary layer simplifications.*

(c) Based upon the assumptions and conditions identified above in part (b), the x-momentum and energy equations have the forms:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left[\frac{\partial u}{\partial y}\right]^2$$

The term

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

is zero for a flat plate and the term

$$\frac{\nu}{c_p}\left[\frac{\partial u}{\partial y}\right]^2$$

is negligible for low velocities or a fluid with small viscosity. For such conditions, the *x-momentum* and *energy equations* have the same form:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

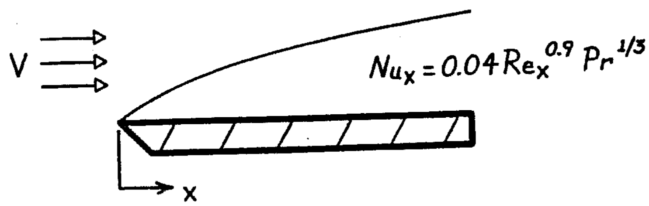
These equations establish the *analogy* between momentum and heat transfer. ◁

PROBLEM 6.30

KNOWN: Local Nusselt number correlation for flow over a roughened surface.

FIND: Ratio of average heat transfer coefficient to local coefficient.

SCHEMATIC:



ANALYSIS: The local convection coefficient is obtained from the prescribed correlation,

$$h_x = Nu_x \frac{k}{x} = 0.04 \frac{k}{x} Re_x^{0.9} Pr^{1/3}$$

$$h_x = 0.04\, k \left[\frac{V}{\nu}\right]^{0.9} Pr^{1/3} \frac{x^{0.9}}{x} \equiv C_1 x^{-0.1}.$$

To determine the average heat transfer coefficient for the length zero to x,

$$\bar{h}_x \equiv \frac{1}{x}\int_0^x h_x\, dx = \frac{1}{x} C_1 \int_0^x x^{-0.1}\, dx$$

$$\bar{h}_x = \frac{C_1}{x}\frac{x^{0.9}}{0.9} = 1.11\, C_1 x^{-0.1}.$$

Hence, the ratio of the average to local coefficient is

$$\frac{\bar{h}_x}{h_x} = \frac{1.11\, C_1 x^{-0.1}}{C_1 x^{-0.1}} = 1.11. \quad \triangleleft$$

COMMENTS: Note that $\overline{Nu}_x / Nu_x$ is also equal to 1.11. Note, however, that

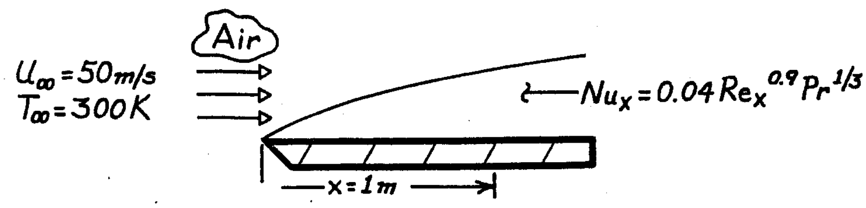
$$\overline{Nu}_x \neq \frac{1}{x}\int_0^x Nu_x\, dx.$$

PROBLEM 6.39

KNOWN: Heat transfer correlation associated with parallel flow over a rough flat plate. Velocity and temperature of air flow over the plate.

FIND: Surface shear stress 1m from the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Modified Reynolds analogy is applicable, (2) Constant properties.

PROPERTIES: *Table A-4*, Air (300K, 1atm): $\nu = 15.89\times10^{-6} m^2/s$, $Pr = 0.71$, $\rho = 1.16 kg/m^3$.

ANALYSIS: Applying the Chilton-Colburn analogy

$$\frac{C_f}{2} = St_x Pr^{2/3} = \frac{Nu_x}{Re_x\, Pr} Pr^{2/3} = \frac{0.04\, Re_x^{0.9}\, Pr^{1/3}}{Re_x\, Pr} Pr^{2/3}$$

$$\frac{C_f}{2} = 0.04\, Re_x^{-0.1}$$

where

$$Re_x = \frac{u_\infty x}{\nu} = \frac{50 m/s \times 1m}{15.89\times10^{-6} m^2/s} = 3.15\times10^6 .$$

Hence, the friction coefficient is

$$C_f = 0.08 (3.15\times10^6)^{-0.1} = 0.0179 = \tau_s/(\rho u_\infty^2/2)$$

and the surface shear stress is

$$\tau_s = C_f(\rho u_\infty^2/2) = 0.0179 \times 1.16 kg/m^3\ (50 m/s)^2/2$$

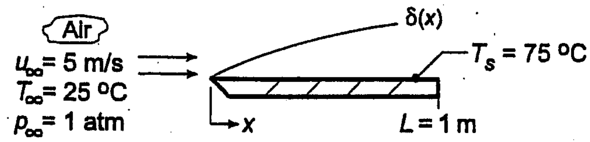
$$\tau_s = 25.96 kg/m\cdot s^2 = 25.96 N/m^2. \quad \triangleleft$$

COMMENTS: Note that turbulent flow will exist at the designated location.

KNOWN: Temperature, pressure and velocity of atmospheric air in parallel flow over a plate of prescribed length and temperature.

FIND: (a) Boundary layer thickness, surface shear stress and heat flux at trailing edge, (b) Drag force and total heat transfer per unit width of plate, and (c) Plot the parameters of part (a) as a function of distance from the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Critical Reynolds number is 5×10^5, (2) Flow over top and bottom surface.

PROPERTIES: *Table A.4*, Air (T_f = 323 K, 1 atm): $\rho = 1.085\ \text{kg/m}^3$, $\nu = 18.2 \times 10^{-6}\ \text{m}^2/\text{s}$, k = 0.028 W/m·K, Pr = 0.707.

ANALYSIS: (a) Calculate the Reynolds number to determine nature of flow,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{5\,\text{m/s} \times 1\,\text{m}}{18.2 \times 10^{-6}\,\text{m}^2/\text{s}} = 2.75 \times 10^5.$$

Hence, the flow is laminar, and at x = L, using Eqs. 7.19 and 7.20,

$$\delta = 5L\,Re_L^{-1/2} = 5 \times 1\,\text{m}\big/\left(2.75 \times 10^5\right)^{1/2} = 9.5\,\text{mm} \qquad <$$

$$\tau_{s,L} = \left(\rho u_\infty^2/2\right)0.664\,Re_L^{-1/2} = \frac{1.085}{2}\frac{\text{kg}}{\text{m}^3}(5\,\text{m/s})^2\,0.664\big/\left(2.75 \times 10^5\right)^{1/2} = 0.0172\,\text{N/m}^2 \qquad <$$

Using the appropriate correlation, Eq. 7.23,

$$Nu_L = \frac{h_L L}{k} = 0.332\,Re_L^{1/2}\,Pr^{1/3} = 0.332\left(2.75 \times 10^5\right)^{1/2}(0.707)^{1/3} = 155.1$$

$$h_L = 155.1(0.028\,\text{W/m}\cdot\text{K})/1\,\text{m} = 4.34\,\text{W/m}^2\cdot\text{K}$$

Hence, the heat flux is

$$q_s''(L) = h_L(T_s - T_\infty) = 4.34\,\text{W/m}^2\cdot\text{K}(75°\text{C} - 25°\text{C}) = 217\,\text{W/m}^2 \qquad <$$

(b) The drag force per unit plate width is $D' = 2L\bar{\tau}_{s,L}$ where the factor of two is included to account for both sides of the plate. Hence, from Eq. 7.30, with

$$\bar{\tau}_{s,L} = \left(\rho u_\infty^2/2\right)1.328\,Re_L^{-1/2} = \left(1.085\,\text{kg/m}^3/2\right)(5\,\text{m/s})^2\,1.328\left(2.75 \times 10^5\right)^{-1/2} = 0.0343\,\text{N/m}^2$$

the drag is

$$D' = 2(1\,\text{m})0.0343\,\text{N/m}^2 = 0.0686\,\text{N/m} \qquad <$$

For laminar flow, the average value $\bar{h}_L$ over the distance 0 to L is twice the local value, h_L,

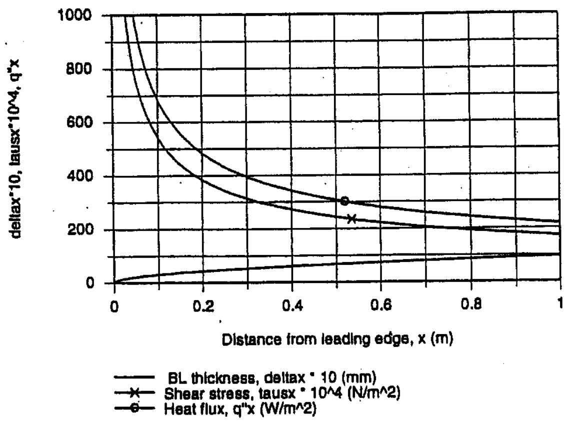
$$\bar{h}_L = 2h_L = 8.68\,\text{W/m}^2\cdot\text{K}$$

Continued...

The total heat transfer rate per unit width of the plate is

$$q' = 2L\bar{h}_L(T_s - T_\infty) = 2(1\,m)8.68\,W/m^2 \cdot K(75-25)^\circ C = 868\,W/m$$ <

(c) Using IHT with the equations of part (a), the boundary layer thickness, surface shear stress and heat flux as a function of distance form the leading edge were calculated and are plotted below.



COMMENTS: (1) The velocity boundary layer is very thin at the leading edge and increases with increasing distance. The local shear stress and heat flux are very large near the leading edge and decrease with increasing distance. The shapes of the two curves are similar.

(2) A copy of the IHT Workspace used to generate the above plot is shown below.

```
// Boundary layer thickness, delta
delta = 5 * x * Rex ^-0.5
delta_mm = delta * 1000
delta_plot = delta_mm * 10          // Scaling parameter for convenience in plotting

// Surface shear stress, tausx
tausx = (rho * uinf^2 / 2) * 0.664 * Rex^-0.5
tausx_plot = tausx * 10000          // Scaling parameter for convenience in plotting

// Heat flux, q"x
q"x = hx * (Ts - Tinf)
Nux = 0.332 * Rex^0.5 * Pr^(1/3)
Nux = hx * x / k

// Reynolds number
Rex = uinf * x / nu

// Properties Tool: Air
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
rho = rho_T("Air",Tf)               // Density, kg/m^3
nu = nu_T("Air",Tf)                 // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)                   // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)                 // Prandtl number

// Assigned variables
Tinf = 25 + 273                     // Airstream temperature, K
Ts = 75 + 273                       // Surface temperature, K
uinf = 5                            // Airstream velocity, m/s
Tf = 323                            // Film temperature, K
x = 1                               // Distance from leading edge, m
```