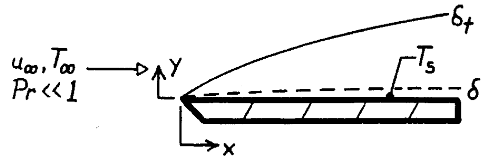
PROBLEM 7.6

KNOWN: Liquid metal in parallel flow over a flat plate.

FIND: An expression for the local Nusselt number.

SCHEMATIC:



ASSUMPTIONS: (1) Steady, incompressible flow, (2) $\delta \gg \delta_t$, hence $u(y) \sim u_\infty$, (3) Boundary layer approximations are valid, (4) Constant properties.

ANALYSIS: The boundary layer energy equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}.$$

Since $u(y) = u_\infty$, it follows that $v = 0$ and the energy equation becomes

$$u_\infty \frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \text{or} \qquad \frac{\partial T}{\partial x} = \frac{\alpha}{u_\infty}\frac{\partial^2 T}{\partial y^2}.$$

Boundary Conditions: $T(x,0) = T_s$, $T(x,\infty) = T_\infty$.

Initial Condition: $T(0,y) = T_\infty$.

The differential equation is analogous to that for transient one-dimensional conduction in a plane wall, and the conditions are analogous to those of Fig. 5.17, Case (1). Hence the solution is given by Eqs. 5.55 and 5.56. Substituting y for x, x for t, T_∞ for T_i, and α/u_∞ for α, the boundary layer temperature and the surface heat flux become

$$\frac{T(x,y) - T_s}{T_\infty - T_s} = \text{erf}\left[\frac{y}{2(\alpha\, x/u_\infty)^{1/2}}\right]$$

$$q_s'' = \frac{k(T_s - T_\infty)}{(\pi\,\alpha\, x/u_\infty)^{1/2}}.$$

Hence, with

$$Nu_x \equiv \frac{hx}{k} = \frac{q_s''x}{(T_s - T_\infty)k}$$

find

$$Nu_x = \frac{x}{(\pi\alpha x/u_\infty)^{1/2}} = \frac{(xu_\infty)^{1/2}}{\pi^{1/2}(k/\rho c_p)^{1/2}} = \frac{1}{\pi^{1/2}}\left(\frac{\rho u_\infty x}{\mu}\cdot\frac{c_p\mu}{k}\right)^{1/2}$$

$$Nu_x = 0.564\,(Re_x\,Pr)^{1/2} = 0.564\,Pe^{1/2} \qquad \triangleleft$$

where $Pe = Re \cdot Pr$ is the Peclet number.

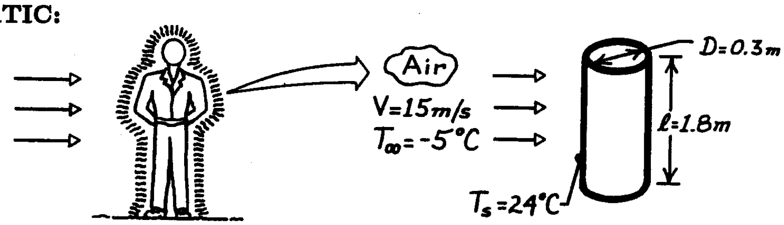
COMMENTS: Because k is very large, axial conduction effects may not be negligible. That is, the $\alpha\,\partial^2 T/\partial x^2$ term of the energy equation may be important.

PROBLEM 7.57

KNOWN: Person, approximated as a cylinder, is subjected to prescribed convection conditions.

FIND: Heat rate from body for given temperature conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Person can be approximated by cylindrical form having uniform surface temperature, (3) Negligible heat loss from cylinder top and bottom surfaces, (4) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air ($T_\infty = 268\,K$, 1 atm): $\nu = 13.04\times10^{-6}\,m^2/s$, $k = 23.74\times10^{-3}\,W/m\cdot K$, $Pr = 0.725$; ($T_s = 297\,K$, 1 atm): $Pr = 0.707$.

ANALYSIS: The heat transfer rate from the cylinder, approximating the person, is given as

$$q = \bar{h} A_s (T_s - T_\infty)$$

where $A_s = \pi D \ell$ and $\bar{h}_m$ must be estimated from a correlation appropriate to cross-flow over a cylinder. Use the Zhukauskas relation,

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = C\, Re_D^m\, Pr^n (Pr/Pr_s)^{1/4}$$

and calculate the Reynold's number,

$$Re_D = \frac{VD}{\nu} = \frac{15\,m/s \times 0.3\,m}{13.04\times10^{-6}\,m^2/s} = 345{,}092\,.$$

From Table 7.4, find $C = 0.076$ and $m = 0.7$. Since $Pr < 10$, $n = 0.37$, giving

$$\overline{Nu}_D = 0.076\,(345{,}092)^{0.7}\,0.725^{0.37}\left(\frac{0.725}{0.707}\right)^{1/4} = 511$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = \frac{511 \times 23.74\times10^{-3}\,W/m\cdot K}{0.3\,m} = 40.4\,W/m\cdot K\,.$$

The heat transfer rate is

$$q = 40.4\,W/m^2\cdot K\,(\pi \times 0.3\,m \times 1.8\,m)\,(24-(-5))\,^\circ C = 1988\,W\,. \qquad \triangleleft$$

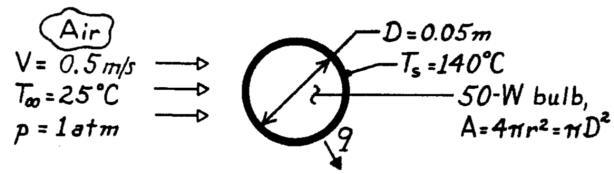
COMMENTS: Note carefully at which temperatures properties must be evaluated for the Zhukauskas correlation.

PROBLEM 7.66

KNOWN: Conditions associated with airflow over a spherical light bulb of prescribed diameter and surface temperature.

FIND: Heat loss by convection.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature.

PROPERTIES: *Table A-4*, Air (T_f = 25°C, 1 atm): $\nu = 15.71 \times 10^{-6}$ m²/s, k = 0.0261 W/m·K, Pr = 0.71, $\mu = 183.6 \times 10^{-7}$ N·s/m²; *Table A-4*, Air (T_s = 140°C, 1 atm): $\mu = 235.5 \times 10^{-7}$ N·s/m².

ANALYSIS: The heat rate by convection is

$$q = \bar{h}(\pi D^2)(T_s - T_\infty)$$

where $\bar{h}$ may be estimated from the Whitaker relation

$$\bar{h} = \frac{k}{D}[2 + (0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3})\,Pr^{0.4}\,(\mu/\mu_s)^{1/4}]$$

where

$$Re_D = \frac{VD}{\nu} = \frac{0.5\text{ m/s} \times 0.05\text{ m}}{15.71 \times 10^{-6}\text{ m}^2/\text{s}} = 1591\,.$$

Hence,

$$\bar{h} = \frac{0.0261\text{ W/m·K}}{0.05\text{ m}}\left\{2 + [0.4(1591)^{1/2} + 0.06(1591)^{2/3}](0.71)^{0.4}\left(\frac{183.6}{235.5}\right)^{1/4}\right\}$$

$$\bar{h} = 11.4\text{ W/m}^2\text{·K}$$

and the heat rate is

$$q = 11.4\frac{\text{W}}{\text{m}^2\text{·K}}\,\pi(0.05\text{ m})^2\,(140\text{–}25)^\circ\text{C} = 10.3\text{ W}\,. \quad \triangleleft$$

COMMENTS: (1) The low value of $\bar{h}$ suggests that heat transfer by free convection may be significant and hence that the total loss by convection exceeds 10.3 W.

(2) The surface of the bulb also dissipates heat to the surrounding by radiation. Further, in an actual light bulb, there is also heat loss by conduction through the socket.

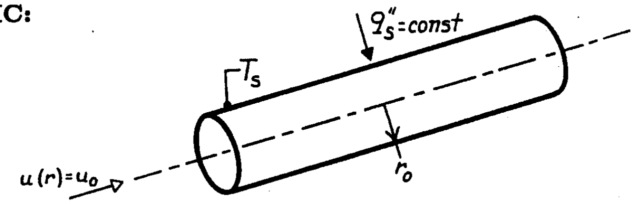
(3) The correlation has been used outside its range of application $(\mu/\mu_s) < 1$.

PROBLEM 8.16

KNOWN: Laminar, slug flow in a circular tube with uniform surface heat flux.

FIND: Temperature distribution and Nusselt number.

SCHEMATIC:



ASSUMPTIONS: (1) Steady, incompressible flow, (2) Constant properties, (3) Fully developed, laminar flow, (4) Uniform surface heat flux.

ANALYSIS: With $v = 0$ for fully developed flow and $\partial T/\partial x = dT_m/dx = \text{const.}$ from Eqs. 8.33 and 8.40, the energy equation, Eq. 8.48, reduces to

$$u_o \frac{dT_m}{dx} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right).$$

Integrating twice, it follows that

$$T(r) = \frac{u_o}{\alpha}\frac{dT_m}{dx}\frac{r^2}{4} + C_1 \ell n(r) + C_2 .$$

Since $T(0)$ must remain finite, $C_1 = 0$. Hence, with $T(r_o) = T_s$

$$C_2 = T_s - \frac{u_o}{\alpha}\frac{dT_m}{dx}\frac{r_o^2}{4} \qquad\qquad T(r) = T_s - \frac{u_o}{4\alpha}\frac{dT_m}{dx}\left(r_o^2 - r^2\right). \qquad \triangleleft$$

From Eq. 8.27, with $u_m = u_o$,

$$T_m = \frac{2}{r_o^2}\int_0^{r_o} Tr\,dr = \frac{2}{r_o^2}\int_0^{r_o}\left[T_s r - \frac{u_o}{4\alpha}\frac{dT_m}{dx}\left(rr_o^2 - r^3\right)\right]dr$$

$$T_m = \frac{2}{r_o^2}\left[T_s\frac{r_o^2}{2} - \frac{u_o}{4\alpha}\frac{dT_m}{dx}\left(\frac{r_o^4}{2} - \frac{r_o^4}{4}\right)\right] = T_s - \frac{u_o r_o^2}{8\alpha}\frac{dT_m}{dx}.$$

From Eq. 8.28 and Fourier's law,

$$h = \frac{q_s''}{T_s - T_m} = \frac{k\left.\frac{\partial T}{\partial r}\right|_{r_o}}{T_s - T_m}$$

Hence,

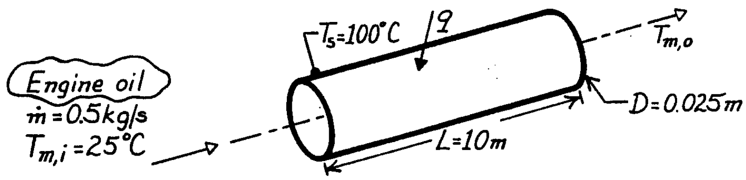
$$h = \frac{k\left(\frac{u_o r_o}{2\alpha}\right)\frac{dT_m}{dx}}{\frac{u_o r_o^2}{8\alpha}\frac{dT_m}{dx}} = \frac{4k}{r_o} = \frac{8k}{D} \qquad\qquad Nu_D = \frac{hD}{k} = 8 . \qquad \triangleleft$$

PROBLEM 8.21

KNOWN: Flow rate and inlet temperature of engine oil in a tube of prescribed length, diameter, and surface temperature.

FIND: Total heat transfer and oil outlet temperature with and without the assumption of fully developed flow.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic energy, potential energy and flow work changes, (3) Constant properties.

PROPERTIES: *Table A-5*, Engine oil, ($\bar{T}_m \approx 340K$): $\rho = 860\ kg/m^3$, $c_p = 2076\ J/kg\cdot K$, $\mu = 5.31\times10^{-2}\ kg/s\cdot m$, $k = 0.139\ W/m\cdot K$, $Pr = 793$.

ANALYSIS: From Eqs. 8.42b and 8.37,

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{\pi DL}{\dot{m} c_p}\bar{h}\right) = 100°C - 75°C\exp\left(-\frac{\pi 0.025m \times 10m}{0.5\,kg/s \times 2076\,J/kg\cdot K}\bar{h}\right)$$

$$T_{m,o} = 100°C - 75°C\exp(-0.000757\times\bar{h})$$

$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) = 0.5\,kg/s \times 2076\,J/kg\cdot K\,(T_{m,o} - 25°C).$$

With $Re_D = 4\,\dot{m}/\pi D\mu = 4(0.5\ kg/s)/\pi(0.025\ m)0.0531\ kg/s\cdot m = 480$ the flow is laminar. Considering the *thermal entry (te) region*, it follows from Eq. 8.56 that

$$\bar{h} = \frac{k}{D}\left[3.66 + \frac{0.0668\,(D/L)Re_D Pr}{1+0.04[(D/L)Re_D Pr]^{2/3}}\right] = \frac{0.139\,W/m\cdot K}{0.025\,m}\left[3.66 + \frac{0.0668\,(940)}{1+0.04\,(940)^{2/3}}\right] = 92.5\,W/m^2\cdot K.$$

$$T_{m,o(te)} = 100°C - 75°C\exp(-0.000757\times92.5) = 100°C - 69.9°C = 30.1°C \quad \triangleleft$$

$$q_{(te)} = 1038\,W/K\,(30.1-25)°C = 5290\,W. \quad \triangleleft$$

If *fully developed* (fd) conditions are assumed for the entire tube,

$$\bar{h} = \frac{k}{D}3.66 = \frac{0.139\,W/m\cdot K}{0.025\,m}3.66 = 20.3\,W/m^2\cdot K$$

$$T_{m,o(fd)} = 100°C - 73.9°C = 26.1°C \quad \triangleleft$$

$$q_{(fd)} = 1038\,W/K\,(26.1-25)°C = 1190\,W. \quad \triangleleft$$

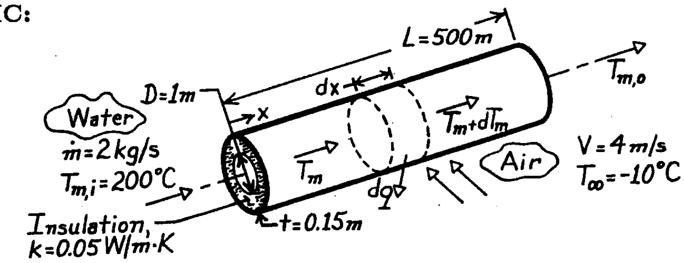
COMMENTS: The assumption of fully developed conditions throughout the tube leads to a large error in the calculation of $\bar{h}$ and hence q. Note that $x_{fd,t} \approx 0.05\,D\,Re_D\,Pr = 0.05(0.025\,m)480(793) = 476\,m$, which is much larger than the tube length. The calculations should be repeated with properties evaluated at $\bar{T}_m = 300K$.

PROBLEM 8.64

KNOWN: Flow conditions associated with water passing through a pipe and air flowing over the pipe.

FIND: (a) Differential equation which determines the variation of the mixed-mean temperature of the water, (b) Heat transfer per unit length of pipe at the inlet and outlet temperature of the water.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature drop across the pipe wall, (2) Negligible radiation exchange between outer surface of insulation and surroundings, (3) Fully developed flow throughout pipe, (4) Negligible potential and kinetic energy and flow work effects.

PROPERTIES: *Table A-6,* Water ($T_{m,i}=200°C$): $c_{p,w}=4500\ J/kg{\cdot}K$, $\mu_w = 134{\times}10^{-6} N{\cdot}s/m^2$, $k_w = 0.665 W/m{\cdot}K$, $Pr_w = 0.91$; *Table A-4,* Air ($T_\infty = -10°C$): $\nu_a = 12.6{\times}10^{-6} m^2/s$, $k_a = 0.023\ W/m{\cdot}K$, $Pr_a = 0.71$, $Pr_s \approx 0.7$.

ANALYSIS: (a) Following the development of Section 8.3.1 and applying an energy balance to a differential element in the water, we obtain

$$\dot{m}\, c_{p,w}\, T_m - dq - \dot{m}\, c_{p,w}(T_m + dT_m) = 0\ .$$

Hence $\quad dq = -\dot{m}\, c_{p,w}\, dT_m$

where $\quad dq = U_i\, dA_i (T_m - T_\infty) = U_i \pi D\, dx (T_m - T_\infty)$.

Substituting into the energy balance, it follows that

$$\frac{dT_m}{dx} = -\frac{U_i \pi D}{\dot{m}\, c_p}(T_m - T_\infty)\ . \qquad (1) \lhd$$

The overall heat transfer coefficient based on the inside surface area may be evaluated from Eq. 3.30 which, for the present conditions, reduces to

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{D}{2k}\ell n\left(\dfrac{D+2t}{D}\right) + \dfrac{D}{D+2t}\dfrac{1}{h_o}}\ . \qquad (2)$$

For the *inner water flow,* Eq. 8.6 gives

$$Re_D = \frac{4\dot{m}}{\pi D \mu_w} = \frac{4 \times 2\, kg/s}{\pi(1m) \times 134{\times}10^{-6}\, kg/s{\cdot}m} = 19{,}004\ .$$

Continued .....

Hence the flow is turbulent. With the assumption of fully developed conditions, it follows from Eq. 8.60 that

$$h_i = \frac{k_w}{D} \times 0.023\,Re_D^{4/5}\,Pr_w^{0.3} . \qquad (3)$$

For the *external air flow*

$$Re_D = \frac{V(D+2t)}{\nu} = \frac{4\,m/s(1.3m)}{12.6\times10^{-6}\,m^2/s} = 4.13\times10^5 .$$

Using Eq. 7.31 to obtain the outside convection coefficient,

$$h_o = \frac{k_a}{(D+2t)} \times 0.076\,Re_D^{0.7}\,Pr_a^{0.37}\,(Pr_a/Pr_s)^{1/4} . \qquad (4)$$

(b) The heat transfer per unit length of pipe at the inlet is

$$q' = \pi D\,U_i(T_{m,i}-T_\infty) . \qquad (5)$$

From Eqs. (3 and 4),

$$h_i = \frac{0.665\,W/m\cdot K}{1m} \times 0.023(19{,}004)^{4/5}\,(0.91)^{0.3} = 39.4\,W/m^2\cdot K$$

$$h_o = \frac{0.023\,W/m\cdot K}{(1.3m)} \times 0.076(4.13\times10^5)^{0.7}\,(0.71)^{0.37}\,(1)^{1/4} = 10.1\,W/m^2\cdot K .$$

Hence from Eq. (2)

$$U_i = \left[\frac{1}{39.4\,W/m^2\cdot K} + \frac{1m}{0.1\,W/m\cdot K}\,\ell n\left(\frac{1.3}{1}\right) + \frac{1}{1.3} \times \frac{1}{10.1\,W/m^2\cdot K}\right]^{-1} = 0.37\,W/m^2\cdot K$$

and from Eq. (5)

$$q' = \pi(1m)\,(0.37\,W/m^2\cdot K)\,(200+10)\,^\circ C = 244\,W/m . \qquad \triangleleft$$

Since U_i is a constant, independent of x, Eq. (1) may be integrated from $x = 0$ to $x = L$. The result is analogous to Eq. 8.42b and may be expressed as

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot m\,c_{p,w}}\,U_i\right) = \exp\left(-\frac{\pi \times 1m \times 500m}{2\,kg/s \times 4500\,J/kg\cdot K} \times 0.37\,W/m^2\cdot K\right)$$

Hence $$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = 0.937.$$

$$T_{m,o} = T_\infty + 0.937(T_{m,i}-T_\infty) = 187\,^\circ C . \qquad \triangleleft$$

COMMENTS: The largest contribution to the denominator on the right-hand side of Eq. (2) is made by the conduction term (the insulation provides 96% of the total resistance to heat transfer). For this reason the assumption of fully developed conditions throughout the pipe has a negligible effect on the calculations. Since the reduction in T_m is small (13 °C), little error is incurred by evaluating all properties of water at $T_{m,i}$.