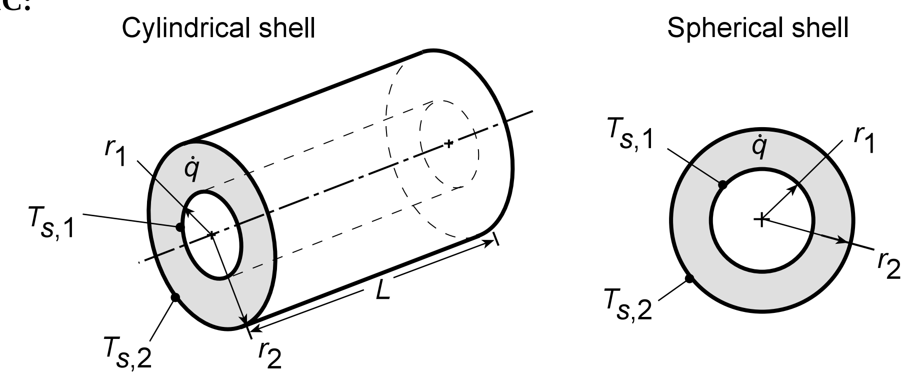
PROBLEM 3.70

KNOWN: Cylindrical and spherical shells with uniform heat generation and surface temperatures.

FIND: Radial distributions of temperature, heat flux and heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Uniform heat generation, (3) Constant k.

ANALYSIS: (a) For the *cylindrical shell*, the appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

The general solution is

$$T(r)=-\frac{\dot{q}}{4k}r^2+C_1\ln r+C_2$$

Applying the boundary conditions, it follows that

$$T(r_1)=T_{s,1}=-\frac{\dot{q}}{4k}r_1^2+C_1\ln r_1+C_2$$

$$T(r_2)=T_{s,2}=-\frac{\dot{q}}{4k}r_2^2+C_1\ln r_2+C_2$$

which may be solved for

$$C_1=\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]\Big/\ln(r_2/r_1)$$

$$C_2=T_{s,2}+(\dot{q}/4k)r_2^2-C_1\ln r_2$$

Hence,

$$T(r)=T_{s,2}+(\dot{q}/4k)\left(r_2^2-r^2\right)+\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]\frac{\ln(r/r_2)}{\ln(r_2/r_1)}$$ <

With $q''=-k\,dT/dr$, the heat flux distribution is

$$q''(r)=\frac{\dot{q}}{2}r-\frac{k\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]}{r\ln(r_2/r_1)}$$ <

Continued...

Similarly, with q = q'' A(r) = q'' (2πrL), the heat rate distribution is

$$q(r) = \pi L \dot{q} r^2 - \frac{2\pi L k\left[(\dot{q}/4k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right]}{\ln(r_2/r_1)} \qquad <$$

(b) For the *spherical shell*, the heat equation and general solution are

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

$$T(r) = -(\dot{q}/6k)r^2 - C_1/r + C_2$$

Applying the boundary conditions, it follows that

$$T(r_1) = T_{s,1} = -(\dot{q}/6k)r_1^2 - C_1/r_1 + C_2$$

$$T(r_2) = T_{s,2} = -(\dot{q}/6k)r_2^2 - C_1/r_2 + C_2$$

Hence,

$$C_1 = \left[(\dot{q}/6k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right] \Big/ \left[(1/r_1) - (1/r_2)\right]$$

$$C_2 = T_{s,2} + (\dot{q}/6k)r_2^2 + C_1/r_2$$

and

$$T(r) = T_{s,2} + (\dot{q}/6k)\left(r_2^2 - r^2\right) - \left[(\dot{q}/6k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right]\frac{(1/r) - (1/r_2)}{(1/r_1) - (1/r_2)} \qquad <$$

With q''(r) = - k dT/dr, the heat flux distribution is

$$q''(r) = \frac{\dot{q}}{3}r - \frac{\left[(\dot{q}/6)\left(r_2^2 - r_1^2\right) + k\left(T_{s,2} - T_{s,1}\right)\right]}{(1/r_1) - (1/r_2)}\frac{1}{r^2} \qquad <$$

and, with q = $q''\left(4\pi r^2\right)$, the heat rate distribution is

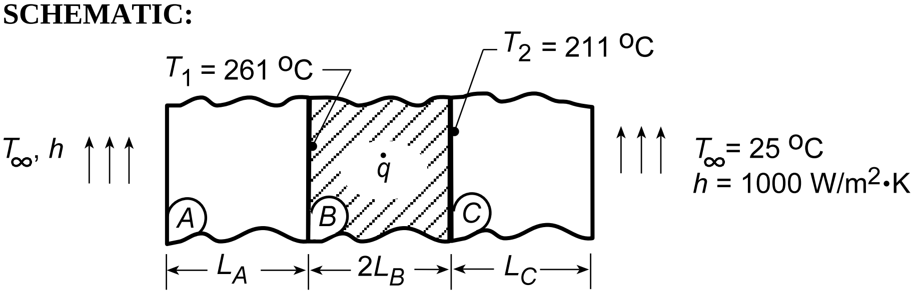
$$q(r) = \frac{4\pi\dot{q}}{3}r^3 - \frac{4\pi\left[(\dot{q}/6)\left(r_2^2 - r_1^2\right) + k\left(T_{s,2} - T_{s,1}\right)\right]}{(1/r_1) - (1/r_2)} \qquad <$$

PROBLEM 3.73

KNOWN: Composite wall with outer surfaces exposed to convection process.

FIND: (a) Volumetric heat generation and thermal conductivity for material B required for special conditions, (b) Plot of temperature distribution, (c) T_1 and T_2, as well as temperature distributions corresponding to loss of coolant condition where h = 0 on surface A.

SCHEMATIC:



L_A = 30 mm
L_B = 30 mm
L_C = 20 mm
k_A = 25 W/m·K
k_C = 50 W/m·K

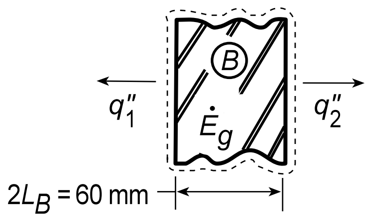
ASSUMPTIONS: (1) Steady-state, one-dimensional heat transfer, (2) Negligible contact resistance at interfaces, (3) Uniform generation in B; zero in A and C.

ANALYSIS: (a) From an energy balance on wall B,

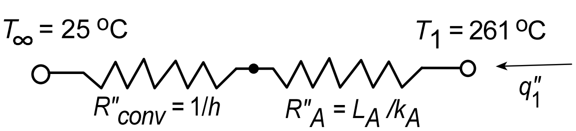
$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

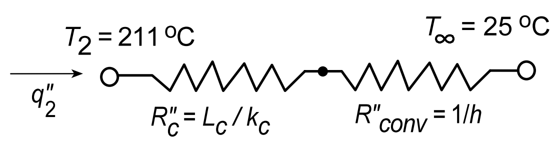
$$-q_1'' - q_2'' + 2\dot{q}L_B = 0$$

$$\dot{q}_B = \left(q_1'' + q_2''\right)/2L_B \ .$$



To determine the heat fluxes, q_1'' and q_2'', construct thermal circuits for A and C:





$$q_1'' = \left(T_1 - T_\infty\right)/\left(1/h + L_A/k_A\right)$$

$$q_1'' = \left(261-25\right)^\circ C\Bigg/\left(\frac{1}{1000\,W/m^2\cdot K} + \frac{0.030\,m}{25\,W/m\cdot K}\right)$$

$$q_1'' = 236^\circ C/\left(0.001+0.0012\right)m^2\cdot K/W$$

$$q_1'' = 107,273\,W/m^2$$

$$q_2'' = \left(T_2 - T_\infty\right)/\left(L_C/k_C + 1/h\right)$$

$$q_2'' = \left(211-25\right)^\circ C\Bigg/\left(\frac{0.020\,m}{50\,W/m\cdot K} + \frac{1}{1000\,W/m^2\cdot K}\right)$$

$$q_2'' = 186^\circ C/\left(0.0004+0.001\right)m^2\cdot K/W$$

$$q_2'' = 132,857\,W/m^2$$

Using the values for q_1'' and q_2'' in Eq. (1), find

$$\dot{q}_B = \left(106,818 + 132,143\,W/m^2\right)\Big/2\times 0.030\,m = 4.00\times 10^6\,W/m^3 \ .$$ <

To determine k_B, use the general form of the temperature and heat flux distributions in wall B,

$$T(x) = -\frac{\dot{q}_B}{2k_B}x^2 + C_1 x + C_2 \qquad q_x''(x) = -k_B\left[-\frac{\dot{q}}{k_B}x + C_1\right] \qquad (1,2)$$

there are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions,

Continued...

PROBLEM 3.73 (Cont.)

$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \qquad \text{where } T_1 = 261°C \qquad (3)$$

$$T(+L_B) = T_2 = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1 L_B + C_2 \qquad \text{where } T_2 = 211°C \qquad (4)$$

$$q''_x(-L_B) = -q''_1 = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] \qquad \text{where } q''_1 = 107{,}273 \text{ W/m}^2 \qquad (5)$$

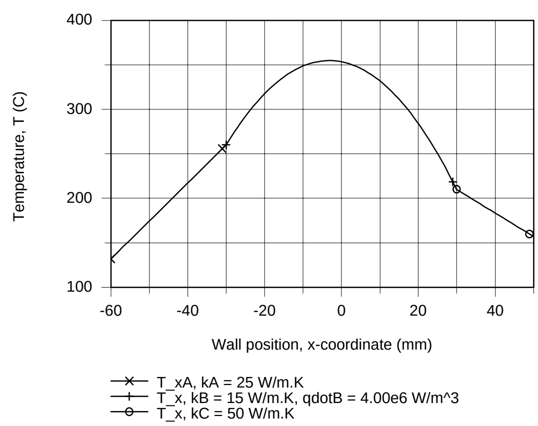
Using IHT to solve Eqs. (3), (4) and (5) simultaneously with $\dot{q}_B = 4.00 \times 10^6$ W/m^3, find

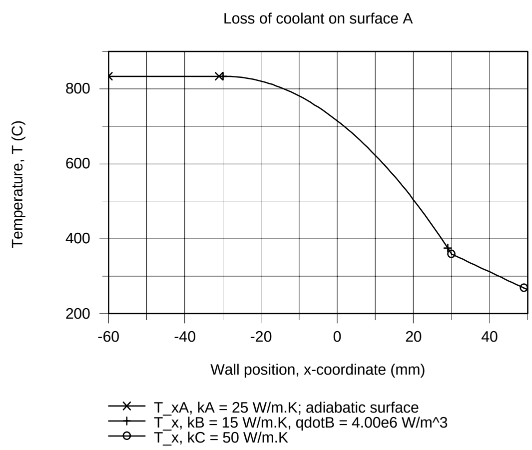
$$k_B = 15.3 \text{ W/m} \cdot \text{K}$$ <

(b) Following the method of analysis in the *IHT Example 3.6, User-Defined Functions*, the temperature distribution is shown in the plot below. The important features are (1) Distribution is quadratic in B, but non-symmetrical; linear in A and C; (2) Because thermal conductivities of the materials are different, discontinuities exist at each interface; (3) By comparison of gradients at x = -L_B and +L_B, find $q''_2 > q''_1$.

(c) Using the same method of analysis as for Part (c), the temperature distribution is shown in the plot below when h = 0 on the surface of A. Since the left boundary is adiabatic, material A will be isothermal at T_1. Find

T_1 = 835°C $\qquad$ T_2 = 360°C <



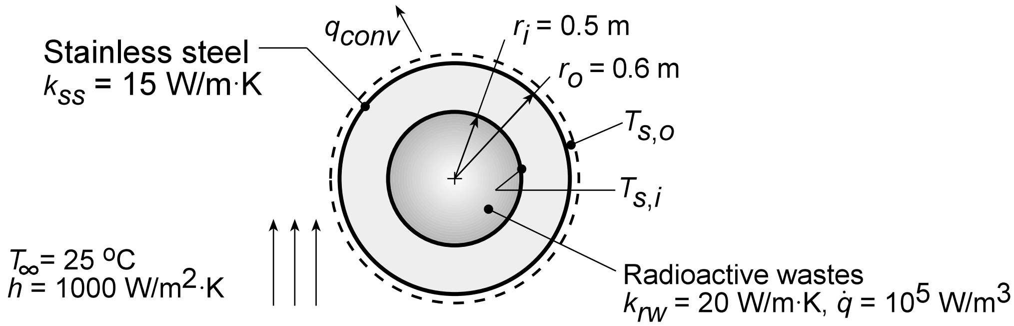


PROBLEM 3.95

KNOWN: Dimensions and thermal conductivity of a spherical container. Thermal conductivity and volumetric energy generation within the container. Outer convection conditions.

FIND: (a) Outer surface temperature, (b) Container inner surface temperature, (c) Temperature distribution within and center temperature of the wastes, (d) Feasibility of operating at twice the energy generation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional radial conduction.

ANALYSIS: (a) For a control volume which includes the container, conservation of energy yields $\dot{E}_g - \dot{E}_{out} = 0$, or $\dot{q}V - q_{conv} = 0$. Hence

$$\dot{q}(4/3)\left(\pi r_i^3\right) = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

and with $\dot{q} = 10^5$ W/m^3,

$$T_{s,o} = T_\infty + \frac{\dot{q}r_i^3}{3hr_o^2} = 25^\circ C + \frac{10^5\,W/m^2\,(0.5\,m)^3}{3000\,W/m^2\cdot K\,(0.6\,m)^2} = 36.6^\circ C.$$ <

(b) Performing a surface energy balance at the outer surface, $\dot{E}_{in} - \dot{E}_{out} = 0$ or $q_{cond} - q_{conv} = 0$. Hence

$$\frac{4\pi k_{ss}\left(T_{s,i} - T_{s,o}\right)}{(1/r_i) - (1/r_o)} = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

$$T_{s,i} = T_{s,o} + \frac{h}{k_{ss}}\left(\frac{r_o}{r_i} - 1\right)r_o\left(T_{s,o} - T_\infty\right) = 36.6^\circ C + \frac{1000\,W/m^2\cdot K}{15\,W/m\cdot K}(0.2)0.6\,m\left(11.6^\circ C\right) = 129.4^\circ C.$$ <

(c) The heat equation in spherical coordinates is

$$k_{rw}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \dot{q}r^2 = 0.$$

Solving,

$$r^2\frac{dT}{dr} = -\frac{\dot{q}r^3}{3k_{rw}} + C_1 \qquad \text{and} \qquad T(r) = -\frac{\dot{q}r^2}{6k_{rw}} - \frac{C_1}{r} + C_2$$

Applying the boundary conditions,

$$\left.\frac{dT}{dr}\right|_{r=0} = 0 \qquad \text{and} \qquad T(r_i) = T_{s,i}$$

$$C_1 = 0 \qquad \text{and} \qquad C_2 = T_{s,i} + \dot{q}r_i^2/6k_{rw}.$$

Continued...

Hence

$$T(r) = T_{s,i} + \frac{\dot{q}}{6k_{rw}}\left(r_i^2 - r^2\right) \qquad <$$

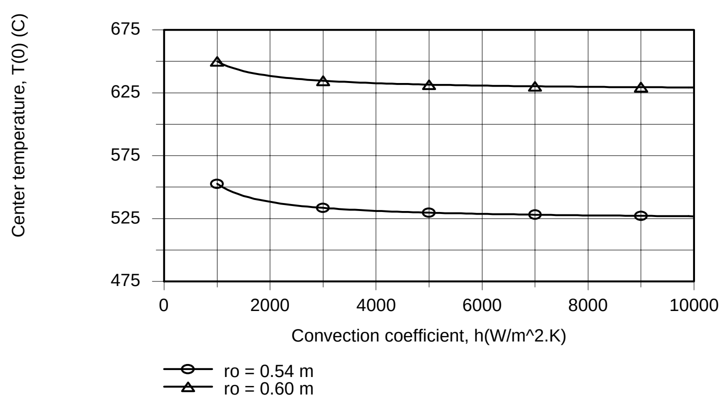
At r = 0,

$$T(0) = T_{s,i} + \frac{\dot{q}r_i^2}{6k_{rw}} = 129.4^\circ\text{C} + \frac{10^5\ \text{W/m}^3\,(0.5\,\text{m})^2}{6(20\,\text{W/m}\cdot\text{K})} = 337.7^\circ\text{C} \qquad <$$

(d) The feasibility assessment may be performed by using the IHT model for one-dimensional, steady-state conduction in a solid sphere, with the surface boundary condition prescribed in terms of the total thermal resistance

$$R''_{tot,i} = \left(4\pi r_i^2\right)R_{tot} = R''_{cnd,i} + R''_{cnv,i} = \frac{r_i^2\left[(1/r_i) - (1/r_o)\right]}{k_{ss}} + \frac{1}{h}\left(\frac{r_i}{r_o}\right)^2$$

where, for r_o = 0.6 m and h = 1000 $W/m^2\cdot K$, $R''_{cnd,i} = 5.56 \times 10^{-3}\ m^2\cdot K/W$, $R''_{cnv,i} = 6.94 \times 10^{-4}\ m^2\cdot K/W$, and $R''_{tot,i} = 6.25 \times 10^{-3}\ m^2\cdot K/W$. Results for the center temperature are shown below.



Clearly, even with $r_o = 0.54$ m = $r_{o,min}$ and h = 10,000 $W/m^2\cdot K$ (a practical upper limit), T(0) > 475°C and the desired condition can not be met. The corresponding resistances are $R''_{cnd,i} = 2.47 \times 10^{-3}\ m^2\cdot K/W$, $R''_{cnv,i} = 8.57 \times 10^{-5}\ m^2\cdot K/W$, and $R''_{tot,i} = 2.56 \times 10^{-3}\ m^2\cdot K/W$. The conduction resistance remains dominant, and the effect of reducing $R''_{cnv,i}$ by increasing h is small. *The proposed extension is not feasible.*

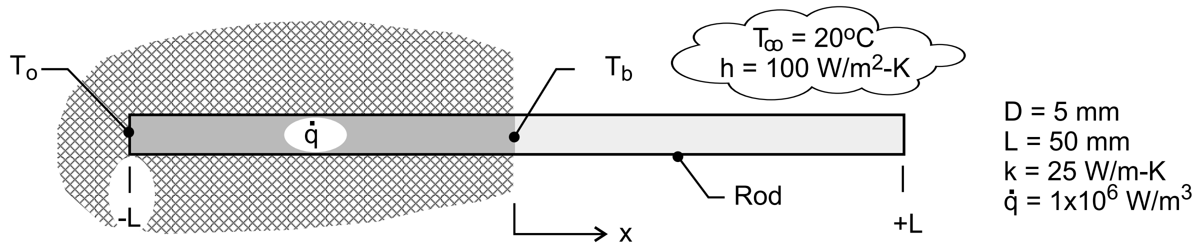
COMMENTS: A value of $\dot{q} = 1.79 \times 10^5\ W/m^3$ would allow for operation at T(0) = 475°C with r_o = 0.54 m and h = 10,000 $W/m^2\cdot K$.

PROBLEM 3.112

KNOWN: Rod (D, k, 2L) inserted into a perfectly insulating wall, exposing one-half of its length to an airstream (T_∞, h). An electromagnetic field induces a uniform volumetric energy generation $(\dot{q})$ in the imbedded portion.

FIND: (a) Derive an expression for T_b at the base of the exposed half of the rod; the exposed region may be approximated as a very long fin; (b) Derive an expression for T_o at the end of the imbedded half of the rod, and (c) Using numerical values, plot the temperature distribution in the rod and describe its key features. Does the rod behave as a very long fin?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in imbedded portion of rod, (3) Imbedded portion of rod is perfectly insulated, (4) Exposed portion of rod behaves as an infinitely long fin, and (5) Constant properties.

ANALYSIS: (a) Since the exposed portion of the rod ($0 \le x \le +L$) behaves as an infinite fin, the fin heat rate using Eq. 3.80 is

$$q_x(0) = q_f = M = (hPkA_c)^{1/2}(T_b - T_\infty) \qquad (1)$$

From an energy balance on the imbedded portion of the rod,

$$q_f = \dot{q}A_cL \qquad (2)$$

Combining Eqs. (1) and (2), with $P = \pi D$ and $A_c = \pi D^2/4$, find

$$T_b = T_\infty + q_f(hPkA_c)^{-1/2} = T_\infty + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \qquad (3) <$$

(b) The imbedded portion of the rod ($-L \le x \le 0$) experiences one-dimensional heat transfer with uniform $\dot{q}$. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \qquad <$$

(c) The temperature distribution T(x) for the rod is piecewise parabolic and exponential,

$$T(x) - T_b = \frac{\dot{q}L^2}{2k}\left(\frac{x}{L}\right)^2 \qquad -L \le x \le 0$$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \exp(-mx) \qquad 0 \le x \le +L$$

Continued …..

PROBLEM 3.112 (Cont.)

The gradient at x = 0 will be continuous since we used this condition in evaluating T_b. The distribution is shown below with T_o = 105.4°C and T_b = 55.4°C.