PROBLEM 5.23

KNOWN: Initial and final temperatures of a niobium sphere. Diameter and properties of the sphere. Temperature of surroundings and/or gas flow, and convection coefficient associated with the flow.

FIND: (a) Time required to cool the sphere exclusively by radiation, (b) Time required to cool the sphere exclusively by convection, (c) Combined effects of radiation and convection.

SCHEMATIC:

ASSUMPTIONS: (1) Uniform temperature at any time, (2) Negligible effect of holding mechanism on heat transfer, (3) Constant properties, (4) Radiation exchange is between a small surface and large surroundings.

ANALYSIS: (a) If cooling is exclusively by radiation, the required time is determined from Eq. (5.18). With \( V = \pi D^3 / 6 \), \( A_{s,r} = \pi D^2 \), and \( \varepsilon = 0.1 \),

\[
t = \frac{8600 \text{ kg/m}^3 (290 \text{ J/kg} \cdot \text{K}) 0.01 \text{m}}{24(0.1)5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (298 \text{K})^3} \left\{ \ln \left( \frac{298 + 573}{298 - 573} \right) - \ln \left( \frac{298 + 1173}{298 - 1173} \right) \right\} + 2 \left[ \tan^{-1} \left( \frac{573}{298} \right) - \tan^{-1} \left( \frac{1173}{298} \right) \right]
\]

\[
t = 6926 \text{s} \left\{ 1.153 - 0.519 + 2 (1.091 - 1.322) \right\} = 1190 \text{s} \quad (\varepsilon = 0.1)
\]

If \( \varepsilon = 0.6 \), cooling is six times faster, in which case,

\[
t = 199 \text{s} \quad (\varepsilon = 0.6)
\]

(b) If cooling is exclusively by convection, Eq. (5.5) yields

\[
t = \frac{\rho c D}{6 h} \ln \left( \frac{T_i - T_\infty}{T_f - T_\infty} \right) = \frac{8600 \text{ kg/m}^3 (290 \text{ J/kg} \cdot \text{K}) 0.01 \text{m}}{1200 \text{ W/m}^2 \cdot \text{K}} \ln \left( \frac{875}{275} \right)
\]

\[
t = 24.1 \text{s}
\]

(c) With both radiation and convection, the temperature history may be obtained from Eq. (5.15).

\[
\rho \left( \pi D^3 / 6 \right) c \frac{dT}{dt} = -\pi D^2 h \left( T - T_\infty \right) + \varepsilon \sigma \left( T^4 - T_{\text{sur}}^4 \right)
\]

Integrating numerically from \( T_i = 1173 \text{ K} \) at \( t = 0 \) to \( T = 573 \text{K} \), we obtain

\[
t = 21.0 \text{s}
\]

Continued …..
PROBLEM 5.23 (Cont.)

Cooling times corresponding to representative changes in $\varepsilon$ and $h$ are tabulated as follows

<table>
<thead>
<tr>
<th>$h$(W/m$^2$·K)</th>
<th>200</th>
<th>200</th>
<th>20</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>0.6</td>
<td>1.0</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$t$(s)</td>
<td>21.0</td>
<td>19.4</td>
<td>102.8</td>
<td>9.1</td>
</tr>
</tbody>
</table>

For values of $h$ representative of forced convection, the influence of radiation is secondary, even for a maximum possible emissivity of 1.0. Hence, to accelerate cooling, it is necessary to increase $h$. However, if cooling is by natural convection, radiation is significant. For a representative natural convection coefficient of $h = 20$ W/m$^2$·K, the radiation flux exceeds the convection flux at the surface of the sphere during early to intermediate stages of the transient.

**COMMENTS:**
(1) Even for $h$ as large as 500 W/m$^2$·K, $Bi = h (D/6)/k = 500$ W/m$^2$·K (0.01m/6)/63 W/m·K = 0.013 < 0.1 and the lumped capacitance model is appropriate. 
(2) The largest value of $h_r$ corresponds to $T_i =1173$ K, and for $\varepsilon = 0.6$ Eq. (1.9) yields $h_r = 0.6 \times 5.67 \times 10^{-8}$ W/m$^2$·K$^4$ ($1173 + 298$)K ($1173^2 + 298^2$)K$^2 = 73.3$ W/m$^2$·K.
PROBLEM 5.33

KNOWN: Configuration, initial temperature and charging conditions of a thermal energy storage unit.

FIND: Time required to achieve 75% of maximum possible energy storage and corresponding minimum and maximum temperatures.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible radiation exchange with surroundings.

ANALYSIS: For the system, find first

\[ \text{Bi} = \frac{hL}{k} = \frac{100 \text{ W/m}^2 \cdot \text{K} \times 0.025 \text{ m}}{0.7 \text{ W/m} \cdot \text{K}} = 3.57 \]

indicating that the lumped capacitance method cannot be used.

**Groebner chart, Fig. D.3:**

\[ Q/Q_0 = 0.75 \]

\[ \alpha = \frac{k}{\rho c} = \frac{0.7 \text{ W/m} \cdot \text{K}}{1900 \text{ kg/m}^3 \times 800 \text{ J/kg} \cdot \text{K}} = 4.605 \times 10^{-7} \text{ m}^2/\text{s} \]

\[ \text{Bi}^2F_0 = \frac{h^2\alpha t}{k^2} = \left(\frac{100 \text{ W/m}^2\text{K}}{0.7 \text{ W/m} \cdot \text{K}}\right)^2 \times \left(4.605 \times 10^{-7} \text{ m}^2/\text{s}\right) \times t (\text{s}) = 9.4 \times 10^{-3} t \]

Find \( \text{Bi}^2F_0 = 11 \), and substituting numerical values

\[ t = 11/9.4 \times 10^{-3} = 1170 \text{s}. \]

**Heisler chart, Fig. D.1:**

\[ T_{\text{min}} \] is at \( x = 0 \) and \( T_{\text{max}} \) at \( x = L \), with

\[ F_0 = \frac{\alpha t}{L^2} = \frac{4.605 \times 10^{-7} \text{ m}^2/\text{s} \times 1170 \text{ s}}{(0.025 \text{ m})^2} = 0.86 \quad \text{Bi}^{-1} = 0.28. \]

From Fig. D.1, \( \theta_0^* = 0.33 \). Hence,

\[ T_0 = T_\infty + 0.33(T_1 - T_\infty) = 600^\circ \text{C} + 0.33\left(-575^\circ \text{C}\right) = 410^\circ \text{C} = T_{\text{min}}. \]

From Fig. D.2, \( \theta/\theta_0 \approx 0.33 \) at \( x = L \), for which

\[ T_{x=L} = T_\infty + 0.33(T_0 - T_\infty) = 600^\circ \text{C} + 0.33\left(-190^\circ \text{C}\right) = 537^\circ \text{C} = T_{\text{max}}. \]

**COMMENTS:** Comparing masonry (m) with aluminum (Al), see Problem 5.10, \((\rho c)_m > (\rho c)_A\) and \(k_{Al} > k_m\). Hence, the aluminum can store more energy and can be charged (or discharged) more quickly.
From printed solution \( \text{Bi} = 3.57 \)

Approx solution: (plane wall)

\[ \Theta_o^* = C_1 \exp \left( -\frac{x^2}{\theta_0} \right) \quad \text{(midplane temp)} \quad (5.41) \]

\[ \Theta^* = \Theta_o^* \cos \left( \frac{x}{\theta_0} \right) \quad (5.40b) \]

\[ \frac{\Theta}{\Theta_o} = 1 - \frac{\sin \frac{x}{\theta_0}}{\frac{x}{\theta_0}} \Theta_o^* \quad (5.46) \]

From Table 5.1, \( \text{Bi} = 3.57 \); plane wall:

\[ \theta_0 = 1.2336 \]

\[ C_1 = 1.2207 \]

\[ \frac{\Theta}{\Theta_o} = 0.75 = 1 - \frac{\sin \left( \frac{1.2336}{1.2336} \right)}{1.2336} \Theta_o^* \]

\[ 0.76498 \Theta_o^* = 0.25 \]

\[ \Theta_o^* = 0.3268 \]

Plug into 5.41:

\[ 0.3268 = 1.2207 \exp \left( -\left( \frac{1.2336}{1.2336} \right)^2 \Theta_o \right) \]

\[ 1.3178 = 1.5218 \Theta_o \]

\[ \Theta_o = 0.8660 \]

\( \text{(Fourier } \# \text{ is } > 0.2, \text{ so our use of the single term approximation is justified)} \)
\[ F_0 = \frac{\alpha t}{L^2} \]

\[ \alpha = 4.605 \times 10^{-7} \ \text{m}^2 / \text{s} \quad \text{(from printed solution)} \]

\[ 0.8660 = \frac{4.605 \times 10^{-7} \ \text{m}^2}{(0.025 \ \text{m})^2} \cdot t \]

\[ t = 1175 \ \text{s} \]

Minimum temp will occur at \( x = 0 \)

\[ \theta_0^* = \frac{T - T_\infty}{T_i - T_\infty} \]

\[ T = T_\infty + \theta_0^* (T_i - T_\infty) \]

\[ = 600 + 0.3268 (25 - 600) \]

\[ T_{\text{min}} = 412.1 \ \text{°C} \]

\( T_{\text{max}} \) at \( x = L \ (x^* = 1) \)

\[ \theta^* = 0.3268 \cos(1.2336(1)) \]

\[ = 0.1081 \]

\[ T_{\text{min}} = T_\infty + \theta^* (T_i - T_\infty) \]

\[ = 600 + 0.1081 (25 - 600) \]

\[ T_{\text{min}} = 537.8 \ \text{°C} \]
PROBLEM 5.34

KNOWN: Thickness, properties and initial temperature of steel slab. Convection conditions.

FIND: Heating time required to achieve a minimum temperature of 550°C in the slab.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible radiation effects, (3) Constant properties.

ANALYSIS: With a Biot number of \( \frac{hL}{k} = \frac{(250 \text{ W/m}^2 \cdot \text{K} \times 0.05 \text{ m})}{48 \text{ W/m} \cdot \text{K}} = 0.260 \), a lumped capacitance analysis should not be performed. At any time during heating, the lowest temperature in the slab is at the midplane, and from the one-term approximation to the transient thermal response of a plane wall, Eq. (5.41), we obtain

\[
\theta^* = \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{(550 - 800) \degree C}{(200 - 800) \degree C} = 0.417 = C_1 \exp\left(-\frac{\zeta^2}{\xi_1} \text{Fo}\right)
\]

With \( \zeta_1 = 0.488 \text{ rad} \) and \( C_1 = 1.0396 \) from Table 5.1 and \( \alpha = k / \rho c = 1.115 \times 10^{-5} \text{ m}^2 / \text{s} \),

\[
-\frac{\zeta^2}{\xi_1} \left(\alpha t / L^2 \right) = \ln(0.401) = -0.914
\]

\[
t = \frac{0.914 L^2}{\zeta_1^2 \alpha} = \frac{0.841(0.05 \text{ m})^2}{(0.488)^2 \times 1.115 \times 10^{-5} \text{ m}^2 / \text{s}} = 861 \text{s}
\]

COMMENTS: The surface temperature at \( t = 861 \text{s} \) may be obtained from Eq. (5.40b), where

\[
\theta^* = \theta^*_0 \cos\left(\zeta_1 x^* \right) = 0.417 \cos(0.488 \text{ rad}) = 0.368.
\]

Hence, \( T(L,792s) = T_s = T_\infty + 0.368(T_i - T_\infty) \)

= \( 800 \degree C - 221 \degree C = 579 \degree C \). Assuming a surface emissivity of \( \varepsilon = 1 \) and surroundings that are at \( T_{\text{sur}} = T_\infty = 800 \degree C \), the radiation heat transfer coefficient corresponding to this surface temperature is

\[
h_r = \varepsilon \sigma \left( T_s + T_{\text{sur}} \right) \left( \frac{T_s^2}{T_{\text{sur}}^2} \right) = 205 \text{ W} / \text{m}^2 \cdot \text{K}.
\]

Since this value is comparable to the convection coefficient, radiation is not negligible and the desired heating will occur well before \( t = 861 \text{s} \).
**Addendum to Problem 5.34**

This problem could also have been solved using the charts in the back of I&D.

Using chart D.1 with,

\[ \theta_o^* = \frac{(T_o - T_{in})}{(T_i - T_{in})} = \frac{(550 - 800)}{(200-800)} = .417 \]

\[ \text{Bi} = \frac{hL}{k} = \frac{[(250 \text{ W/m}^2\text{K})(.05 \text{ m})]}{48 \text{ W/mK}} = .26 \text{ so } \text{Bi}^{-1} = 3.85 \]

From chart D.1, get \( \text{Fo} = 3.8 \), so \( t = 852 \)

Also, regardless of whether you use the charts in I&D or the tables in I&D, both those sources of information **assume \( \text{Fo} > .2 \) so that the one term approximate solution remains valid. If you can not calculate \( \text{Fo} \) in the beginning of the problem, you must calculate it at the end to justify your use of the 1 term approximation.**

\[ \text{Fo} = \frac{(\alpha \cdot t)}{L^2} = 3.84 > .2 \text{ so one term approximation is ok.} \]
PROBLEM 5.50

KNOWN: Long pyroceram rod, initially at a uniform temperature of 900 K, and clad with a thin metallic tube giving rise to a thermal contact resistance, is suddenly cooled by convection.

FIND: (a) Time required for rod centerline to reach 600 K, (b) Effect of convection coefficient on cooling rate.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Thermal resistance and capacitance of metal tube are negligible, (3) Constant properties, (4) \( \text{Fo} \geq 0.2 \).

PROPERTIES: Table A-2, Pyroceram (\( \bar{T} = (600 + 900) \text{K}/2 = 750 \text{K} \)): \( \rho = 2600 \text{ kg/m}^3, c = 1100 \text{ J/kg} \cdot \text{K}, k = 3.13 \text{ W/m} \cdot \text{K} \).

ANALYSIS: (a) The thermal contact and convection resistances can be combined to give an overall heat transfer coefficient. Note that \( R_{t,c} \) [m·K/W] is expressed per unit length for the outer surface. Hence, for \( h = 100 \text{ W/m}^2 \cdot \text{K} \),

\[
U = \frac{1}{l/h + R_{t,c} (\pi D)} = \frac{1}{l/100 \text{ W/m}^2 \cdot \text{K} + 0.12 \text{ m} \cdot \text{K}/\text{W} (\pi \times 0.020 \text{ m})} = 57.0 \text{ W/m}^2 \cdot \text{K}.
\]

Using the approximate series solution, Eq. 5.50c, the Fourier number can be expressed as

\[
\text{Fo} = -\left(\frac{1}{\zeta^2}\right) \ln \left(\frac{\theta^*_o}{C_1}\right).
\]

From Table 5.1, find \( \zeta_i = 0.5884 \text{ rad} \) and \( C_1 = 1.0441 \) for

\[
\text{Bi} = \frac{U r_o}{k} = \frac{57.0 \text{ W/m}^2 \cdot \text{K} (0.020 \text{ m}/2)}{3.13 \text{ W/m} \cdot \text{K}} = 1.82.
\]

The dimensionless temperature is

\[
\theta^*_o (0, \text{Fo}) = \frac{T(0,t) - T_{\infty}}{T_i - T_{\infty}} = \frac{(600 - 300) \text{ K}}{(900 - 300) \text{ K}} = 0.5.
\]

Substituting numerical values to find \( \text{Fo} \) and then the time \( t \),

\[
\text{Fo} = -\frac{1}{(0.5884)^2} \ln \left(\frac{0.5}{1.0441}\right) = 2.127
\]

\[
t = \frac{\text{Fo} \cdot \rho \cdot c}{k} = \frac{t_o^2}{\alpha} = \frac{t_o^2 \rho c}{k}
\]

\[
t = 2.127 (0.020 \text{ m}/2)^2 \frac{2600 \text{ kg}/\text{m}^3 \times 1100 \text{ J/kg} \cdot \text{K}}{3.13 \text{ W/m} \cdot \text{K}} = 194 \text{ s}.
\]

(b) The following temperature histories were generated using the IHT Transient conduction Model for a Cylinder.

Continued...
While enhanced cooling is achieved by increasing $h$ from 100 to 500 W/m$^2$·K, there is little benefit associated with increasing $h$ from 500 to 1000 W/m$^2$·K. The reason is that for $h$ much above 500 W/m$^2$·K, the contact resistance becomes the dominant contribution to the total resistance between the fluid and the rod, rendering the effect of further reductions in the convection resistance negligible. Note that, for $h$ = 100, 500 and 1000 W/m$^2$·K, the corresponding values of $U$ are 57.0, 104.8 and 117.1 W/m$^2$·K, respectively.

**COMMENTS:** For Part (a), note that, since $Fo = 2.127 > 0.2$, Assumption (4) is satisfied.
**Addendum to Problem 5.50**

Note that the \( Bi = \frac{Ur_o}{k} = 0.182 \) which is not much less than one so the lumped capacitance method cannot be used.

Since you used either the tables in I&D or the charts in I&D to solve this problem, you must justify the assumption that one term can be used to accurately approximate the solution. Since you were looking for \( t \) and couldn't calculate the \( Fo \) in the beginning, you must check to see that \( Fo > 0.2 \) at the end, after you find \( t \).

\[ Fo = 2.127 > 0.2 \] so it was ok to use the one term approximate solution.

You could also have used the charts to do this problem.

Using chart D.4 with

\[ \theta_o^* = \frac{T(0,t) - T_{inf}}{(T_i - T_{inf})} = \frac{(600 - 300)K}{(900-300)K} = 0.5 \]

\[ Bi = \frac{Ur_o}{k} = 0.182 \text{ so } Bi^{-1} = 5.49 \]

Find \( Fo = 2.13 \) from Chart D.4, \( t = 195 \text{ s} \).
PROBLEM 5.63

KNOWN: Sphere quenching in a constant temperature bath.

FIND: (a) Plot T(0,t) and T(r_o,t) as function of time, (b) Time required for surface to reach 415 K, t', (c) Heat flux when T(r_o, t') = 415 K, (d) Energy lost by sphere in cooling to T(r_o, t') = 415 K, (e) Steady-state temperature reached after sphere is insulated at t = t', (f) Effect of h on center and surface temperature histories.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Uniform initial temperature.

ANALYSIS: (a) Calculate Biot number to determine if sphere behaves as spatially isothermal object,

\[ Bi = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{75 \text{ W/m}^2 \cdot \text{K}}{1.7 \text{ W/m} \cdot \text{K}} = 0.22. \]

Hence, temperature gradients exist in the sphere and T(r,t) vs. t appears as shown above.

(b) The Heisler charts may be used to find t' when T(r_o, t') = 415 K. Using Fig. D.8 with r/r_o = 1 and Bi^1 = k/hr_o = 1.7 W/m-K/(75 W/m^2-K \times 0.015 m) = 1.51, \( \theta (1, t')/\theta_o \approx 0.72 \). In order to enter Fig. D.7, we need to determine \( \theta_o / \theta_i \), which is

\[ \frac{\theta_o}{\theta_i} = \frac{\theta (1, t')}{\theta_o} \approx \frac{(415 - 320)\text{K}}{(800 - 320)\text{K}} / 0.72 = 0.275. \]

Hence, for Bi = 1.51, Fo = \( \alpha t'/r_o^2 \approx 0.87 \) and

\[ t' = Fo \frac{r_o^2}{\alpha} = Fo \cdot \frac{f \rho c_p}{k} \cdot r_o \approx 0.87 \frac{400 \text{ kg/m}^3 \times 1600 \text{ J/kg} \cdot \text{K}}{1.7 \text{ W/m} \cdot \text{K}} \times (0.015 \text{ m})^2 = 74 \text{s}. \]

(c) The heat flux at the outer surface at time t' is given by Newton's law of cooling

\[ q^* = h\left[ T(r_o, t') - T_\infty \right] = 75 \text{ W/m}^2 \cdot \text{K} \left[ 415 - 320 \right] \text{K} = 7125 \text{ W/m}^2. \]

The manner in which \( q^* \) is calculated indicates that energy is leaving the sphere.

(d) The energy lost by the sphere during the cooling process from t = 0 to t' can be determined from the Groeber chart, Fig. D.9. With Bi = 1/1.51 = 0.67 and Bi^2Fo = (1/1.51)^2 \times 0.87 = 0.4, the chart yields \( Q/Q_o \approx 0.75 \). The energy loss by the sphere with \( V = (\pi d^3)/6 \) is therefore

\[ Q = 0.75Q_o = 0.85 \rho \left( \pi D^3/6 \right) c_p \left( T_i - T_\infty \right) \]

\[ Q = 0.75 \times 400 \text{ kg/m}^3 \left( \pi \left[ 0.030 \text{ m} \right]^3 / 6 \right) 1600 \text{ J/kg} \cdot \text{K} \left( 800 - 320 \right) \text{K} = 3257 \text{J}. \]

Continued...
PROBLEM 5.63 (Cont.)

(e) If at time \( t' \) the surface of the sphere is perfectly insulated, eventually the temperature of the sphere will be uniform at \( T(\infty) \). Applying conservation of energy to the sphere over a time interval, \( E_{\text{in}} - E_{\text{out}} = \Delta E \equiv E_{\text{final}} - E_{\text{initial}} \). Hence, \(-Q = \rho c V [T(\infty) - T_\infty] - Q_o\), where \( Q_o \equiv \rho c V [T_i - T_\infty] \). Dividing by \( Q_o \) and regrouping, we obtain

\[
T(\infty) = T_\infty + (1 - Q/Q_o) (T_i - T_\infty) = 320 \, \text{K} + (1 - 0.75)(800 - 320) \, \text{K} = 440 \, \text{K}
\]

(f) Using the IHT Transient Conduction Model for a Sphere, the following graphical results were generated.

The quenching process is clearly accelerated by increasing \( h \) from 75 to 200 W/m²·K and is virtually completed by \( t \approx 100 \) s for the larger value of \( h \). Note that, for both values of \( h \), the temperature difference \([T(0,t) - T(r_o,t)]\) decreases with increasing \( t \). Although the surface heat flux for \( h = 200 \) W/m²·K is initially larger than that for \( h = 75 \) W/m²·K, the more rapid decline in \( T(r_o,t) \) causes it to become smaller at \( t = 30 \) s.

COMMENTS: 1. There is considerable uncertainty associated with reading \( Q/Q_o \) from the Groeber chart, Fig. D.9, and it would be better to use the one-term approximation solutions of Section 5.6.2. With \( Bi = 0.662 \), from Table 5.1, find \( \zeta_1 = 1.319 \) rad and \( C_1 = 1.188 \). Using Eq. 5.50, find \( Fo = 0.852 \) and \( t' = 72.2 \) s. Using Eq. 5.52, find \( Q/Q_o = 0.775 \) and \( T(\infty) = 428 \) K.

2. Using the Transient Conduction/Sphere model in IHT based upon multiple-term series solution, the following results were obtained: \( t' = 72.1 \) s; \( Q/Q_o = 0.7745 \), and \( T(\infty) = 428 \) K.
(b) \[ Bi = \frac{\pi T_0}{k} = \frac{(45)(0.015)}{1.7} = 0.662 \]

Approx. Solution (sphere):
\[ \Theta_0^* = C_1 \exp\left(-\frac{r^*}{\Theta_0^*}\right) \quad (5.50c) \]
\[ \Theta^* = \Theta_0^* \left( \frac{1}{\Theta_0^*} \right) \sin(\delta_1 r^*) \quad (5.50b) \]
\[ \Theta_0^* = 1 - \frac{3}{\delta_1^2} \left[ \sin(\delta_1) - \delta_1 \cos(\delta_1) \right] \quad (5.52) \]

From Table 5.1, \( Bi = 0.662 \), sphere:
\[ \delta_1 = 1.319 \]
\[ C_1 = 1.188 \]

Find time when \( T_{\text{surf}} = 415 \text{ K} \)

at surface \( r^* = 1 \)

\[ \Theta_{\text{surf}}^* = \frac{T_{\text{surf}} - T_w}{T_i - T_w} = \frac{415 - 320}{800 - 320} = 0.198 \]

Plug into 5.50b:
\[ 0.198 = \Theta_0^* \left( \frac{1}{1.319} \right) \sin(1.319(1)) \]

\[ \Theta_0^* = 0.2697 \]
Problem 5.63 (cont)

Plug into 5.50c:

\[ 0.2697 = 1.188 \exp (- (1.319)^4 F_0) \]

\[ F_0 = 0.8523 \]

\[ F_0 = \frac{\alpha t}{L^2} \]

\[ 0.8523 = \frac{1.7}{(400)(1600)} \frac{t}{(0.015)^2} \]

\[ t = 72.2 \mu s \]

(e):

\[ \frac{Q}{Q_0} = 1 - 3(0.2697) \left[ \sin (1.319) - 1.319 \cos (1.319) \right] \]

\[ \frac{Q}{Q_0} = 0.774 \]

\[ T(\infty) = T_\infty + (1 - \frac{Q}{Q_0})(T_i - T_\infty) \]

\[ = 320 + (1 - 0.774)(800 - 320) \]

\[ T(\infty) = 428 K \]
PROBLEM 5.74

KNOWN: Tile-iron, 254 mm to a side, at 150°C is suddenly brought into contact with tile over a subflooring material initially at \( T_i = 25 \)°C with prescribed thermophysical properties. Tile adhesive softens in 2 minutes at 50°C, but deteriorates above 120°C.

FIND: (a) Time required to lift a tile after being heated by the tile-iron and whether adhesive temperature exceeds 120°C, (2) How much energy has been removed from the tile-iron during the time it has taken to lift the tile.

SCHEMATIC:

ASSUMPTIONS: (1) Tile and subflooring have same thermophysical properties, (2) Thickness of adhesive is negligible compared to that of tile, (3) Tile-subflooring behaves as semi-infinite solid experiencing one-dimensional transient conduction.

PROPERTIES: Tile-subflooring (given): \( k = 0.15 \) W/m·K, \( \rho c_p = 1.5 \times 10^6 \) J/m\(^3\)·K, \( \alpha = \frac{k}{\rho c_p} = 1.00 \times 10^{-7} \) m\(^2\)/s.

ANALYSIS: (a) The tile-subflooring can be approximated as a semi-infinite solid, initially at a uniform temperature \( T_i = 25 \)°C, experiencing a sudden change in surface temperature \( T_s = T(0,t) = 150 \)°C. This corresponds to Case 1, Figure 5.7. The time required to heat the adhesive \( (x_o = 4 \) mm\) to 50°C follows from Eq. 5.57

\[
\frac{T(x_o, t_o) - T_s}{T_i - T_s} = \text{erf} \left( \frac{x_o}{2(\alpha t_o)^{1/2}} \right)
\]

\[
\frac{50 - 150}{25 - 150} = \text{erf} \left( \frac{0.004 \text{m}}{2 \left( 1.00 \times 10^{-7} \text{m}^2/\text{s} \times t_o \right)^{1/2}} \right)
\]

\[
0.80 = \text{erf} \left( 6.325 t_o^{-1/2} \right)
\]

\[ t_o = 48.7 \text{s} = 0.81 \text{ min} \]

using error function values from Table B.2. Since the softening time, \( \Delta t_s \), for the adhesive is 2 minutes, the time to lift the tile is

\[ t_\ell = t_o + \Delta t_s = (0.81 + 2.0) \text{ min} = 2.81 \text{ min} . \]

To determine whether the adhesive temperature has exceeded 120°C, calculate its temperature at \( t_\ell = 2.81 \) min; that is, find \( T(x_o, t_\ell) \)

\[
\frac{T(x_o, t_\ell) - 150}{25 - 150} = \text{erf} \left( \frac{0.004 \text{m}}{2 \left( 1.0 \times 10^{-7} \text{m}^2/\text{s} \times 2.81 \times 60 \text{s} \right)^{1/2}} \right)
\]

Continued...
PROBLEM 5.74 (Cont.)

\[ T(x_0, t) - 150 = -125 \text{erf}(0.4880) = 125 \times 0.5098 \]

\[ T(x_0, t) = 86^\circ \text{C} \] <

Since \( T(x_0, t) < 120^\circ \text{C} \), the adhesive will not deteriorate.

(b) The energy required to heat a tile to the lift-off condition is

\[ Q = \int_0^{t_f} q''_x (0, t) \cdot A_s \, dt. \]

Using Eq. 5.58 for the surface heat flux \( q''_x (t) = q''_x (0, t) \), find

\[ Q = \int_0^{t_f} \frac{k(T_s - T_i)}{(\pi \alpha)^{1/2}} A_s \cdot \frac{dt}{t^{1/2}} = \frac{2k(T_s - T_i)}{(\pi \alpha)^{1/2}} A_s t_f^{1/2} \]

\[ Q = \frac{2 \times 0.15 \text{ W/m} \cdot \text{K} (150 - 25)^\circ \text{C}}{(\pi \times 1.00 \times 10^{-7} \text{ m}^2/\text{s})^{1/2}} \times (0.254 \text{ m})^2 \times (2.81 \times 60 \text{s})^{1/2} = 56 \text{ kJ} \] <

COMMENTS: (1) Increasing the tile-iron temperature would decrease the time required to soften the adhesive, but the risk of burning the adhesive increases.

(2) From the energy calculation of part (b) we can estimate the size of an electrical heater, if operating continuously during the 2.81 min period, to maintain the tile-iron at a near constant temperature. The power required is

\[ P = \frac{Q}{t_f} = \frac{56 \text{ kJ}}{2.81 \times 60 \text{s}} = 330 \text{ W}. \]

Of course a much larger electrical heater would be required to initially heat the tile-iron up to the operating temperature in a reasonable period of time.
PROBLEM 5.80

KNOWN: Very thick plate, initially at a uniform temperature, $T_i$, is suddenly exposed to a surface convection cooling process ($T_\infty$, $h$).

FIND: (a) Temperatures at the surface and 45 mm depth after 3 minutes, (b) Effect of thermal diffusivity and conductivity on temperature histories at $x = 0, 0.045$ m.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Plate approximates semi-infinite medium, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: (a) The temperature distribution for a semi-infinite solid with surface convection is given by Eq. 5.60.

$$ T(x,t) - T_i = \text{erfc} \left( \frac{x}{2(\alpha t)^{1/2}} \right) \left[ \exp \left( \frac{hx}{k} + \frac{h^2\alpha t}{k^2} \right) \right] \text{erfc} \left( \frac{x}{2(\alpha t)^{1/2}} + \frac{h(\alpha t)^{1/2}}{k} \right). $$

At the surface, $x = 0$, and for $t = 3$ min = 180s,

$$ T(0,180s) - 325^\circ C = \text{erfc}(0) - \left[ \exp \left( \frac{100^2 W^2/m^4 K^2 \times 5.6 \times 10^{-6} m^2/s \times 180s}{20 W/m \cdot K} \right) \right] $$

$$ \times \left[ \text{erfc} \left( 0 + \frac{100 W/m \cdot K \times 5.6 \times 10^{-6} m^2/s \times 180s^{1/2}}{20 W/m \cdot K} \right) \right] $$

$$ = 1 - \left[ \exp(0.02520) \right] \times \left[ \text{erfc}(0.159) \right] = 1 - 1.02552 \times (1 - 0.178) $$

$$ T(0,180s) = 325^\circ C - (15 - 325)^\circ C \cdot (1 - 1.0255 \times 0.822) $$

At the depth $x = 0.045$ m, with $t = 180$s,

$$ T(0.045m,180s) - 325^\circ C = \text{erfc} \left( \frac{0.045 m}{2 \left( 5.6 \times 10^{-6} m^2/s \times 180s \right)^{1/2}} \right) \left[ \exp \left( \frac{100 W/m \cdot K \times 0.045 m}{20 W/m \cdot K} + 0.02520 \right) \right] $$

$$ \times \left[ \text{erfc} \left( \frac{0.045 m}{2 \left( 5.6 \times 10^{-6} m^2/s \times 180s \right)^{1/2}} \right) + 0.159 \right] $$

$$ = \text{erfc} \left( 0.7087 \right) + \left[ \exp(0.225 + 0.0252) \right] \times \left[ \text{erfc} \left( 0.7087 + 0.159 \right) \right]. $$

$$ T(0.045m,180s) = 325^\circ C + (15 - 325)^\circ C \left[ (1 - 0.684) - 1.284(1 - 0.780) \right] = 315^\circ C \quad \text{Continued...} \)
(b) The IHT Transient Conduction Model for a Semi-Infinite Solid was used to generate temperature histories, and for the two locations the effects of varying $\alpha$ and $k$ are as follows.

For fixed $k$, increasing alpha corresponds to a reduction in the thermal capacitance per unit volume ($\rho c_p$) of the material and hence to a more pronounced reduction in temperature at both surface and interior locations. Similarly, for fixed $\alpha$, decreasing $k$ corresponds to a reduction in $\rho c_p$ and hence to a more pronounced decay in temperature.

**COMMENTS:** In part (a) recognize that Fig. 5.8 could also be used to determine the required temperatures.
Addendum to Problem 5.80

This problem could also be solved using Figure 5.8

at x = 0
Using Figure 5.8 with,

\[ \frac{x}{2(\alpha t)^{1/2}} = 0 \]
\[ \frac{h(\alpha t)^{1/2}}{k} = .1588 \]

From Figure 5.8, \( \frac{(T(0,t) - T_i)}{(T_{\inf} - T_i)} = .165 \) so \( T(0,t) = 274 \) C

at x = 45 mm
Using Figure 5.8 with,

\[ \frac{x}{2(\alpha t)^{1/2}} = .7087 \]
\[ \frac{h(\alpha t)^{1/2}}{k} = .1588 \]

From Figure 5.8, \( \frac{(T(.045m,t) - T_i)}{(T_{\inf} - T_i)} = .03 \) so \( T(.045m,t) = 316 \) C
**PROBLEM 5.88**

**KNOWN:** Initial temperature of fire clay brick which is cooled by convection.

**FIND:** Center and corner temperatures after 50 minutes of cooling.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Homogeneous medium with constant properties, (2) Negligible radiation effects.

**PROPERTIES:** Table A-3, Fire clay brick (900K): \( \rho = 2050 \text{ kg/m}^3 \), \( k = 1.0 \text{ W/m-K} \), \( c_p = 960 \text{ J/kg-K} \), \( \alpha = 0.51 \times 10^{-6} \text{ m}^2/\text{s} \).

**ANALYSIS:** From Fig. 5.11(h), the center temperature is given by

\[
T(0,0,0,t) - T_\infty = P_1(0,t) \times P_2(0,t) \times P_3(0,t)
\]

where \( P_1 \), \( P_2 \) and \( P_3 \) must be obtained from Fig. D.1.

- \( L_1 = 0.03 \text{ m} \): \( \text{Bi}_1 = \frac{h L_1}{k} = 1.50 \) \( \text{Fo}_1 = \frac{\alpha t}{L_1^2} = 1.70 \)
- \( L_2 = 0.045 \text{ m} \): \( \text{Bi}_2 = \frac{h L_2}{k} = 2.25 \) \( \text{Fo}_2 = \frac{\alpha t}{L_2^2} = 0.756 \)
- \( L_3 = 0.10 \text{ m} \): \( \text{Bi}_3 = \frac{h L_3}{k} = 5.0 \) \( \text{Fo}_3 = \frac{\alpha t}{L_3^2} = 0.153 \)

Hence from Fig. D.1,

\( P_1(0,t) \approx 0.22 \quad P_2(0,t) \approx 0.50 \quad P_3(0,t) \approx 0.85 \).

Hence,

\[
\frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} \approx 0.22 \times 0.50 \times 0.85 = 0.094
\]

and the center temperature is

\[
T(0,0,0,t) \approx 0.094(1600 - 313) \text{ K} + 313 \text{ K} = 434 \text{ K}.
\]

Continued .....
PROBLEM 5.88 (Cont.)

The corner temperature is given by

\[ \frac{T(L_1, L_2, L_3, t) - T_\infty}{T_1 - T_\infty} = P(L_1, t) \times P(L_2, t) \times P(L_3, t) \]

where

\[ P(L_1, t) = \frac{\theta(L_1, t)}{\theta_0} \cdot P_1(0, t), \text{ etc.} \]

and similar forms can be written for \( L_2 \) and \( L_3 \). From Fig. D.2,

\[ \frac{\theta(L_1, t)}{\theta_0} \approx 0.55 \quad \frac{\theta(L_2, t)}{\theta_0} \approx 0.43 \quad \frac{\theta(L_3, t)}{\theta_0} \approx 0.25. \]

Hence,

\[ P(L_1, t) \approx 0.55 \times 0.22 = 0.12 \]
\[ P(L_2, t) \approx 0.43 \times 0.50 = 0.22 \]
\[ P(L_3, t) \approx 0.85 \times 0.25 = 0.21 \]

and

\[ \frac{T(L_1, L_2, L_3, t) - T_\infty}{T_1 - T_\infty} \approx 0.12 \times 0.22 \times 0.21 = 0.0056 \]

or

\[ T(L_1, L_2, L_3, t) \approx 0.0056(1600 - 313) + 313 \text{K}. \]

The corner temperature is then

\[ T(L_1, L_2, L_3, t) \approx 320 \text{K}. \]

**COMMENTS:**  
(1) The foregoing temperatures are overpredicted by ignoring radiation, which is significant during the early portion of the transient.  
(2) Note that, if the time required to reach a certain temperature were to be determined, an iterative approach would have to be used. The foregoing procedure would be used to compute the temperature for an assumed value of the time, and the calculation would be repeated until the specified temperature were obtained.
Addendum to Problem 5.88

For L3, Fo = .153 < .2 so the one term approximation will not be very accurate. Consequently, the charts are very hard to use since you really shouldn't be using the I&D charts unless Fo < .2. In this case, since Fo is close to .2 the error will probably not be too large, so the first term can still be used to get a very rough approximate answer.

The best solution however, is to use the exact solution (not the one term approximation) and use the charts handed out in class that work for any Fo since these charts are derived from the exact solution.